**Universität Leipzig**  
**Wirtschaftswissenschaftliche Fakultät**

**MASTER VWL – PRÜFUNG (WDH./ RESIT)**

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<th>DATUM:</th>
<th>25.09.2012</th>
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<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
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<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
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**ERLÄUTERUNGEN (EXPLANATION)**

1. Die Klausur besteht aus *sechs Aufgaben*. Hiervon sind *vier Aufgaben* zu bearbeiten! Sollten Sie alle sechs Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of *six exercises*. Of these six exercises *four exercises* have to be edited. If you have edited all six exercises, the first four exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt **120 Minuten** zur Verfügung. (To process the exam you have **120 minutes** available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSENEN HILFSMITTEL:** keine

**Punkte:**

**DATUM, UNTERSCHRIFT DES PRÜFERS:**
Exercise 1: Optimal consumption in a simple two-period household setup (20 points)

Consider an agent who lives for two periods. This agent has an initial endowment of financial wealth, denoted as $\Omega$. Financial wealth can be used to finance consumption in the first period of life ($C_1$) or can be invested at the capital market at a (fixed) interest rate $r$. The return can then be used to finance consumption in the second period of life ($C_2$). The instantaneous utility function is of the CIES-type such that lifetime utility is given by

$$U = \frac{(C_1)^{1-\sigma} - 1}{1-\sigma} + \frac{1}{1+\rho} \frac{(C_2)^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma, \rho > 0$ denote constant preference parameters. We further assume that the agent under consideration chooses a time path of consumption $\{C_1, C_2\}$ such that life-time utility is being maximized.

(a) Determine the level of consumption in the first period and in the second period of life, i.e. $C_1$ and $C_2$, as functions of $\Omega$, $r$, $\rho$, and $\sigma$.

(b) How do $C_1$ and $C_2$ change in response to changes in $r$ given that $\sigma = 1$? Provide a concise economic interpretation of your result.

Exercise 2: New Keynesian Theory: the household’s intertemporal problem (20 points)

Consider an infinitely lived representative household who is assumed to maximize its life-time utility where the instantaneous utility function depends on consumption ($C_t$), real money holdings ($M_t / P_t$), and working time ($N_t$). The household has access to a perfect capital market. Buying and holding bonds yields a nominal interest rate $\gamma_t$. For simplicity, we abstract from uncertainty. Time is discrete. The household’s intertemporal problem may be stated as

$$\max \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_1^{1-\sigma}}{1-\sigma} + \frac{\phi}{1-b} \frac{M_t}{P_t} \right] - \chi_N^{t+\eta},$$

s.t. $C_t + \frac{M_t}{P_t} + B_t \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t+1}}{P_t} + (1 + \gamma_{t+1}) \frac{B_{t+1}}{P_t}$

where $\beta, \sigma, \phi, b, \chi, \eta$ denote constant preference parameters, $P_t$ the price level, $M_t$ nominal money balances, $B_t$ bond holdings, $W_t$ the nominal wage rate, and $\gamma_t$ the nominal interest rate earned by holding bonds, respectively.

(a) Set up the first-order conditions for optimal $C_t$, $M_t$, and $N_t$. (For simplicity you may want to assume that the household solves its intertemporal problem sequentially and focus on period $t = 0$ only.)

(b) Provide a concise economic interpretation of every first-order condition.
**Exercise 3: Increasing returns to scale and imperfect product market competition**

(20 points)

The economy comprises two sectors. In the perfectly competitive final output sector there is mass one of identical firms. The output technology reads

\[ Y(j) = \left( \int_{i=0}^{1} x(i)^{\lambda} \, di \right)^{\frac{1}{\lambda}} \quad \text{with} \quad 0 < \lambda < 1; \ j \in [0,...,1] \]

In the monopolistically competitive intermediate goods sector there is mass one of identical firms. Each firm has access to the following technology

\[ x(i) = K(i)^{\alpha} L(i)^{\beta} \quad \text{with} \quad \alpha, \beta > 0; \ \alpha + \beta > 1; \ i \in [0,...,1] \]

On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market. Households are the owners of the firms. Total earnings of the representative household are given by

\[ \text{Earnings} = \pi_{Y(j)} + \pi_{x(i)} + rK + wL \]

Factor markets are perfectly competitive.

Is a rewarding scheme according to the rule “factor price equals marginal productivity” possible in this economy? Provide a concise economic reasoning.

*(Notation: \( Y(j) \): output of firm \( j \) in the final output sector; \( A \): constant technology parameter; \( K \): capital; \( L \): labor; \( x(i) \): output of firm \( i \) in the intermediate goods sector; \( \pi_{Y(j)} \): profits earned by the typical \( Y(j) \)-firm; \( \pi_{x(i)} \): profits earned by the typical \( x(i) \)-firm; \( r \): interest rate, \( w \): wage rate)*
Exercise 4: Investment demand of firms (20 points)

Consider a firm that produces a homogenous final output good \( Y_t \) under perfect competition. The output technology reads

\[
Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}
\]

The firm's planning horizon is infinity. There are investment costs (capital adjustment costs), denoted as \( IC_t \). Installing the amount of \( I_t \) additional capital goods (gross investment) requires the following amount of final output

\[
IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right]
\]

It is assumed that the firm maximizes the present value of its cash flow (or entrepreneurial residual income) subject to a capital accumulation equation, \( K_{t+1} = I_t + (1-\delta)K_t \), i.e. the firm solves the following dynamic problem

\[
\max \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t (Y_t - w_t L_t - IC_t)
\]

s.t. \( K_{t+1} = I_t + (1-\delta)K_t \)

\[
IC_t = I_t \left[ 1 + \theta \left( \frac{I_t}{K_t} \right)^\eta \right]
\]

\( K_0 = \text{given} \)

(a) Consider the production technology \( Y_t = A(K_t)^\alpha (L_t)^{1-\alpha} \). Assume that \( K_t \) increases by 1 percent. By how much does, ceteris paribus, \( Y_t \) change in proportional terms?

