## ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus fünf Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle fünf Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of five exercises. Of these five exercises four exercises have to be edited. If you have edited all five exercises, the first four exercises will be scored.)
2. Zur Bearbeitung stehen insgesamt 120 Minuten zur Verfügung. (To process the exam you have 120 minutes available.)
3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)
4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

## ZUGELASSENE HILFSMITTEL: 
*Programmsoftware Excel auf bereitgestelltem Rechner*

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**DATUM, UNTERSCHRIFT DES PRÜFERS:**
Exercise 1: Labor productivity in a Dixit-Stiglitz (1977) type model (20 points)

Consider a static economy with two sectors. Production technologies are as follows:

- Technology in x-sector: \( x_i = L_i \) for all \( i \in \{1, \ldots, N\} \)
- Technology in Y-sector: \( Y = \sum_{i=1}^{N} x_i^\alpha, \; 0 < \alpha < 1 \)

where \( x_i \) denotes the amount of intermediate good produced by firm \( i \), \( L_i \) the amount of labor employed by firm \( i \), \( N \) the number of goods / firms (every firm produces exactly one intermediate good), and \( Y \) is final output. Production takes place according to the following sequence: (i) \( x_i \)-production; (ii) \( Y \)-production. On the household side there is a continuum of mass one of identical households. Every household is endowed with \( L \) units of time which are supplied inelastically to the labor market. Total labor supply therefore is \( L^S = L \). Total labor demand is \( N_L = \sum_{i=1}^{N} L_i \) (in symmetric equilibrium this is simply \( NL_i \)). We also assume that there is an infinite number of possible varieties \( x_i \). Setting up a new firm is associated with fixed costs \( F > 0 \).

**Definition:** Let \( P \) denote the price of the \( Y \)-good, \( p_i \) the price of \( x_i \), and \( w \) the wage rate. A (general) equilibrium in this economy consists of quantities \( \{L_i, x_i\} \) and prices \( \{P, p_i, w\} \) such that (i) \( Y \)-producers maximize profits, (ii) \( x_i \)-producers maximize profits, (iii) the labor market clears, i.e. \( L_D = L_S \), and (iv) the intermediate goods market clears.

(a) Determine the number of \( x_i \)-firms, denoted by \( N \), in equilibrium. Discuss how \( N \) depends on \( F \). Provide a concise economic interpretation.

(b) How does labor productivity \( Y/L \) depend on \( F \)? Provide a concise economic reasoning.

Exercise 2: The Solow model - numerical evaluation (20 points)

Consider a simple Solovian economy (no population growth, no technical change), which is described by the following components

- Technology: \( Y(t) = AK(t)^\alpha L(t)^{1-\alpha} \)
- Capital accumulation equation: \( \dot{K}(t) = sY(t) - \delta K(t) \) with \( K(0) = K_0 \)

where \( Y(t) \) denotes output at time \( t \in \mathbb{R} \) (i.e. time is continuous), \( A > 0, \; 0 < \alpha < 1 \) constant technology parameters, \( K(t) \) the stock of physical capital, \( L(t) \) the stock of population (=labor input), \( 0 < s < 1 \) the saving rate, and \( \delta > 0 \) the capital depreciation rate.

(a) Determine the level of per capita output \( y = Y/L \) and per capita consumption \( c = C/L \) in the steady state analytically and provide a concise economic interpretation of your results.

(b) Assume that (i) the economy is in a steady state initially; (ii) the (initial) set of parameters is \( \alpha = 0.3, \; s = 0.1, \; A = 1, \; \delta = 0.1 \); and (iii) the saving rate increases permanently from \( s = 0.1 \) to \( s = 0.15 \).

(b.1) Calculate (using the software Excel) the time path of per capita output \( y = Y/L \) and per capita consumption \( c = C/L \) for the first 100 time periods following the shock in the saving rate and provide a graphical representation. (THIS SOLUTION MUST BE HANDED OUT TO THE STAFF AS A FILE.)

