## ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus fünf Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle fünf Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of five exercises. Of these five exercises four exercises have to be edited. If you have edited all five exercises, the first four exercises will be scored.)
2. Zur Bearbeitung stehen insgesamt 120 Minuten zur Verfügung. (To process the exam you have 120 minutes available.)
3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)
4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

## ZUGELASSENE HILFSMITTEL: KEINE
Exercise 1: Ricardian equivalence (20 points)

Consider an economy which is populated by a large number of identical households. The economy lasts for two periods (period 1 and 2) only. Intertemporal welfare of the typical household is given by

\[ U = \ln \left( C_1 \right) + \frac{1}{1+\rho} \ln \left( C_2 \right) \]

The household has a fixed income stream \( \{Y_1, Y_2\} \). Capital markets are perfect. Both the households and government have perfect foresight. The stream of government expenditures \( \{G_1, G_2\} \) is fixed.

(a) Non-distortionary taxation (8 points)

The periodic budget constraints of the representative household are given by

period 1: \( A_1 + C_1 = (1+r)A_1 + (1-\tau_1)Y_1 \)

period 2: \( A_2 + C_2 = (1+r)A_2 + (1-\tau_2)Y_2 \)

The periodic budget constraints of the government read as follows

period 1: \( B_1 - B_0 = rB_0 + G_1 - \tau_1 Y_1 \)

period 2: \( B_2 - B_1 = rB_1 + G_2 - \tau_2 Y_2 \)

Derive the consolidated budget constraint of the whole economy and explain the intermediate steps concisely. Provide a clear economic interpretation of the result. Explain the economic logic behind the Ricardian equivalence proposition.

(b) Distortionary taxation (8 points)

Now assume that the government levies a (proportional) comprehensive income tax such that tax revenues read \( T_i = \tau_i (Y_i + rB_i) \) for \( i \in \{1, 2\} \). It can be readily shown that the consolidated budget constraint of the whole economy is then given by

\[ C_1 + \frac{C_2}{1+(1-\tau_2)r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+(1-\tau_2)r} \]

Determine the level of consumption in the first and in the second period. Additionally, also determine the level of household saving in the first period. Provide a concise economic interpretation of the result.

(c) Critical assumptions (4 points)

What are the critical assumptions underlying the Ricardian equivalence proposition? Is Ricardian equivalence likely to hold in reality? Provide a very concise reasoning.

(Notation: \( A_i \): household’s financial wealth in period \( i \in \{1, 2\} \); \( B_i \): government bonds; \( C_i \): consumption; \( Y_i \): household income; \( G_i \): government purchases; \( \tau_i \): tax rates; \( r > 0 \): constant interest rate; \( \rho \geq 0 \): time preference rate)
Exercise 2: Increasing returns to scale and competitive equilibrium (20 points)

(a) Increasing returns to scale in a standard neoclassical economy (4 points)

Consider a one-sector economy. There is continuum of length one of identical firms. Each firm has access to the following technology

\[ Y(i) = AK(i)^\alpha L(i)^\beta \quad \text{with} \quad A > 0; \, \alpha + \beta > 1; \, i \in [0,\ldots,1] \]

On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market.

Is a rewarding scheme according to the rule “factor price equals marginal productivity” possible in this economy? Provide a concise economic reasoning.

(b) Increasing returns to scale under external economies (6 points)

Consider a perfectly competitive, one-sector economy. There is continuum of length one of identical firms. Each firm has access to the following technology

\[ Y(i) = AK(i)^\alpha L(i)^\beta \bar{K}^a \bar{L}^b \quad \text{with} \quad A > 0; \, \alpha + \beta = 1; \, a, b > 0; \, i \in [0,\ldots,1] \]

where \( \bar{K} = \int_{i=0}^{1} K(i) di \) and \( \bar{L} = \int_{i=0}^{1} L(i) di \) denote the average (across firms) levels of capital and labor, respectively.

On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market.

Is a rewarding scheme according to the rule “factor price equals marginal productivity” possible in this economy? Provide a concise economic reasoning.

