Universität Leipzig
Wirtschaftswissenschaftliche Fakultät

MASTER VWL – PRÜFUNG (WDH./ RESIT)

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<th>DATUM:</th>
<th>10.10.2013</th>
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<tr>
<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
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<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
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<td>PRÜFUNGS-NR.:</td>
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<td>STUDIENGANG:</td>
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<td>NAME, VORNAME:</td>
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<td>UNTERSCHRIFT DES STUDENTEN:</td>
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ERLÄUTERUNGEN (EXPLANATION)

(1) Die Klausur besteht aus sechs Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle sechs Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of six exercises. Of these six exercises four exercises have to be edited. If you have edited all six exercises, the first four exercises will be scored.)

(2) Zur Bearbeitung stehen insgesamt 120 Minuten zur Verfügung. (To process the exam you have 120 minutes available.)

(3) Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

(4) Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

ZUGELASSENE HILFSMITTEL: keine

Punkte:

| DATUM, UNTERSCHRIFT DES PRÜFERS: |             |
Exercise 1: Convergence in the Neoclassical Growth Model (20 points)

Consider a simple Solovian economy (no population growth, no technical change), which is described by the following components

\[
Y(t) = AK(t)^\alpha L(t)^{1-\alpha}
\]

\[
\dot{K}(t) = sY(t) - \delta K(t) \quad \text{with} \quad K(0) = K_0
\]

where \(Y(t)\) denotes output at time \(t \in \mathbb{R}\) (i.e. time is continuous), \(A > 0, 0 < \alpha < 1\) constant technology parameters, \(K(t)\) the stock of physical capital, \(L(t)\) the stock of population (=labor input), \(0 < s < 1\) the saving rate, and \(\delta > 0\) the capital depreciation rate.

(a) Determine the local rate of convergence at which physical capital per capita \(k (= K/L)\) converges to its steady state level. (Hint: It may be helpful to linearize the differential equation in \(K(t)\) by applying a first-order Taylor approximation.)

(b) Provide a sound economic interpretation of the local rate of convergence.

(c) Explain how the rate of convergence can be transformed into the half-life.

Exercise 2: Increasing returns to scale and imperfect product market competition (20 points)

The static economy under study comprises two sectors. In the perfectly competitive final output sector there is mass one of identical firms. The output technology reads

\[
Y(j) = \left( \frac{1}{j!} \int_{i=0}^{j} x(i)^{\lambda} di \right)^{1/\lambda} \quad \text{with} \quad 0 < \lambda < 1; \ j \in [0,\ldots,1]
\]

In the monopolistically competitive intermediate goods sector there is mass one of identical firms. Each firm has access to the following technology

\[
x(i) = K(i)^\alpha L(i)^\beta \quad \text{with} \quad \alpha, \beta > 0; \ \alpha + \beta > 1; \ i \in [0,\ldots,1]
\]

On the household side there is mass one of identical households who own the capital stock \(K\) and are endowed with \(L\) units of labor, which are supplied inelastically to the labor market. Households are the owners of the firms. Total earnings of the representative household are given by

\[
\text{Earnings} = \pi_{Y(j)} + \pi_{x(i)} + rK + wL
\]

Factor markets are perfectly competitive.

Show whether or not a competitive rewarding scheme according to the rule “factor price equals marginal productivity” is consistently possible in this economy? Provide a concise economic reasoning.

(Notation: \(Y(j)\): output of firm \(j\) in the final output sector; \(A\): constant technology parameter; \(K\): capital; \(L\): labor; \(x(i)\): output of firm \(i\) in the intermediate goods sector; \(\pi_{Y(j)}\): profits earned by the typical \(Y(j)\)-firm; \(\pi_{x(i)}\): profits earned by the typical \(x(i)\)-firm; \(r\): interest rate, \(w\): wage rate)
Exercise 3: New Keynesian Economics: convex price adjustment cost (20 points)

