### ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

### ZUGELASSENE HILFSMITTEL: keine
Exercise 1: Productive government expenditures - Barro (1990) model (20 points)

Consider a perfectly competitive economy. There is a continuum of length one of identical firms, indexed by \( i \in [0,1] \). Output technology of firm \( i \) is given by

\[
Y_i = G^\beta K_i^\alpha L_i^{1-\alpha},
\]

where \( Y_i \) denotes final output produced by firm \( i \), \( G \) a public input (productive government expenditures), \( K_i \) the stock of physical capital employed by firm \( i \), \( L_i \) the amount of labor employed by firm \( i \), and \( 0 < \alpha, \beta < 1 \).

It is assumed that the government runs a balanced budget, i.e.

\[
G = \tau Y,
\]

where \( 0 < \tau < 1 \) denotes a linear tax rate and \( Y = \int_0^1 Y_i \, di \) denotes aggregate income, respectively.

(a) Set up, by eliminating \( G \), the reduced-form output technology \( Y = F(K,L) \) and determine the degree of returns to scale in the private input factors \( K \) and \( L \). (5 points)

(b) Determine the competitive wage rate and the competitive interest rate in this economy. (5 points)

(c) Assuming that the Keynes-Ramsey rule holds, how does the growth rate of consumption change in response to \( \tau \)? Provide a concise and model-based economic reasoning. (10 points)
Exercise 2: AK growth model - growth rate and saving rate (20 points)

Consider an infinitely living consumer-producer agent, often described by the metaphor "Robinson Crusoe", who is endowed with an initial amount of capital $K_0$. Capital, denoted as $K$, can be accumulated by abstaining from consumption and it depreciates at a constant rate $\delta > 0$. There is no technical change. The production function for final output is

$$Y(t) = AK(t),$$

where $Y(t)$ denotes output at time $t \in \mathbb{R}$ and $A > 0$ a constant technology parameter, respectively.

The agent is assumed to maximize intertemporal welfare as given by

$$U_0 = \int_0^\infty \ln(C(t)) e^{-\rho t} dt,$$

where $C(t)$ denotes the level of consumption at time $t$ and $\rho > 0$ the discount rate.

(a) Derive the steady state growth rate of output $Y$, capital $K$, and consumption $C$. (8 points)

Remark: A convenient possibility to solve dynamic optimization problems consists in the application of the maximum principle. This requires to set up the (current-value) Hamiltonian function, which may be stated as $H = u(C(t)) + \lambda(t)[Y(t) - \delta K(t) - C(t)]$, where $u(C(t))$ denotes the instantaneous utility function and $\lambda(t)$ a co-state variable (current-value shadow price). The necessary first-order conditions may then be expressed as follows: $H_{C(t)} = 0$ and $\dot{\lambda}(t) = -H_{K(t)} + \rho \lambda(t)$ for all $t$.

(b) Provide a concise economic interpretation of the Keynes-Ramsey rule. (4 points)

(c) Determine the saving rate defined as follows: $s = \frac{Y - C}{Y}$. (8 points)
Exercise 3: Miscellaneous (20 points)

(1) Goods demand and supply price in the Dixit-Stiglitz model (10 points)

Consider a static economy with imperfect product market competition. There are two sectors. The final output sector produces a final output good, denoted as $X$, by employing a set of intermediate goods, denoted as $x_i$ with $i \in \{1,...,N\}$. The intermediate goods sector employs labor $L_i$ to produce intermediate goods $x_i$. Production technologies are as follows:

- **Final output:**
  \[
  X = \left( \sum_{i=1}^{N} x_i^\alpha \right)^{1/\alpha} \quad \text{with} \quad 0 < \alpha < 1
  \]

- **Intermediate goods:**
  \[
  x_i = L_i \quad \text{for all} \quad i \in \{1,...,N\}
  \]

(a) Derive the demand for intermediate goods $x_i$ for all $i \in \{1,...,N\}$, as articulated by the typical $X$-producer.

(b) Determine the optimal supply price the typical $x_i$-producer sets.

(2) Savings and goods demand in the OLG model (10 points)

Consider a dynamic model of a perfectly competitive, small open economy. On the household side there are overlapping generations. Time is discrete. Each individual lives for two periods ("youth" and "retirement") only. Life-time utility of an individual born at $t \in \mathbb{N}$ is given by

\[
U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}),
\]

where $c_{1,t}$ denotes the amount of goods consumed by a young individual born at the beginning of period $t$ and, similarly, $c_{2,t+1}$ is the amount of goods consumed by an old individual born at $t$, and $0 < \beta < 1$. Each individual supplies one unit of labor when young and divides the resulting labor income between current consumption and savings. In the second period, the individual consumes the savings and the interest payments. Hence, the representative household solves the following problem

\[
\max_{c_{1,t}, c_{2,t+1}} U_t \quad \text{s.t.} \quad w_t \geq c_{1,t} + \frac{c_{2,t+1}}{1+r},
\]

where $w_t$ denotes the wage rate and $r$ the constant interest rate.

(a) Derive the savings rate and the amount of first-period consumption of young individuals born at $t$.

(b) How does the savings rate depend on the interest rate $r$? Provide a concise economic interpretation.