**ERLÄUTERUNGEN (EXPLANATION)**

1. Die Klausur besteht aus sechs Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle sechs Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of six exercises. Of these six exercises four exercises have to be edited. If you have edited all six exercises, the first four exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt **120 Minuten** zur Verfügung. (To process the exam you have **120 minutes** available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSENE HILFSMITTEL:** keine

**Punkte:**

**DATUM, UNTERSCHRIFT DES PRÜFERS:**
Exercise 1: Optimal capital subsidization (20 points)

Consider a perfectly competitive economy in the form of a deterministic, time-continuous Ramsey model. There is a continuum of length one of identical, infinitely lived households. Each household is endowed with one unit of time per period, which is supplied inelastically to the labor market. The representative household is assumed to solve the following problem

\[
\max_{\{c(t)\}} \int_0^\infty \ln[c(t)]e^{-\rho t}dt \\
\text{s.t. } a(t) = r(t)a(t) + w(t) - c(t) - T(t), \\
a(0) = a_0
\]

where \(a(t)\) denotes financial wealth, \(r(t)\) the competitive interest rate, \(w(t)\) the competitive wage rate, \(c(t)\) denotes consumption, and \(T(t)\) a lump sum tax.

There is a continuum of length one of identical final output firms. Each firm has access to the following technology

\[
Y(i,t) = K(i,t) ^ {\alpha} L(i,t)^{1-\alpha},
\]

where \(Y(i,t)\) denotes final output of firm \(i\) at time \(t\), \(K(i,t)\) capital employed by firm \(i\), \(L(i,t)\) the amount of labor employed by firm \(i\), \(\bar{K}(t) = \int_{i=0}^1 K(i,t)di\) the average level (across firms) of capital, and \(0 < \alpha < 1, \beta \geq 0\) constant technology parameters. Profit of the individual firm \(\pi(i,t)\) may be expressed as:

\[
\pi(i,t) = p_i Y(i,t) - (1-s_K)[r(t) + \delta]K(i,t) - w(t)L(i,t)
\]

where \(0<s_K<1\) denotes a capital subsidization rate. Finally, we assume that the government runs a balanced budget such that

\[
T(t) = s_K [r(t) + \delta]K(t)
\]

where \(K(t)\) is the aggregate stock of capital given by \(K(t) = \int_{i=0}^1 K(i,t)di\).

(a) Determine the first-best subsidization rate \(s_K\), i.e. the subsidization rate that maximizes intertemporal welfare. (You can take for granted that the solution of the household’s optimization problem involves the following Keynes-Ramsey rule: \(\dot{c} = c(r - \rho)\))

(b) Comment briefly on the different steps which are necessary in the process to determine the optimal capital subsidy \(s_K\).
Exercise 2: A simple Dixit-Stiglitz (1977) type model (20 points)

Consider a static economy with imperfect product market competition. There are two sectors. The final output sector produces a final output good, denoted by $X$, by employing a set of intermediate goods, denoted by $x_i$ with $i \in [1, \ldots, N]$. The intermediate goods sector employs labor $L_i$ to produce intermediate goods $x_i$. Production technologies are as follows:

**Final output:** $X = \sum_{i=1}^{N} x_i^\alpha$ with $0 < \alpha < 1$

**Intermediate goods:** $x_i = L_i$ for all $i \in \{1, \ldots, N\}$

On the household side there is a continuum of mass one of identical households. Every household is endowed with $L$ units of time which are supplied inelastically to the labor market. Total labor supply is hence given by $L$. Total labor demand may be expressed as $\sum_{i=1}^{N} L_i$.

We also assume that there is an infinite number of possible varieties $x_i$. Setting up a new intermediate goods firm is associated with fixed costs $F > 0$ in units of $X$, implying that profits of the intermediate goods firms read: $\pi_i = p_i x_i - w L_i - F$.

(a) Determine the number of $x_i$-firms, denoted as $N$, in equilibrium. Discuss how $N$ depends on $F$. Provide a concise economic interpretation.

(b) How does production of the final output good $X$ depend on the number of firms $N$? Provide a concise economic interpretation.
Exercise 3: House Prices Dynamics (20 points)

Empirical data show that the aggregate house price (in real terms) has risen substantially in most industrialized countries since 1950. To discuss this phenomenon, we consider a simple dynamic model of the housing sector. The economy considered is perfectly competitive. The demand side for housing services is described by:

<table>
<thead>
<tr>
<th>equilibrium - housing market</th>
<th>$v^H(t) \cdot H(t) = Y^H(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no-arbitrage condition</td>
<td>$\frac{\dot{p}^H(t) + v^H(t)}{p^H(t)} = r$</td>
</tr>
</tbody>
</table>

where $v^H(t)$ denotes the price for housing services at time $t \in \mathbb{R}$ (i.e. time is continuous), $H(t) > 0$ the housing stock (equal to the number of housing services), $Y^H(t)$ the (exogenous) demand for housing services in units of final output, $p^H(t)$ the price of houses, and $r > 0$ the exogenously given interest rate.

The supply side is described by:

| production technology for houses | $F \left( Z^H, X \right) = \left[ Z(t)^H \right]^\alpha X(t)^\beta$ |
| equation of motion of houses     | $\dot{H}(t) = F \left( Z^H, X \right) - \delta \cdot H(t)$ |

where $Z^H(t)$ denotes the amount of land devoted to the production of houses, $X$ the amount of residential structures, $\delta > 0$ the depreciation rate of the housing stock, and $\alpha, \beta > 0$ denote constant technology parameters. The land price, $p^Z(t)$, and the price of residential structures, $p^X(t)$, are exogenously given.