(c) Increasing returns to scale under imperfect product market competition (10 points)

The economy comprises two sectors. In the perfectly competitive final output sector there is mass one of identical firms. The output technology reads

\[ Y(j) = \left( \int_{i=0}^{1} x(i)^\lambda \right)^{\frac{1}{\lambda}} \quad \text{with} \quad 0 < \lambda < 1; \, j \in [0,\ldots,1] \]

In the monopolistically competitive intermediate goods sector there is mass one of identical firms. Each firm has access to the following technology

\[ x(i) = K(i)^\alpha L(i)^\beta \quad \text{with} \quad \alpha, \beta > 0; \, \alpha + \beta > 1; \, i \in [0,\ldots,1] \]

On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market. Households are the owners of the firms. Total earnings of the representative household are given by

\[ \text{Earnings} = \pi_{Y(j)} + \pi_{x(i)} + rK + wL \]

Factor markets are perfectly competitive.

Is a rewarding scheme according to the rule “factor price equals marginal productivity” possible in this economy? Provide a concise economic reasoning.

(Notation: \( Y(j) \): output of firm \( j \) in the final output sector; \( A \): constant technology parameter; \( K \): capital; \( L \): labor; \( x(i) \): output of firm \( i \) in the intermediate goods sector; \( \pi_{Y(j)} \): profits earned by the typical \( Y(j) \)-firm; \( \pi_{x(i)} \): profits earned by the typical \( x(i) \)-firm; \( r \): interest rate, \( w \): wage rate)
Exercise 3: New Classical Macroeconomics (20 points)

Part 1: Lucas imperfect-information model

The aggregate goods demand curve and the aggregate goods supply curve (Lucas supply curve) are given by

\[ y_t = m_t - p_t \]

\[ y_t = b\left[p_t - E(p_{t+1})\right] \]

Money supply may be expressed as follows

\[ m_t = m_{t-1} + c + u_t \]

(Notation: \( y_t \): logarithm of aggregate goods demand or goods supply in period \( t \); \( b, c > 0 \): constant coefficients; \( m_t \): logarithm of the stock of money; \( p_t \): logarithm of price level; \( u_t \): white noise error term, i.e. \( E(u_t) = 0 \) for all \( t \), \( V(u_t) = \text{const.} \) for all \( t \) and \( \text{Cov}(u_t, u_{t-1}) = 0 \) for all \( t \) and \( i \)).

(a) Macroeconomic equilibrium (5 points)

Determine the level of output and price level in equilibrium. Provide a concise economic interpretation of the result.

(b) Phillips-curve relationship and Lucas Critique (5 points)

Determine the implied equilibrium rate of inflation. Provide a concise economic interpretation with regard to the implied Phillips-curve relationship and Lucas Critique.

Part 2: Real business cycle theory

Consider a perfectly neoclassical, one-sector model economy. Final output firms have access to the following technology

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]

with \( A_t = A_0^p \cdot e^\epsilon \) where \( A_0 = \bar{A} \)

(Notation: \( Y_t \): output in period \( t \); \( A \): total factor productivity; \( \bar{A} > 0 \): a constant; \( K_t \): stock of capital; \( L_t \): labor input; \( 0 < p < 1 \): constant parameter; \( \epsilon_t \): white noise error term, i.e. \( E(\epsilon_t) = 0 \) for all \( t \), \( V(\epsilon_t) = \text{const.} \) for all \( t \) and \( \text{Cov}(\epsilon_t, \epsilon_{t-1}) = 0 \) for all \( t \) and \( i \); \( 0 < s < 1 \): saving rate)

(a) Output fluctuations (1) (3 points)

Assume that (i) \( K_t = K \), (ii) \( L_t = L \) for all \( t \) and \( p = 0 \). Question: Is this model able to reproduce (i) volatility in output fluctuations and (ii) persistent deviations of output from trend? Provide a concise economic reasoning.

(b) Output fluctuations (2) (5 points)

Assume now that (i) the saving rate \( s \) is constant and the depreciation rate is 100 percent; (ii) \( 0 < p < 1 \), and (iii) \( L_s = L_s(w/p) \) with \( \partial L_s/\partial (w/p) > 0 \). Question: Is this model able to reproduce (i) volatility in output fluctuations and (ii) persistent deviations of output from trend? Provide a concise economic reasoning.

(c) Critical discussion of RBC approach (2 points)

Discuss the RBC theory critically. To which extent is the RBC model not suitable to understand fluctuations in output and employment in the real world? What can nonetheless be learned?
Exercise 4: Research and development (R&D) in macroeconomic models (20 points)

Part 1: A static macroeconomic model with R&D

Consider a static macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[
\begin{align*}
\text{R&D} & : A = \eta L_a \\
\text{Intermediate goods} & : x(i) = k(i) \quad \text{for all } i \in \{0, \ldots, A\} \\
\text{Final output} & : Y = L_y^{\gamma-a} \int_{i=0}^{A} x(i)^{\alpha} \, di
\end{align*}
\]

(a) The decentralized allocation of labor (7 points)

Determine the amount of labor allocated to R&D, i.e. \(L_a\), in market equilibrium.