(b.2) Provide a concise economic interpretation of the results.
Exercise 3: The stochastic Ramsey model (20 points)

Consider a simple consumer-producer economy. The economic environment of the infinitely lived representative consumer-producer agent is described by the following model components:

- **Instantaneous utility**: \[ u(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \sigma > 0 \]
- **Technology**: \[ Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1 \]
- **Dynamics of technology**: \[ A_t = A_{t-1} e^{\varepsilon}, \quad 0 < \gamma < 1 \]

where \( C_t \) denotes consumption at time \( t \in \mathbb{N} \) (i.e. time is discrete), \( Y_t \) output that can be used for consumption or physical capital investment, \( A_t \) a stochastic TFP parameter, \( K_t \) the stock of physical capital, \( L_t \) the fixed amount of labor input, and \( \varepsilon_t \) a white noise error term with \( E(\varepsilon_t) = 0 \) for all \( t \), \( V(\varepsilon_t) = \text{const.} \) for all \( t \) and \( \operatorname{Cov}(\varepsilon_t, \varepsilon_{t-i}) = 0 \) for all \( t \) and \( i \).

The timing of events within every period is as follows: The shock materializes, then the agent decides on consumption. We assume that the representative consumer-producer agent maximizes expected intertemporal utility.

(a) Set up the dynamic optimization problem of the representative consumer-producer agent.

(b) Determine the first-order conditions for a maximum of expected intertemporal utility and provide a sound economic interpretation of these first-order conditions.

(c) Which stylized business cycle facts can the model explain, which cannot be explained?

Exercise 4: New Keynesian theory - sticky prices (20 points)

Consider an infinitely lived firm that acts under monopolistic competition and produces a (differentiated) consumption good \( c_{jt} \). The demand schedule and the production function are as follows

- **Demand schedule**: \[ c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^\theta Y_t \]
- **Production function**: \[ c_{jt} = AL_{jt} \]

where \( c_{jt} \) is the amount of consumption good \( j \) at time \( t \in \mathbb{N} \) (i.e. time is discrete), \( P_{jt} \) denotes the price of \( c_{jt} \), \( P_t \) a price index, \( \theta > 1 \) a constant parameter, \( Y_t \) household income, \( A > 0 \) a constant technology parameter, and \( L_{jt} \) the amount of labor employed by firm \( j \).

We assume that there are nominal price rigidities according to Calvo (1983). Each period there is a constant probability \( 0 < \omega < 1 \) that the firm is not allowed to adjust its goods price.

(a) Set up the firm's decision problem by assuming that the firm maximizes the expected present discounted value of the entire stream of future profits.

(b) Provide a sound economic interpretation of your result. Consider also the limiting case \( \omega = 0 \).
Exercise 5: Miscellaneous (20 points)

(1) R&D-based growth without scale effects (15 points)

Consider a simple economy which produces a final output good $Y$ according to the aggregate output technology $Y = A L_Y^\alpha$, where $A$ is a (potentially) time-varying technology parameter, $0<\alpha<1$ a constant technology parameter, and $L_Y$ the amount of labor employed in $Y$-production. The R&D technology reads as follows

$$\dot{A} = \eta A^\phi L_A^\gamma$$

where $\dot{A} := \frac{dA}{dt}$, $\eta>0$, $0<\phi<1$, $0<\gamma\leq1$ and $L_A$ denotes the amount of labor employed in the R&D sector. The labor market is assumed to clear at each point in time, i.e. $L = L_A + L_Y$. Total labor supply grows at an exogenous growth rate, i.e. $L = L_0 e^{nt}$, where $n \geq 0$ and $t \in \mathbb{R}$ denotes the continuous time index.

(a) Determine the steady state growth rate of final output $Y$.

(b) Compare the model under study to the Romer (1990) model. Which of these models is empirically more plausible with regard to the scale effect implication?

(c) Is public policy effective with respect to controlling the long-run growth rate? Provide a brief reasoning for your answer.

(2) Capital market equilibrium (5 points)

Consider a model economy with two assets. First, there is an equity share, its value is denoted as $v(t)$, which pays a dividend of $\pi(t)$ each period. Second, there is a bond which pays a (constant) rate of return of $r$.

(a) Set up the capital market equilibrium condition or, equivalently, the condition which describes the absence of arbitrage opportunities. Provide a concise economic interpretation of this capital market equilibrium condition.

(b) Explain the relationship between the capital market equilibrium condition and the following “solution” for $v(t)$: $v(0) = \int_0^\infty e^{-rt}\pi(t)dt$ (you are not expected to solve the differential equation which describes capital market equilibrium).