Die Klausur besteht aus fünf Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle fünf Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of five exercises. Of these five exercises four exercises have to be edited. If you have edited all five exercises, the first four exercises will be scored.)

Zur Bearbeitung stehen insgesamt 120 Minuten zur Verfügung. (To process the exam you have 120 minutes available.)

Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)
**Exercise 1: Contractive monetary policy in a basic dynamic macromodel (20 points)**

Consider the following simple model economy. Firms produce output under perfect competition according to the following technology

\[ Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1 \]

where \( Y_t \) denotes output at time \( t \in \mathbb{N} \) (i.e. time is discrete), \( L_t \) denotes the amount of labor employed in \( Y \)-production. Labor unions have a target real wage which is normalized to one (i.e. \( W_t/P_t=1 \)) and have the power to control the nominal wage accordingly. Hence, labor unions set \( W_t \), negotiated at the beginning of each period \( t \), such that

\[ W_t = P_t^e \]

where \( W_t \) denotes the nominal wage rate and \( P_t^e \) denotes the expected price level prevailing at time \( t \). The aggregate demand schedule is described by the quantity equation of money

\[ M_t V_t = Y_t P_t \]

where \( M_t \) denotes nominal money supply, \( V_t \) the velocity of circulation, and \( P_t \) the price level, respectively. It is also assumed that price expectations are formed according to an adaptive expectations scheme

\[ P_t^e = (1 - \beta) \frac{P_t}{P_{t-1}} + \beta P_t^e, \quad 0 \leq \beta \leq 1 \]

Finally, monetary policy controls the money supply according to

\[ M_t = M^* e_t^\varepsilon \]

where \( M^*>0 \) is a fixed number and \( e_t^\varepsilon \) represents an error term with \( E(e_t^\varepsilon)=0, V(e_t^\varepsilon) = \text{const.} \) and \( \text{Cov}(e_t^\varepsilon, e_{t+i}^\varepsilon) = 0 \) for all \( t \) and \( i \).

(a) Set up the complete dynamic system which describes the evolution of this model economy. *(Remark: It may be useful to form natural logarithms to end up with a linear model specification.)* (5 points)

(b) Characterize the steady state for the endogenous variables \( Y_t, P_t, \) and \( P_t^e \). Provide a concise economic reasoning for the respective steady state solutions. (5 points)

(c) Assume that there is a temporary contractive monetary shock, i.e. \( \varepsilon_t<0 \) and \( \varepsilon_{t+i}=0 \) for \( i>0 \). Describe the resulting dynamic adjustment and provide a concise economic reasoning. (10 points)
Exercise 2: Productive government expenditures in the Barro (1990) model (20 points)

Consider a perfectly competitive economy. There is a continuum of length one of identical firms, indexed by \( i \in [0, \ldots, 1] \). Output technology of firm \( i \) is given by

\[
Y^*_i = G^\beta K^\alpha L^{1-\alpha},
\]

where \( Y^*_i \) denotes final output produced by firm \( i \), \( G \) a public input (productive government expenditures), \( K^i \) the stock of physical capital employed by firm \( i \), \( L^i \) the amount of labor employed by firm \( i \), and \( 0 < \alpha, \beta < 1 \).

It is assumed that the government runs a balanced budget, i.e.

\[
G = \tau Y,
\]

where \( 0 < \tau < 1 \) denotes a linear tax rate and \( Y = \int_0^1 Y_i \, di \) denotes aggregate income, respectively.

(a) Set up, by eliminating \( G \), the reduced-form output technology \( Y = F(K, L) \) and determine the degree of returns to scale in the private input factors \( K \) and \( L \). (5 points)

(b) Determine the competitive wage rate and the competitive interest rate in this economy. (5 points)

(c) Assuming that the Keynes-Ramsey rule holds, how does the growth rate of consumption change in response to \( \tau \)? Provide a concise economic reasoning. (10 points)
Exercise 3: R&D-based growth - the Romer (1990) model (20 points)

Consider a dynamic macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[
\begin{align*}
\text{R&D} & \quad \dot{A} = \eta L_A A \quad \text{with} \quad A(0) = A_0 \\
\text{Intermediate goods} & \quad x(i) = k(i) \quad \text{for all} \quad i \in [0, \ldots, A] \\
\text{Final output} & \quad Y = L^\alpha_A \int_{i=0}^{A} x(i)^{\alpha} \, di
\end{align*}
\]

(a) The reduced-form technology and the specialization effect (5 points)

Define the aggregate stock of capital as \( K := A x \) and note the symmetry implication which says that \( x_i = x \) for all \( i \). Set up the reduced-form technology \( Y = F(K, L, A) \). How does an increase in the number of intermediate goods \( A \) affect total factor productivity? Provide a concise economic reasoning.

(b) Free entry into the R&D sector (5 points)

There is free entry into the R&D-sector. Assuming that the economy is in a steady state, the free-entry condition may be expressed as follows: \( \frac{\pi}{r} \leq \frac{W}{\eta A} \). Provide an economic interpretation of this condition.

