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<th><strong>DATUM:</strong></th>
<th>28.05.2013</th>
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<td><strong>MODUL:</strong></td>
<td>ADVANCED MACROECONOMICS</td>
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<td><strong>PRÜFER:</strong></td>
<td>PROF. DR. THOMAS STEGER</td>
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<td><strong>PRÜFUNGS-NR.:</strong></td>
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<td><strong>STUDIENGANG:</strong></td>
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<td><strong>UNTERSCHRIFT DES STUDENTEN:</strong></td>
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**ERLÄUTERUNGEN (Explanation)**

(1) Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

(2) Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

(3) Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

(4) Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSENE HILFSMITTEL:** keine

**Punkte:**
Exercise 1: A simple Dixit-Stiglitz (1977) type model (20 points)

Consider a static economy with imperfect product market competition. There are two sectors. The final output sector produces a final output, denoted as $X$, by employing a set of intermediate goods, denoted as $x_i$ with $i \in [1, ..., N]$. The intermediate goods sector employs labor $L_i$ to produce intermediate goods $x_i$. Production technologies are as follows:

Final output: $X = \sum_{i=1}^{N} x_i^\alpha$ with $0 < \alpha < 1$

Intermediate goods: $x_i = L_i$ for all $i \in \{1, ..., N\}$

On the household side there is a continuum of mass one of identical households. Every household is endowed with $L$ units of time which are supplied inelastically to the labor market. Total labor supply is hence given by $L$. Total labor demand may be expressed as $\sum_{i=1}^{N} L_i$.

We also assume that there is an infinite number of possible varieties $x_i$. Setting up a new intermediate goods firm is associated with fixed costs $F > 0$ in units of $X$, implying that profits of the intermediate goods firms read: $\pi_i = p_i x_i - w L_i - F$.

(a) Determine the number of $x_i$-firms, denoted as $N$, in equilibrium. Discuss how $N$ depends on $F$. Provide a concise economic interpretation.

(b) How does production of the final output good $X$ depend on the number of firms $N$? Provide a concise economic interpretation.
Exercise 2: Keynes-Ramsey rule under Subsistence Consumption (20 points)
Consider an infinitely living consumer-producer agent, often described by the metaphor "Robinson Crusoe", who is endowed with $L$ units of time per period, which is entirely devoted to output production, and an initial amount of capital $K_0$. Capital can be accumulated by abstaining from consumption and depreciates at a constant rate $\delta > 0$. There is no technical change and no population growth. The production function for final output is

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$$

where $Y(t)$ denotes output at time $t \in \mathbb{R}$, $L(t) = L$ constant labor input, and $0 < \alpha < 1$ a constant technology parameter. The agent is assumed to maximize intertemporal welfare as given by

$$U = \int_0^\infty \left[ \frac{(C(t) - \tilde{C})^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt \right]$$

where $C(t) \geq \tilde{C}$ denotes the level of consumption, $\tilde{C}$ the minimum level of consumption required to stay alive (subsistence level), $\rho > 0$ the discount rate, and $\sigma > 0$ a constant preference parameter.

(a) Determine the Keynes-Ramsey rule of optimal consumption growth.

**Remark:** A convenient possibility to solve dynamic optimization problems consists in the application of the maximum principle. This requires to set up the (current-value) Hamiltonian function, which may be stated as $H = u[C(t)] + \lambda(t)[Y(t) - \delta K(t) - C(t)]$, where $u[C(t)]$ denotes the instantaneous utility function, $\lambda(t)$ a co-state variable (current-value shadow price). The necessary first-order conditions may then be expressed as follows: $H_{C(t)} = 0$ and $\dot{\lambda}(t) = -H_{K(t)} - \rho \lambda(t)$ for all $t$.

(b) Provide a concise economic interpretation of the Keynes-Ramsey rule.
Exercise 3: Miscellaneous (20 points)

(1) Capital mobility in a small, open economy (10 points)

Consider a perfectly competitive, one-sector, small, open economy with (aggregate) final output technology

\[ Y = AK^\alpha L^{1-\alpha} \]

where \( A > 0 \) denotes total factor productivity (TFP), \( K \) the stock of capital, \( L \) the total amount of labor input, and \( 0 < \alpha < 1 \) a constant technology parameter.

The model describes a static economy, i.e. there is no continuous capital accumulation. However, the economy has free access to the international capital market and hence capital may flow into or out of the economy. The international real interest rate, denoted as \( r \), is constant. Labor is immobile.

(a) Assume that TFP \( A \) increases permanently by 10 percent. By how much does GDP (\( Y \)) increase under capital market integration, assuming that the real interest rate \( r \) remains constant?

(b) Provide a concise economic interpretation of your result.

(2) Growth accounting under CES technology (10 points)

Consider the following, aggregate CES technology

\[ Y(t) = \left[ L(t)^\sigma + K(t)^\sigma \right]^{\frac{1}{\sigma}}, \]

where \( Y(t) \) denotes real, aggregate output at time \( t \), \( L(t) \) labor input, \( K(t) \) the stock of physical capital, \( \sigma \leq 1 \) a constant technology parameter, and \( t \in \mathbb{R} \) the time index. Assume we have information about \( \sigma \) (from estimations of the elasticity of substitution between \( K \) and \( L \)), the labor coefficient \( L/Y \), the capital coefficient \( K/Y \), the growth rate of labor input \( \dot{L} \) and the growth rate of physical capital \( \dot{K} \).

(a) Determine \( \dot{Y} \) to answer the question by how much, then, real aggregate output grows in proportional terms?

(b) How does the growth rate \( \dot{Y} \) look like for \( \sigma \) approaching zero?