<table>
<thead>
<tr>
<th>Datum:</th>
<th>04. August 2015</th>
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<tbody>
<tr>
<td>Modul:</td>
<td>ADVANCED MACROECONOMICS</td>
</tr>
<tr>
<td>Prüfer:</td>
<td>PROF. DR. THOMAS STEGER</td>
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<td>PRÜFUNG-NR.:</td>
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<td>Studiengang:</td>
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<td>Name, Vorname:</td>
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<td>Unterschrift des Studenten:</td>
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<tr>
<td>Erläuterungen (Explanations)</td>
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<tr>
<td>(1) Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)</td>
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<td>(2) Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)</td>
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<td>(3) Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)</td>
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<td>(4) Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)</td>
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<td>ZUGELASSENNE HILFSMITTEL: keine</td>
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<td>Punkte:</td>
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<td>Datum, Unterschrift des Prüfers:</td>
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Exercise 1: Investment Demand of Firms (20 points)

Consider a firm that produces a homogenous final output good \( Y_t \) under perfect competition. The output technology reads

\[
Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}
\]

The firm's planning horizon is infinity. There are investment costs (capital adjustment costs), denoted as \( IC_t \). Installing the amount of \( I_t \) capital goods (gross investment) requires the following amount of final output

\[
IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right].
\]

It is assumed that the firm maximizes the present value of its cash flow (or entrepreneurial residual income) subject to a capital accumulation equation, \( K_{t+1} = I_t + (1 - \delta) K_t \), i.e. the firm solves the following dynamic problem

\[
\max_{\{I_t, K_t\}} \sum_{t=0}^{\infty} \frac{1}{1+r} \left( Y_t - w_t L_t - IC_t \right)
\]

s.t. \( K_{t+1} = I_t + (1 - \delta) K_t \)

\[
IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right]
\]

\( K_0 = \text{given} \).

(a) Consider the production technology \( Y_t = A(K_t)^\alpha (L_t)^{1-\alpha} \). Assume that \( K_t \) increases by 1 percent. By how much does, ceteris paribus, \( Y_t \) change in proportional terms?

(b) Determine the firm's investment demand (i.e. the demand for final output devoted to capital investment).

Remark: Investment demand will be a function \( I_t = I_t(q_t, K_t) \), where \( q_t \) denotes the shadow price of installed capital goods. (You are not requested to determine the difference equation which describes the dynamics of \( q_t \).)

(c) Provide a sound economic interpretation of your result.

Notation: \( Y_t \): final output good at time \( t \in \mathbb{N} \); \( A > 0 \): constant technology parameter; \( K_t \): stock of physical capital at \( t \); \( L_t \): labor employed at time \( t \); \( IC_t \): investment costs; \( 0 < \alpha < 1, \theta \geq 0, \eta \geq 1 \): constant technology parameter; \( w_t \): denotes the wage rate, \( r \): fixed interest rate, and \( \delta \geq 0 \): capital depreciation rate
Exercise 2: New Keynesian Theory: the household’s intertemporal problem (20 points)

Consider an infinitely lived representative household who is assumed to maximize its lifetime utility. Instantaneous utility depends on the individual’s (overall) consumption \( C_t \), real money holdings \( M_t / P_t \), and working time \( N_t \). The household has access to a perfect capital market. Buying and holding bonds yields a nominal interest rate \( \gamma_t \). For simplicity, we abstract from uncertainty. Time is discrete. The household’s intertemporal problem may be stated as

\[
\max_{\{C_t,M_t,N_t\}} \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\sigma} \left( \frac{M_t}{P_t} \right)^{1-b} - \frac{N_t^{1+\eta}}{1+\eta} \right]
\]

s.t. \( C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1+\gamma_{t-1}) \frac{B_{t-1}}{P_t} \) \hspace{1cm} (P1)

where \( \beta, \sigma, \phi, b, \chi, \eta \) denote constant preference parameters, \( P_t \) the price level, \( M_t \) nominal money balances, \( B_t \) bond holdings, \( W_t \) the nominal wage rate, and \( \gamma_t \) the nominal interest rate earned by holding bonds, respectively.

