### ERLÄUTERUNGEN (Explanations)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSEN HILFSMITTEL: keine**

### Punkte:

**DATUM, UNTERSCHRIFT DES PRÜFERS:**
Exercise 1: Investment Demand of Firms (20 points)

Consider a firm that produces a homogenous final output good $Y_t$ under perfect competition. The output technology reads

$$Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$$

The firm's planning horizon is infinity. There are investment costs (capital adjustment costs), denoted as $IC_t$. Installing the amount of $I_t$ capital goods (gross investment) requires the following amount of final output

$$IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right].$$

It is assumed that the firm maximizes the present value of its cash flow (or entrepreneurial residual income) subject to a capital accumulation equation, $K_{t+1} = I_t + (1 - \delta)K_t$, i.e. the firm solves the following dynamic problem

$$\max_{\{I_t, K_t\}} \sum_{t=0}^{\infty} \frac{1}{1+r} \left( Y_t - w_tL_t - IC_t \right)$$

s.t. $K_{t+1} = I_t + (1 - \delta)K_t$

$$IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right]$$

$$K_0 = \text{given}.$$

(a) Consider the production technology $Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$. Assume that $K_t$ increases by 1 percent. By how much does, ceteris paribus, $Y_t$ change in proportional terms?

(b) Determine the firm's investment demand (i.e. the demand for final output devoted to capital investment).

Remark: Investment demand will be a function $I_t = I_t(q_t, K_t)$, where $q_t$ denotes the shadow price of installed capital goods. (You are not requested to determine the difference equation which describes the dynamics of $q_t$.)

(c) Provide a sound economic interpretation of your result.

Notation: $Y_t$: final output good at time $t \in \mathbb{R}$; $A > 0$: constant technology parameter; $K_t$: stock of physical capital at $t$; $L_t$: labor employed at time $t$; $IC_t$: investment costs; $0 < \alpha < 1$, $0 \leq \theta, \eta \geq 1$: constant technology parameter; $w_t$: denotes the wage rate, $r$: fixed interest rate, and $\delta \geq 0$: capital depreciation rate
Exercise 2: Simplistic Real-Business-Cycle Model (20 points)

Consider a highly simplified Real-Business-Cycle model (RBC model). There is mass one of identical consumer-producer agents. Individual and instantaneous utility is given by

\[ u(C_t, L_t) = C_t + \gamma(1 - L_t) \]

where \( C_t \) denotes consumption at time \( t \), \( L_t \) working time, and \( 0 < \gamma < 1 \) a constant preference parameter. The output technology is of the following shape

\[ Y_t = (L_t)^\alpha (M_t)^\beta, \quad \alpha + \beta < 1, \]

where \( Y_t \) denotes output, \( M_t \) the amount of raw materials (oil) employed in production, and \( 0 < \alpha, \beta < 1 \) a constant technology parameter. Moreover, the agent can purchase raw materials at price \( p_t^M \) at the world market for raw materials. The dynamics of the raw material price is governed by the following process (with \( p_0^M \) given)

\[ p_t^M = \left( p_{t-1}^M \right)^\eta e^{v_t}, \]

where \( 0 < \eta < 1 \) denotes a constant parameter, and \( v_t \) an identically and independently distributed stochastic variable with \( E(v_t) = 0 \) and \( V(v_t) = \text{const.} \)

(a) Determine the optimal amount of labor as a function of the input price \( p_t^M \).

(b) Assume the economy is in a steady state initially and there is a price shock, i.e. \( v_t > 0 \) at some \( t \), and no further price shock in the future. Describe the dynamic responses of the macroeconomic variables output \( (Y_t) \), consumption \( (C_t) \), and employment \( (L_t) \) and assess the empirical plausibility of these dynamic implications.

(c) Discuss one modification of the simplistic RBC model set up above (by sketching the formal model structure) to improve the implications in a plausible manner.
Exercise 3: Miscellaneous (20 points)

(1) Cobb-Douglas Price Index (10 points)
Consider a perfectly competitive goods sector. There is mass one of identical firms. Every firm produces according to the following standard production technology

\[ Y = K^\alpha L^{1-\alpha}, \]

where \( Y \) denotes output, \( K \) the stock of physical capital, \( L \) the amount of labor, and \( 0 < \alpha < 1 \) a constant technology parameter. Let \( r \) denote the real interest rate and \( w \) the real wage rate. Remark: Do not normalize the goods price to one.

(a) Determine the goods price in equilibrium as a function of input prices \( r \) and \( w \) and model parameters only, i.e. determine \( P = P(r,w) \).

(b) By how much, in proportional terms, does the equilibrium goods price increase, if the interest rate \( r \) increases by 1 percent? Provide a sound economic interpretation of the result.

(2) Efficiency Wages (10 points)
Consider a goods producer that acts under perfect competition. The output technology is given by

\[ Y = A[e(w)L]^{\alpha}, \]

where \( Y \) denotes output, \( e(w) \) the effort function, \( w \) the real wage rate per hour of labor services, \( L \) the amount of labor services (measured in hours), and \( A > 0, 0 < \alpha < 1 \) a constant technology parameter. The effort function is assumed to be of the following shape

\[ e(w) = \begin{cases} B(w - h)^\beta & \text{for } w \geq h \\ 0 & \text{for } w < h \end{cases}, \]

where \( B > 1, \beta > 0 \) denote constant parameters and \( h > 0 \) represents the (constant) value of an outside option (leisure or unemployment insurance).

(a) Determine the wage rate a profit-maximizing firm is willing to pay, assuming that the firm is unconstrained with regard to the amount of labor it wishes to employ.

(b) Assume the economy moves into a recession, as captured by a decline in \( A \). How does the equilibrium wage rate change? Provide a concise economic reasoning.