Consider a firm that produces a differentiated good $X_{jt}$ under monopolistic competition. If the firm adjusts its supply price, it must bear convex price adjustment cost (denoted as $PAC_{jt}$) of the form

$$PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2$$

where $\bar{\pi}$ denotes the steady state inflation factor ($\pi = P_t / P_{t-1}$) and $\phi > 0$ a constant technology parameter. Each firm solves the following dynamic optimization problem

$$\max_{\{p_j\}} \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{p_{jt}}{p_t} - \lambda_t \right) X^D_{jt} - PAC_{jt} \right]$$

subject to

$$X^D_{jt} = \left( \frac{p_{jt}}{p_t} \right)^{-\varepsilon} Y_t$$

$$PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2$$

where $p_{jt}$ denotes the price of good $X_{jt}$ at time $t$, $P_t$ a price index, $\lambda_t$ the real marginal cost, $X^D_{jt}$ the demand for good $j$, $\varepsilon > 0$ a constant parameter, and $Y_t$ is aggregate income.

(a) Determine the difference equation that governs the optimal supply price $p_{jt}$.

(b) Determine the optimal supply price in the steady state.

(c) Discuss briefly the similarities and the differences between this "convex price adjustment setup" and the Calvo (1983) approach.
Exercise 4: Monetary policy in a baseline monetarist model (20 points)

Consider the following simple model economy. Firms produce output under perfect competition according to the following technology

\[ Y_t = L_t^\alpha, \quad 0 < \alpha < 1 \]

where \( Y_t \) denotes output at time \( t \in \mathbb{N} \) (i.e. time is discrete), \( L_t \) denotes the amount of labor employed in \( Y \)-production. Labor unions have a target real wage which is normalized to one (i.e. \( W_t/P_t = 1 \)) and have the power to control the nominal wage accordingly. Hence, labor unions set \( W_t \), negotiated at the beginning of each period \( t \), such that

\[ W_t = P_t^e \]

where \( W_t \) denotes the nominal wage rate and \( P_t^e \) denotes the expected price level prevailing at time \( t \). The aggregate demand schedule is described by the quantity equation of money

\[ M_t V_t = Y_t P_t \]

where \( M_t \) denotes nominal money supply, \( V_t \) the velocity of circulation, and \( P_t \) the price level, respectively. It is also assumed that price expectations are formed according to an adaptive expectations scheme

\[ P_t^e = P_{t-1} \left( \frac{P_t}{P_t^e} \right)^\beta, \quad 0 \leq \beta \leq 1 \]

Finally, monetary policy controls the money supply according to

\[ M_t = M^* e_t \]

where \( M^* > 0 \) is a fixed number and \( e_t \) represents a stochastic variable with \( E(e_t) = 0 \), \( V(e_t) = \text{const.} \) and \( \text{Cov}(e_t, e_{t+i}) = 0 \) for all \( t \) and \( i \).

(a) Set up the complete dynamic system which describes the evolution of this model economy. (Remark: It may be useful to form the natural logarithms to end up with a linear model specification.)

(b) Characterize the steady state for the endogenous variables \( Y_t \), \( P_t \), and \( P_t^e \). Provide a concise economic reasoning for the respective steady state solutions.

(c) Assume that there is a temporary contractive monetary shock, i.e. \( e_t < 0 \) and \( e_{t+i} = 0 \) for \( i > 0 \). Describe the resulting dynamic adjustment and provide a concise economic reasoning.
Exercise 5: Miscellaneous (1) (20 points)

(1) Real business cycle theory (10 points)

Consider a representative consumer-producer agent who has instantaneous utility 
\( u_t = \ln C_t + \eta(1-L_t) \). The agent has access to the following production technology 
\( Y_t = A_t \left( K_t \right)^{\alpha} \left( L_t \right)^{1-\alpha} \). We assume that the shock materializes at first and can be immediately
observed such that the agent knows the state of technology (given by \( A_t \)) when deciding on
optimal labor input.