(c) The steady state growth rate (10 points)

We assume that the Keynes-Ramsey rule, which may be written as \( r = \sigma g + \rho \), holds. Moreover, it can be readily shown that equilibrium profits may be expressed as \( \pi = (1 - \alpha)\alpha L_A^{\alpha-\alpha} x^\alpha \). Given these information, determine the steady state growth rate of final output \( Y \). Provide an economic interpretation of the result.

(Notation: \( A \): “number” of intermediate good types; \( \dot{x} := \frac{dx}{dt} \); \( \eta > 0, 0 < \alpha < 1 \): constant technology parameter; \( L_A \): amount of labor devoted to R&D; \( L_Y \): amount of labor devoted to \( Y \)-production; \( x(i) \): number of intermediate goods of type \( i \); \( k(i) \): capital used in \( x(i) \)-production; \( \pi \): profit of the typical \( x \)-producer; \( g \): steady state growth rate of \( Y, C, K \) and \( A \), where \( C \): consumption, \( K \): stock of capital; \( \sigma, \rho > 0 \): constant preference parameters)
**Exercise 4: Ricardian equivalence (20 points)**

Consider an economy which is populated by a large number of identical households. The economy lasts for two periods (period 1 and 2) only. Intertemporal welfare of the typical household is given by

\[ U = \ln(C_1) + \frac{1}{1+\rho} \ln(C_2) \]

The household has a fixed income stream \( \{Y_1, Y_2\} \). Capital markets are perfect. Both the households and government have perfect foresight. The stream of government expenditures \( \{G_1, G_2\} \) is fixed.

(a) **Non-distortionary taxation (8 points)**

The periodic budget constraints of the representative household are given by

- period 1: \( A_1 + C_1 = (1+r)A_0 + (1-\tau_1)Y_1 \)
- period 2: \( A_2 + C_2 = (1+r)A_1 + (1-\tau_2)Y_2 \)

The periodic budget constraints of the government read as follows

- period 1: \( B_1 - B_0 = rB_0 + G_1 - \tau_1 Y_1 \)
- period 2: \( B_2 - B_1 = rB_1 + G_2 - \tau_2 Y_2 \)

Derive the consolidated budget constraint of the whole economy and explain the intermediate steps concisely. Provide a clear economic interpretation of the result. Explain the economic logic behind the Ricardian equivalence proposition.

(b) **Distortionary taxation (8 points)**

Now assume that the government levies a (proportional) comprehensive income tax such that tax revenues read \( T_i = \tau_i(Y_i + rB_i) \) for \( i \in \{1,2\} \). It can be readily shown that the consolidated budget constraint of the whole economy is then given by

\[ C_1 + \frac{C_2}{1+(1-\tau_2)r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+(1-\tau_2)r} \]

Determine the level of consumption in the first and in the second period. Additionally, also determine the level of household saving in the first period. Provide a concise economic interpretation of the result.

(c) **Critical assumptions (4 points)**

What are the critical assumptions underlying the Ricardian equivalence proposition? Is Ricardian equivalence likely to hold in reality? Provide a very concise reasoning.

(Notation: \( A_i \): household’s financial wealth in period \( i \in \{1,2\} \); \( B_i \): government bonds; \( C_i \): consumption; \( Y_i \): household income; \( G_i \): government purchases; \( \tau_i \): tax rates; \( r > 0 \): constant interest rate; \( \rho \geq 0 \): time preference rate)
Exercise 5: Miscellaneous (20 points)

(1) How large is the value of the elasticity of substitution between $K$ and $L$ in the case of a Cobb-Douglas technology $Y=K^{\alpha}L^{1-\alpha}$, where $Y$ denotes final output, $K$ the stock of physical capital, $L$ the number of workers, and $0<\alpha<1$? (It suffices to merely mention the value of the elasticity of substitution.) (2 points)

(2) Assume that the demand for good $x$ has the following shape $x=p^{-0.1}Y$, where $p$ denotes the price of good $x$ and $Y$ denotes income, respectively. By how much does the demand for good $x$ change if the price declines by 2 percent? (3 points)

(3) Consider a firm that produces a good $x$ according to the technology $x=AL$, where $x$ is output, $A>0$ denotes a constant, and $L$ labor input measured in working hours. Let $p$ denote the price of good $x$ and $w$ the wage rate per working hour. Set up the profit function of this firm and provide a concise explanation. (5 points)

(4) Consider a perfectly neoclassical small open economy with aggregate output technology $Y=K^{\alpha}L^{1-\alpha}$, where $Y$ denotes final output, $K$ the stock of physical capital, $L$ the number of workers, and $0<\alpha<1$. Capital is perfectly mobile, labor is immobile. The exogenous interest rate is denoted by $\bar{r}$. By how much does output per worker $y:=Y/L$ change if the interest rate increases by 1 percent? (10 points)