**Universität Leipzig**  
**Wirtschaftswissenschaftliche Fakultät**  

**MASTER VWL – PRÜFUNG (FINAL EXAM)**

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<th>DATUM:</th>
<th>07.08.2012</th>
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<tr>
<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
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<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
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<td>PRÜFUNGS-NR.:</td>
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<td>STUDIENGANG:</td>
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<td>NAME, VORNAME:</td>
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**ERLÄUTERUNGEN (Explanation)**

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

**ZUGELASSENE HILFSMITTEL:** keine

**Punkte:**

| DATUM, UNTERSCHRIFT DES PRÜFERS: | |
Exercise 1: R&D-based growth - the Romer (1990) model (20 points)

Consider a dynamic macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[
\begin{align*}
\text{R&D} & : \dot{A} = \eta L_A A \quad \text{with} \quad A(0) = A_0 \\
\text{Intermediate goods} & : x(i) = k(i) \quad \text{for all} \quad i \in [0, \ldots, A] \\
\text{Final output} & : Y = L_A^{1-a} \int_{i=0}^{A} x(i)^{\sigma} \, di
\end{align*}
\]

(a) Consider the production technology \( \dot{A} = \eta L_A A \). Assume that \( L_A \) increases by 1 percent. By how much does, ceteris paribus, R&D output \( \dot{A} \) change in proportional terms?

(b) We assume that the Keynes-Ramsey rule holds, i.e. \( r = g + \rho \) (logarithmic preferences). Moreover, it can be shown that equilibrium profits of the typical intermediate goods producer is \( \pi_x = (1 - \alpha) \alpha L_A^{1-a} x^\alpha \). Assume further that there are R&D subsidies such that profits of the typical R&D firm is \( \pi_{R&D} = p_A A - (1-s_A) w_{R&D} L_A \). Given these information, determine the steady state growth rate of final output \( Y \).

(c) Assume that the government increases \( s_A \) from \( s_A = 0 \) to some \( 0 < s_A < 1 \). What happens to the steady state growth rate and intertemporal welfare? (The consideration should be restricted to the steady state. A qualitative discussion is sufficient.) Provide a concise economic reasoning.

(Notation: \( A \): “number” of intermediate good types; \( \dot{x} \) = \( dx/dt \); \( \eta > 0 \), \( 0 < \alpha < 1 \): constant technology parameter; \( L_A \): amount of labor devoted to R&D; \( L_Y \): amount of labor devoted to \( Y \)-production; \( x(i) \): number of intermediate goods of type \( i \); \( k(i) \): capital used in \( x(i) \)-production; \( \pi_x \): profit of the typical \( x \)-producer; \( \pi_{R&D} \): profit of the R&D firm; \( g \): steady state growth rate of \( Y \); \( C, K \) and \( A \), where \( C \): consumption, \( K \): stock of capital; \( \sigma, \rho > 0 \): constant preference parameters; \( 0 \leq s_A < 1 \): R&D subsidy parameter)
Exercise 2: Investment demand of firms (20 points)

Consider a firm that produces a homogenous final output good $Y_t$ under perfect competition. The output technology reads

$$Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}$$

The firm's planning horizon is infinity. There are investment costs (capital adjustment costs), denoted as $IC_t$. Installing the amount of $I_t$ additional capital goods (gross investment) requires the following amount of final output

$$IC_t = I_t \left[1 + \theta \left(\frac{I_t}{K_t}\right)^\eta\right]$$

It is assumed that the firm maximizes the present value of its cash flow (or entrepreneurial residual income) subject to a capital accumulation equation, $K_{t+1} = I_t + (1-\delta)K_t$, i.e. the firm solves the following dynamic problem

$$\max_{\{I_t\}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (Y_t - w_tL_t - IC_t)$$

s.t. $K_{t+1} = I_t + (1-\delta)K_t$

$$IC_t = I_t \left[1 + \theta \left(\frac{I_t}{K_t}\right)^\eta\right]$$

$K_0$ = given

(a) Consider the production technology $Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}$. Assume that $K_t$ increases by 1 percent. By how much does, ceteris paribus, $Y_t$ change in proportional terms?

(b) Determine the firm's investment demand (i.e. the demand for final output devoted to capital investment).

Remark: Investment demand will be a function $I_t = I_t(q_t)$, where $q_t$ denotes the shadow price of installed capital goods. (You are not requested to determine the difference equation which describes the dynamics of $q_t$.)

(c) Provide a sound economic interpretation of your result.

(Notation: $Y_t$: final output good at time $t \in \mathbb{N}$; $A > 0$: constant technology parameter; $K_t$: stock of physical capital at $t$; $L_t$: labor employed at time $t$; $0 < \alpha < 1$, $\theta \geq 0$, $\eta \geq 1$: constant technology parameter; $w_t$: denotes the wage rate, $r$: fixed interest rate, and $\delta \geq 0$: capital depreciation rate)
Exercise 3: Miscellaneous (20 points)

(1) Unemployment (10 points)

There are three major stylized facts of unemployment in advanced economies: (i) there is substantial and persistent unemployment; (ii) employment varies strongly procyclical over the business cycle; (iii) the wage rate varies only mildly procyclical over the business cycle.

(a) Sketch the basic idea, the major assumptions and the main implications of the Shapiro-Stiglitz model. (It may be helpful, though not necessary, to use a graphical exposition.)

(b) To what extent can the Shapiro-Stiglitz model explain the three stylized facts of unemployment mentioned above.

(2) New Keynesian Macroeconomics - sticky prices (10 points)

Consider a firm that acts under monopolistic competition and produces a (differentiated) consumption good \( c_{jt} \). The firm's planning horizon is infinity. The demand schedule and the production function are as follows:

\[
\text{demand schedule} \quad c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} Y_t
\]

\[
\text{production function} \quad c_{jt} = A L_{jt}
\]

where \( c_{jt} \) is the amount of consumption good \( j \) at time \( t \in \mathbb{N} \) (i.e. time is discrete), \( P_{jt} \) denotes the price of \( c_{jt} \), \( P_t \) a price index, \( \theta > 1 \) a constant parameter, \( Y_t \) household income, \( A_t > 0 \) a stochastic technology parameter, and \( L_{jt} \) the amount of labor employed by firm \( j \).

We assume that there are nominal price rigidities according to Calvo (1983). That is, each period there is a constant probability \( 0 \leq \omega \leq 1 \) that the firm is not allowed to adjust its goods price. It can be shown that the optimal supply price at time \( t=0 \) is then given by

\[
\text{optimal supply price} \quad p_{0t}^* = \frac{\theta}{\theta-1} \left( \frac{E_0 \sum_{i=0}^\infty \omega \Delta_{0,t} W_i / A_i \left( \frac{1}{P_i} \right)^\theta}{E_0 \sum_{i=0}^\infty \omega \Delta_{0,i} \left( \frac{1}{P_i} \right)^\theta} \right)
\]

(a) Consider the demand schedule. Assume that the individual goods price \( P_{jt} \) increases by 1 percent. By how much does, ceteris paribus, individual goods demand \( c_{jt} \) change in proportional terms?

(b) Provide a sound economic interpretation of the above stated optimal supply price \( p_{0t}^* \).

(c) Consider the limiting case \( \omega=0 \). How does the optimal supply price look like in this case? Provide a sound economic interpretation of this special result.

(Additional notation: \( E_0 \): expected value conditional on information at \( t=0 \); \( \Delta_{0,t} \): discount factor between period \( \theta \) and \( t \); \( W_i \): nominal wage rate)