# MASTER VWL – PRÜFUNG (MIDTERM EXAM)

<table>
<thead>
<tr>
<th>DATUM:</th>
<th>28.05.2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
</tr>
<tr>
<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
</tr>
<tr>
<td>PRÜFUNGS-NR.:</td>
<td></td>
</tr>
<tr>
<td>STUEDIENGANG:</td>
<td></td>
</tr>
<tr>
<td>NAME, VORNAME:</td>
<td></td>
</tr>
<tr>
<td>UNTERSCHRIFT DES STUDENTEN:</td>
<td></td>
</tr>
</tbody>
</table>

## ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

## ZUGELASSENEN HILFSMITTEL: keine
Exercise 1: Ramsey growth model (20 points)

Consider an infinitely lived consumer-producer agent with intertemporal preferences over consumption as given by

\[ U = \int_{t=0}^{\infty} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \]

where \( c(t) \) denotes consumption at time \( t \in \mathbb{R} \) and \( \sigma, \rho > 0 \) constant preference parameters. The available production technology has the following shape

\[ y(t) = A k(t)^{\alpha}, \]

where \( y(t) \) denotes final output, \( k(t) \) the stock of physical capital, and \( A > 0 \) and \( 0 < \alpha < 1 \) constant technology parameters, respectively. Final output can either be consumed or used to build up the stock of capital. Capital depreciates at constant rate \( \delta > 0 \). Hence, the stock of capital evolves according to

\[ \dot{k}(t) = A k(t)^{\alpha} - \delta k(t) - c(t) \quad \text{with} \quad k(0) = \text{given}. \]

Assignments

1. Determine the Keynes-Ramsey rule of optimal consumption growth and provide a concise economic interpretation.

2. Determine the (interior) steady state \((\bar{k}, \bar{c})\) analytically and discuss the comparative static implications with respect to \( A \).

3. Assume that (i) the economy is in a steady state initially and (ii) \( A \) increases once and for all. Describe the transition from the initial to the new steady state by employing a phase diagram.

Reminder: Dynamic optimization problems can be solved by applying the maximum principle. This requires to set up the (current-value) Hamiltonian function, which may be stated as \( H[x(t), u(t), \mu(t)] := I[x(t), u(t)] + \mu(t) f[x(t), u(t)] \), where \( I[x(t), u(t)] \) denotes the instantaneous objective function, \( f[x(t), u(t)] \) the right-hand side of the dynamic constraint \( \dot{x}(t) = f[x(t), u(t)] \) and \( \mu(t) \) the current-value shadow price. The necessary first-order conditions (apart from boundary conditions) may then be expressed as:

\[ \frac{\partial H(.)}{\partial u(t)} = 0 \quad \forall t \in [0, \ldots, \infty] \quad \text{and} \quad \mu(t) = -\frac{\partial H(.)}{\partial x(t)} + \rho \mu(t) \quad \forall t \in [0, \ldots, \infty]. \]
Exercise 2: R&D-based growth - the Romer (1990) model (20 points)

Consider a dynamic macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[ \dot{A} = \eta L_d A \quad \text{with} \quad A(0) = A_0 \]

Intermediate goods
\[ x(i) = k(i) \quad \text{for all} \quad i \in [0, ..., A] \]

Final output
\[ Y = L^1_y \int_0^A x(i)^a \, d\bar{i} \]

Assignments

1. Consider the production technology \( \dot{A} = \eta L_d A \). Assume that \( \dot{A} \) increases by 1 percent. By how much does, ceteris paribus, R&D output \( \dot{A} \) change in proportional terms?

2. Determine the steady state growth rate of final output \( Y \), assuming that (i) the Keynes-Ramsey rule holds, i.e. \( r = g + \rho \) (logarithmic preferences); (ii) equilibrium profits of the typical intermediate goods producer are \( \pi_i = (1 - \alpha)\alpha L^1_y x^\alpha \); and (iii) there is an R&D subsidy such that profits of the typical R&D firm read \( \pi_{R&D} = p_A \dot{A} - (1 - s_A)w_{R&D} L_A \).

3. The government increases \( s_A \) from \( s_A = 0 \) to some \( 0 < s_A < 1 \). What happens to the steady state growth rate and intertemporal welfare? (A verbal discussion is sufficient.) Provide a concise economic reasoning.

Notation: \( A \): “number” of intermediate good types; \( \dot{x} := dx/dt \); \( \eta > 0, \ 0 < \alpha < 1 \): constant technology parameter; \( L_d \): amount of labor devoted to R&D; \( L_y \): amount of labor devoted to \( Y \)-production; \( x(i) \): number of intermediate goods of type \( i \); \( k(i) \): capital used in \( x(i) \)-production; \( \pi_i \): profit of the typical \( x \)-producer; \( \pi_{R&D} \): profit of the R&D firm; \( g \): steady state growth rate of \( Y, C, K \) and \( A \), where \( C \): consumption, \( K \): stock of capital; \( \sigma, \rho > 0 \): constant preference parameters; \( 0 \leq s_A < 1 \): R&D subsidy parameter
Exercise 3: Miscellaneous (20 points)

(1) Implications of alternative production technologies (10 points)

Consider a static, small open economy. This perfectly competitive economy comprises one sector that produces a final output good, denoted as \( Y \), by employing a set of intermediate goods, denoted as \( x_i \) with \( i \in \{1, \ldots, N\} \). Intermediate goods can be purchased at the going price on the world market for intermediate goods. The production technology reads:

\[
Y = \left( \sum_{i=1}^{N} x_i^\alpha \right)^\gamma \quad \text{with} \quad 0 < \alpha < 1 \quad \text{and} \quad 1 \leq \gamma \leq \frac{1}{\alpha}
\]

(a) Assume \( \gamma = 1 \): Determine the elasticity of substitution between any two \( x_i \) and determine the degree of returns to scale.

(b) Assume \( \gamma = \frac{1}{\alpha} \): Determine the elasticity of substitution between any two \( x_i \) and determine the degree of returns to scale.

(c) Rank the profits earned by the typical firm in the \( Y \)-sector assuming that \( \gamma = \frac{1}{\alpha} \) and \( \gamma = 1 \). Provide a concise economic reasoning.

(2) Savings and goods demand in the OLG model (10 points)

Consider a dynamic model of a perfectly competitive, small open economy. On the household side there are overlapping generations. Time is discrete. Each individual lives for two periods ("youth" and "retirement"). Life-time utility of an individual born at \( t \in \mathbb{N} \) is given by

\[
U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \quad (1)
\]

where \( c_{1,t} \) denotes the amount of goods consumed by a young individual born at the beginning of period \( t \), \( c_{2,t+1} \) is the amount of goods consumed by an old individual born at \( t \), and \( 0 < \beta < 1 \). Each individual supplies one unit of labor when young and divides the resulting labor income between current consumption and savings. In the second period, the individual consumes the savings and the interest payments. The representative household solves the following problem

\[
\max_{c_{1,t}, c_{2,t+1}} U_t \quad \text{s.t.} \quad (1) \quad \text{and} \quad w_t \geq c_{1,t} + c_{2,t+1} \frac{1 + r}{1 + r},
\]

where \( w_t \) denotes the wage rate and \( r \) the constant interest rate.

(a) Determine the savings rate and the amount of first-period consumption of young individuals born at \( t \).

(b) How does the savings rate depend on the interest rate \( r \)? Provide a concise economic reasoning.