(a) Set up the indirect production function for houses, as given by $G \left( p^H, p^Z, p^X \right)$. By how much does the production of new houses respond to a one percent increase in the house price $p^H(t)$?

(b) How does the long-run house price $p^H(t)$ change if the demand for housing services $Y^H(t)$ increases? (You may want to distinguish the case $\alpha + \beta = 1$ from the case $\alpha + \beta < 1$.)

(c) How does the long-run house price $p^H(t)$ change if the interest rate $r$ increases? A qualitative but clearly model based reasoning suffices.
Exercise 4: New Keynesian Economics: convex price adjustment cost (20 points)

Consider a firm that produces a differentiated good $X_{jt}$ under monopolistic competition. If the firm adjusts its supply price, it must bear convex price adjustment cost (denoted as $PAC_{jt}$) of the form:

$$PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2,$$

where $\bar{\pi}$ denotes the steady state inflation factor ($\pi = p_t / p_{t-1}$) and $\phi > 0$ a constant technology parameter. Each firm solves the following dynamic optimization problem:

$$\max_{(p_{jt})} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{p_{jt}}{P_t} - \lambda \right) X_{jt}^D - PAC_{jt} \right]$$

s.t. $X_{jt}^D = \left( \frac{p_{jt}}{P_t} \right)^{\epsilon} Y_t$, $PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2$,

where $p_{jt}$ denotes the price of good $X_{jt}$ at time $t$, $P_t$ a price index, $\lambda$ the real marginal cost, $X_{jt}^D$ the demand for good $j$, $\epsilon > 0$ a constant parameter, and $Y_t$ is aggregate income.

(a) Determine the difference equation that governs the optimal supply price $p_{jt}$.

(b) Determine the optimal supply price in the steady state.

(c) Discuss briefly the similarities and the differences between this "convex price adjustment setup" and the Calvo (1983) approach.
Exercise 5: Miscellaneous (1) (20 points)

(1) Growth accounting under CES technology (10 points)
Consider the following, aggregate CES technology

\[ Y(t) = \left[ L(t)^\sigma + K(t)^\sigma \right]^{\frac{1}{\sigma}}, \]

where \( Y(t) \) denotes real, aggregate output at time \( t \), \( L(t) \) labor input, \( K(t) \) the stock of physical capital, \( \sigma \leq 1 \) a constant technology parameter, and \( t \in \mathbb{R} \) the time index. Assume we have information about \( \sigma \) (from estimations of the elasticity of substitution between \( K \) and \( L \)), the labor coefficient \( L/Y \), the capital coefficient \( K/Y \), the growth rate of labor input \( \dot{L} \) and the growth rate of physical capital \( \dot{K} \).

(a) Determine \( \dot{Y} \) to answer the question by how much, then, real aggregate output grows in proportional terms?

(b) How does the growth rate \( \dot{Y} \) look like for \( \sigma \) approaching zero?

(2) Hypothetical consumption level (10 points)
Assume we know the time path of consumption in response to some policy intervention, denoted by \( c_A(t) \). The associated intertemporal welfare (or life-time utility), denoted by \( U_A \), can then be calculated numerically by evaluating the following intertemporal utility function

\[ U_A = \int_{t=0}^{\infty} \frac{c_A(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \]

where \( \sigma, \rho > 0 \). Given a specific value for \( U_A \) it may be instructive to determine the (hypothetical) constant level of consumption, denoted as \( c_H \), which leads to the same intertemporal welfare \( U_A \).

(a) Determine the (hypothetical) constant level of consumption, denoted as \( c_H \), which leads to the same intertemporal welfare \( U_A \) analytically.

(b) Assume that \( U_A = 20 \), \( \sigma = 0.5 \), \( \rho = 0.1 \). What is the numerical value for \( c_H \)?
Exercise 6: Miscellaneous (2) (20 points)

(1) Allocation of land in a simple two-sector model (12 points)
Consider a perfectly competitive and static economy with two sectors: manufacturing and services. In the manufacturing sector capital $K$ is combined with land $Z^M$ to produce consumption goods $M$. Service producers combine labor $L$ and land $Z^S$ to produce services $S$. The sectoral production functions read as follows

$$M = (K)^{1-\alpha} (Z^M)^{\alpha}$$

$$S = (L)^{1-\beta} (Z^S)^{\beta},$$

where $0 < \alpha, \beta < 1$ denote constant technology parameters. Only the intersectoral allocation of land is endogenous, whereas $K$ and $L$ are fixed. Aggregate income is given by $PY = p^M M + p^S S$, where $P=1$ denotes the price level, $p^M$ the (real) price of the manufacturing good and $p^S$ the (real) price of services. Let $0 < \theta < 1$ denote the share of income devoted to services, i.e. $\theta = \frac{p^S S}{Y}$. Equilibrium in the market for services is then described by

$$p^S S = \theta Y.$$

Total land supply is fixed and normalized to one. The land constraint reads $Z^M + Z^S = 1$.

(a) Determine the amount of land employed in the service sector ($Z^S$).

(b) Provide a concise interpretation of the comparative static implication $\frac{\partial Z^S}{\partial \theta}$.

(2) Unemployment (8 points)
There are three major stylized facts of unemployment in advanced economies: (i) there is substantial and persistent unemployment; (ii) employment varies strongly procyclical over the business cycle; (iii) the wage rate varies only mildly procyclical over the business cycle.

(a) Sketch the basic idea, the major assumptions and the main implications of the Shapiro-Stiglitz model. (It may be helpful, though not necessary, to use a graphical exposition.)

(b) To what extent can the Shapiro-Stiglitz model explain the three stylized facts of unemployment mentioned above.