Notation: 
- \( u_t \): instantaneous utility at time \( t \);
- \( C_t \): consumption;
- \( Y_t \): output;
- \( L_t \): units of time devoted to
production activities (labor input);
- \( K_t \): stock of physical capital;
- \( \eta, \sigma > 0 \): constant preference parameter;
- \( 0 < \alpha < 1 \): constant technology parameter;
- \( \mu_t \): shadow price of physical capital.

(a) Provide a sound economic interpretation of the first-order condition for optimal labor
input, which reads as follows:
\( \frac{\partial u_t}{\partial L_t} = -\alpha \mu_t A_t \left( K_t \right)^{\alpha} \left( L_t \right)^{-\alpha} \).

(b) How does the first-order condition for optimal labor input change if we assume that both
(i) the agent decides on  optimal labor input before the shock materializes and (ii) the
instantaneous utility function has the following shape 
\( u_t = \left( C_t \right)^{\sigma} + \eta(1-L_t)^{\sigma} \). Provide
a concise economic interpretation.

(2) Small-open economy with capital externalities (10 points)

Consider a perfectly competitive, one-sector, small, open economy with (aggregate) final
output technology
\( Y = AK^\alpha L^{1-\alpha} \)

where \( A > 0 \) denotes total factor productivity (TFP), \( K \) the stock of capital, \( L \) the total amount
of labor input, and \( \theta < \alpha < 1 \) a constant technology parameter.

The model describes a static economy, i.e. there is no continuous capital accumulation.
However, the economy has free access to the international capital market and hence capital
may flow into or out of the economy. The international real interest rate, denoted as \( r \), is
constant. Labor is immobile.

(a) Assume that TFP \( A \) increases permanently by 10 percent. By how much does GDP \( (Y) \)
increase under capital market integration, assuming that the real interest rate \( r \) remains
constant?

(b) Provide a concise economic interpretation of your result.
Exercise 6: Miscellaneous (2)  (20 points)

(1) R&D-based growth without scale effects  (12 points)
Consider a simple economy which produces a final output good $Y$ according to the aggregate output technology $Y = AL_Y^\alpha$, where $A$ is a (potentially) time-varying technology parameter, $0<\alpha<1$ a constant technology parameter, and $L_Y$ the amount of labor employed in $Y$-production. The R&D technology reads as follows

$$\dot{A} = \eta A^\phi L_Y^\gamma$$

where $\dot{A} := dA / dt$, $\eta>0$, $0<\phi<1$, $0<\gamma\leq1$ and $L_A$ denotes the amount of labor employed in the R&D sector. The labor market is assumed to clear at each point in time, i.e. $L=L_A+L_Y$. Total labor supply grows at an exogenous growth rate, i.e. $L=L_0e^{nt}$, where $n\geq0$ and $t\in\mathbb{R}$ denotes the continuous time index.

(a) Determine the steady state growth rate of final output $Y$.
(b) Compare the model under study to the Romer (1990) model. Which of these models is empirically more plausible with regard to the scale effect implication?
(c) Is public policy effective with respect to controlling the long-run growth rate? Provide a brief reasoning for your answer.

(2) Hypothetical consumption level  (8 points)
Assume we know the time path of consumption in response to some policy intervention, denoted as $c_A(t)$. The associated intertemporal welfare (or life-time utility), denoted as $U_A$, can then be calculated numerically by evaluating the following intertemporal utility function

$$U_A = \int_0^\infty c_A(t)^{1-\sigma} - \frac{1}{1-\sigma} e^{-\rho t} dt,$$

where $\sigma, \rho>0$. Given a specific value for $U_A$ it may be instructive to determine the (hypothetical) constant level of consumption, denoted as $c_H$, which leads to the same intertemporal welfare $U_A$.

(a) Determine the (hypothetical) constant level of consumption, denoted as $c_H$, which leads to the same intertemporal welfare $U_A$ analytically.
(b) Assume that $U_A = 20$, $\sigma = 0.5$, $\rho = 0.1$. What is the numerical value for $c_H$?