(b) Determine the firm's investment demand (i.e. the demand for final output devoted to capital investment).

Remark: Investment demand will be a function \( I_t = I_t(q_t) \), where \( q_t \) denotes the shadow price of installed capital goods. (You are not requested to determine the difference equation which describes the dynamics of \( q_t \).)

(c) Provide a sound economic interpretation of your result.

(Notation: \( Y_t \): final output good at time \( t \in \mathbb{N} \); \( A > 0 \): constant technology parameter; \( K_t \): stock of physical capital at \( t \); \( L_t \): labor employed at time \( t \); \( 0 < \alpha < 1 \), \( \theta \geq 0 \), \( \eta \geq 1 \): constant technology parameter; \( w_t \): denotes the wage rate, \( r \): fixed interest rate, and \( \delta \geq 0 \): capital depreciation rate)
Exercise 5: Miscellaneous (1) (20 points)

(1) R&D-based growth without scale effects (14 points)

Consider a simple economy which produces a final output good $Y$ according to the aggregate output technology $Y = AL_Y^a$, where $A$ is a (potentially) time-varying technology parameter, $0 < \alpha < 1$ a constant technology parameter, and $L_Y$ the amount of labor employed in $Y$-production. The R&D technology reads as follows

$$\dot{A} = \eta A^\theta L_A^\gamma$$

where $\dot{A} := \frac{dA}{dt}$, $\eta > 0$, $0 < \phi < 1$, $0 < \gamma \leq 1$ and $L_A$ denotes the amount of labor employed in the R&D sector. The labor market is assumed to clear at each point in time, i.e. $L = L_A + L_Y$. Total labor supply grows at an exogenous growth rate, i.e. $L = L_0 e^{nt}$, where $n \geq 0$ and $t \in \mathbb{R}$ denotes the continuous time index.

(a) Determine the steady state growth rate of final output $Y$.

(b) Compare the model under study to the Romer (1990) model. Which of these models is empirically more plausible with regard to the scale effect implication?

(c) Is public policy effective with respect to controlling the long-run growth rate? Provide a brief reasoning for your answer.

(2) Business cycle fluctuations (6 points)

The output gap of modern economies typically exhibits volatility and persistent deviation from its mean (which is zero by construction). One step in economic theorizing consists in the identification of minimal economic structures which are consistent with this empirical pattern. Let us denote the output gap by $x_t$, where $t$ denotes a discrete time index.

(a) Imagine a reduced form-model which leads to a first-order, linear difference equation in $x_t$, i.e. $x_t = a \cdot x_{t-1}$, where $a$ represents a constant coefficient. Which features of business cycle fluctuations can be explained, which cannot be explained? Comment on the required value of $a$.

(b) Imagine a reduced form model which leads to a second-order, linear difference equation in $x_t$, i.e. $x_t = a_1 \cdot x_{t-1} + a_2 \cdot x_{t-2}$, where $a_1, a_2$ represent constant coefficients. Which features of business cycle fluctuations can be explained? (A qualitative discussion suffices.)
Exercise 6: Miscellaneous (2) (20 points)

(1) The rate of convergence in the Solow model (14 points)

The reduced form of the well-known Solow model may be expressed as follows

\[
\dot{k}(t) = s k(t)^\alpha - (n + \xi) k(t) \quad \text{with} \quad k(0) = k_0,
\]

where \(0 < \alpha < 1\) denotes a constant technology parameter, \(0 < s < 1\) the (time invariant) saving rate, \(n \geq 0\) the population growth rate, \(\xi \geq 0\) the rate of technical progress, \(t\) a continuous time index, \(\dot{k}(t) := \frac{dk(t)}{dt}\), and \(k(t)\) the stock of physical capital per effective units of labor (i.e. \(k := K / AL\)), respectively.

(a) Determine the steady state value of \(k(t)\).

(b) Determine the rate of convergence at which \(k(t)\) converges to its steady state value.

(2) Capital market equilibrium (6 points)

Consider a model economy with two assets. First, there is an equity share, its value is denoted as \(v(t)\), which pays a dividend of \(\pi(t)\) each period. Second, there is a bond which pays a (constant) rate of return of \(r\).

(a) Set up the capital market equilibrium condition or, equivalently, the condition which describes the absence of arbitrage opportunities. Provide a concise economic interpretation of this capital market equilibrium condition.

(b) Explain the relationship between the capital market equilibrium condition and the following “solution” for \(v(t)\): 
\[
v(0) = \int_{-\infty}^{\infty} e^{-\tau} \pi(t) dt
\]
(you are not expected to solve the differential equation which describes capital market equilibrium).