# UNIVERSITÄT LEIPZIG

## WIRTSCHAFTSWISSENSCHAFTLICHE FAKULTÄT

### MASTER VWL – PRÜFUNG (MIDTERM EXAM)

<table>
<thead>
<tr>
<th>DATUM:</th>
<th>31.05.2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
</tr>
<tr>
<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
</tr>
<tr>
<td>PRÜFUNGS-NR.:</td>
<td></td>
</tr>
<tr>
<td>STUDIENGANG:</td>
<td></td>
</tr>
<tr>
<td>NAME, VORNAME:</td>
<td></td>
</tr>
<tr>
<td>UNTERSCHRIFT DES STUDENTEN:</td>
<td></td>
</tr>
</tbody>
</table>

**ERLÄUTERUNGEN (Explanation)**

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)
2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)
3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)
4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSENE HILFSMITTEL:** keine

**Punkte:**

| DATUM, UNTERSCHRIFT DES PRÜFERS: |   |
Exercise 1: The Keynes-Ramsey rule of optimal consumption (20 points)
Consider a household within a perfectly competitive economy. The household is endowed with $L > 0$ units of time per period, which are inelastically supplied to the labor market, and possesses financial wealth $a(t)$, which is rented to firms. Household income is given by $X(t) = w(t)L + r(t)a(t)$, where $w(t)$: wage rate; $r(t)$: interest rate; $t \in \mathbb{R}^+$: time index. The instantaneous utility function reads $u[c(t)] = \ln[c(t)]$, where $\ln(x)$ denotes the natural logarithm of $x > 0$. It is assumed that the representative household maximizes intertemporal welfare as given by ($\rho > 0$ denotes the time preference rate):

$$U = \int_{t=0}^{\infty} \ln[c(t)] e^{-\rho t} dt$$

(a) Determine the Keynes-Ramsey rule of optimal consumption.

(Remark: A convenient possibility to solve dynamic optimization problems consists in the application of the maximum principle. This requires to set up the Hamiltonian function, which may be stated as $H := u[c(t)] + \lambda(t)[X(t) - c(t)]$, where $\lambda(t)$ denotes a co-state variable (current-value shadow price). The necessary first-order conditions may then be expressed as follows: $H_{c(t)} = 0$ and $\dot{\lambda}(t) = -H_{a(t)} + \rho \lambda(t)$.)

(b) What is the value of the intertemporal elasticity of substitution (IES) between consumption at any two points in time? (The IES need not be determined, just mention its value.)

(c) Provide a concise economic interpretation of the Keynes-Ramsey rule.

Exercise 2: Multiplier-Accelerator Model and RBC Model (20 points)
Business cycle models aim at explaining the stylized facts of the business cycle. In the lecture we have discussed, among others, the Multiplier-Accelerator Model and the RBC model.

(a) Describe very concisely the formal structure of the Multiplier-Accelerator Model and the RBC Model.

(b) Compare both models according to the following criteria (keep the exposition brief):
   (b.1) What are the major impulses that give rise to business cycles?
   (b.2) How can the basic propagation mechanisms be described?
   (b.3) Which role do demand and supply play within the respective models?
Exercise 3: Miscellaneous (20 points)

(1) Small-open economy with capital externalities (10 points)

Consider a simple, static, neoclassical, small-open economy. There is a continuum of length one of identical firms indexed by $i \in [0,1]$. The production function of the representative firm reads

$$Y_i = K_i^K \bar{K}^\beta L_i^{1-\alpha}, \quad 0 < \alpha, \beta < 1$$

where $Y_i$ denotes output of firm $i$, $K_i$ capital employed by firm $i$, $L_i$ labor employed by firm $i$, and $\bar{K} = \int_{i=0}^{1} K_i \, di$ the average stock of capital. Firms have access to the world capital market and can borrow capital at a constant (real) interest rate $r > 0$.

(a) Determine the optimal capital investment subsidy $s_K$ (we assume that capital subsidies can be financed by lump sum taxes).

(b) Explain concisely why welfare increases in response to the implementation of optimal $s_K$.

(2) Hypothetical consumption level (10 points)

Assume we know the time path of consumption in response to some policy intervention, denoted as $c_A(t)$. The associated intertemporal welfare (or life-time utility), denoted as $U_A$, can then be calculated numerically by evaluating the following intertemporal utility function

$$U_A = \int_{t=0}^{\infty} \frac{c_A(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} \, dt,$$

where $\sigma, \rho > 0$. Given a specific value for $U_A$ it may be instructive to determine the (hypothetical) constant level of consumption, denoted as $c_H$, which leads to the same intertemporal welfare $U_A$.

(a) Determine the (hypothetical) constant level of consumption, denoted as $c_H$, which leads to the same intertemporal welfare $U_A$ analytically.

(b) Assume that $U_A = 20$, $\sigma = 0.5$, $\rho = 0.1$. What is the numerical value for $c_H$?