(b) The first-best allocation of labor (3 points)

Determine the socially optimal amount of labor allocated to R&D.

Part 2: The Romer (1990) model

Consider a dynamic macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[
\begin{align*}
\dot{A} & = \eta L_x A \quad \text{with } A(0) = A_0 \\
\text{Intermediate goods} & : x(i) = k(i) \quad \text{for all } i \in \{0, \ldots, A\} \\
\text{Final output} & : Y = L_y^{\gamma-a} \int_{i=0}^{A} x(i)^{\alpha} \, di
\end{align*}
\]

(a) Free entry into the R&D sector (3 points)

There is free entry into the R&D-sector. Assuming that the economy is in a steady state, the free-entry condition may be expressed as follows: \(\frac{\pi}{r} \leq \frac{w}{\eta A}\). Provide an economic interpretation of this condition.

(b) The steady state growth rate (7 points)

Equilibrium on the production side requires \(w_L = w_{R&D}\). Equilibrium on the consumer side is described by the Keynes-Ramsey rule: \(r = \sigma g + \rho\). It can be readily shown that equilibrium profits may be expressed as \(\pi = (1 - \alpha)\alpha L_y^{\gamma-a} x^\alpha\). Given these information, determine the steady state growth rate of final output \(Y\). Provide an economic interpretation of the result.

(Notation: \(A\): “number” of intermediate good types; \(\dot{x} = \frac{dx}{dt}\); \(\eta > 0\), \(0 < \alpha < 1\): constant technology parameter; \(L_a\): amount of labor devoted to R&D; \(L_y\): amount of labor devoted to \(Y\)-production; \(x(i)\): number of intermediate goods of type \(i\); \(k(i)\): capital used in \(x(i)\)-production; \(\pi\): profit of the typical \(x\)-producer; \(g\): steady state growth rate of \(Y, C, K\) and \(A\), where \(C\): consumption, \(K\): stock of capital; \(\alpha, \rho > 0\): constant preference parameters)
Exercise 5: Miscellaneous (20 points)

(1) Capital market equilibrium (5 points)

Consider a model economy with two assets. First, there is an equity share, its value is denoted as $v(t)$, which pays a dividend of $\pi(t)$ each period. Second, there is a bond which pays a (constant) rate of return of $r$. Capital market equilibrium requires that the following condition holds: $v(t) + \pi(t) = rv(t)$.

(a) Provide a concise economic interpretation of this capital market equilibrium condition.
(b) Explain the relationship between the capital market equilibrium condition and the following “solution” for $v(t)$: $v(0) = \int_{0}^{\infty} e^{-rt} \pi(t) dt$ (you are not expected to solve the above stated differential equation).

(2) International capital mobility (5 points)

International capital market integration did not induce capital movements from rich to poor countries as implied by standard neoclassical theory. Lucas (1990) discussed this observation in his paper entitled “Why doesn’t capital flow from rich to poor countries”.

(a) Describe the two basic possibilities that were discussed by Lucas in the above mentioned paper.
(b) How can imperfect property rights contribute to a better understanding of the so-called Lucas Paradox?

(3) Unemployment (5 points)

There are three major stylized facts of unemployment in advanced economies: (i) there is substantial and persistent unemployment; (ii) employment varies strongly procyclical over the business cycle; (iii) the wage rate varies only mildly procyclical over the business cycle.

(a) Sketch the basic idea, the major assumptions and the main implications of the Shapiro-Stiglitz model. (It may be helpful, though not necessary, to use a graphical exposition.)
(b) To what extent can the Shapiro-Stiglitz model explain the three stylized facts of unemployment mentioned above.

(4) Growth in small open economies (5 points)

It is well known that the Ramsey model runs into severe difficulties when it comes to explaining growth in the open economy context. The literature has produced a number of modifications of the basic model in order to avoid most of these implausible implications.

(a) Explain how capital market imperfections help to avoid some of the counterfactual implications of the Ramsey model.
(b) Describe the dynamic optimization problem of the representative firm under capital adjustment costs. How should the process of capital reallocation, in response to capital market integration, be described under capital adjustment costs?