The individual’s (overall) consumption \( C_t \) is described by the following constant elasticity of substitution (CES) index (or subutility function)

\[
C_t = \left[ \int_0^1 c_j^{\theta-1} \beta^j \right]^{\frac{1}{\theta-1}}, \text{ where } c_j \text{ denotes the level of differentiated consumption good } j \text{ at time } t.
\]

a. Consider the intertemporal optimization problem, i.e. problem (P1). Set up the first-order conditions for optimal \( C_t, M_t, \) and \( N_t \).

b. Provide a concise economic interpretation of every first-order condition.

c. Set up the intratemporal consumption problem and explain this optimization problem concisely.
Exercise 3: Miscellaneous (20 points)

(1) The Financial Accelerator model (10 points)

Consider an economy populated by mass one of firms indexed by $j \in [0, ..., 1]$. Given the size of the investment project $Q_j K^j_{t+1}$, the (gross) return to capital can be expressed as $\omega_j^j R^j_{t+1} Q_j K^j_{t+1}$. Firms can finance their investment projects either internally or by borrowing from financial intermediaries. The financial intermediary cannot directly observe the success of the investment project. The contract between firm $j$ and the financial intermediary specifies $\bar{\omega}^j$ and $Z^j_{t+1}$ such that

- for $\omega^j \geq \bar{\omega}^j$ the firm repays $Z^j_{t+1} B^j_{t+1}$ to the financial intermediary and

- for $\omega^j < \bar{\omega}^j$ the firm cannot repay the contractual return and declares default. In this case, the financial intermediary pays monitoring costs of $\mu \omega^j R^k_{t+1} Q_j K^j_{t+1}$ to observe the return to the investment project and gets to keep what it finds.

a. Write the expected gross return of a loan to firm $j$ down, taking $R^k_{t+1}$ as given, that the financial intermediary can reap. Explain the different components in detail.

b. Write the expected profit of firm $j$ down, taking $R^k_{t+1}$ as given. Explain the different components in detail.

Notation

<table>
<thead>
<tr>
<th>$Q_j$</th>
<th>price of capital in purchased in period $t$</th>
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<tbody>
<tr>
<td>$K^j_{t+1}$</td>
<td>quantity of capital goods employed in period $t+1$</td>
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<tr>
<td>$\omega^j$</td>
<td>idiosyncratic shocks with continuous p.d.f. $f(\omega^j)$ and associated c.d.f. $F(\omega^j)$ and with</td>
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<tr>
<td>$R^k_{t+1}$</td>
<td>aggregate shock to capital productivity</td>
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<td>$B^j_{t+1}$</td>
<td>firm’s $j$ borrowing</td>
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<tr>
<td>$Z^j_{t+1}$</td>
<td>gross rate borrowing rate</td>
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Exercise 3: Miscellaneous (20 points) - continued

(2) Bubbles in the OLG model – the Tirole (1985) model (10 points)

Consider a dynamic model of a closed economy with asset price bubbles. On the household side there are overlapping generations. Time is discrete. There is no population growth and no technological growth. Each individual lives for two periods. Optimal savings are given by

\[ s_t = \frac{\beta}{1 + \beta} w_t, \]

where \( \beta \) is the discount factor. Savings can be allocated either to productive capital, \( k_t \), or to an asset with zero dividends, \( b_t \). The variable \( b_t \) denotes the aggregate value of the zero-dividend assets (The quantity of the zero-dividend assets is normalized to one such that \( b_t \) equals also the price of the zero-dividend asset). Perfect competition in the firm sector results in wages \( w_t = (1 - \alpha)k_t^{\alpha} \) and interest rates \( r_t = \alpha k_t^{\alpha-1} - \delta \). The central two equations of the model are

\[ \frac{b_{t+1}}{b_t} = 1 + r_{t+1} \quad (1) \]

\[ k_{t+1} = s_t - b_t \quad (2) \]

a. Provide a concise economic interpretation of both equations.

b. Determine the two non-trivial steady states of the economy. (In the trivial steady state both \( k \) and \( b \) are zero.)

c. The necessary condition for feasible bubbles is: \( 0 \leq b_t < s_t \). Which further necessary condition has to hold such that feasible bubbles can exist in a steady state of this economy?