Consider a firm that exists for two periods only (period $t = 0$ and period $t = 1$) and acts under monopolistic competition. It produces a differentiated consumption good $c_{jt}$. The demand schedule and the production technology are as follows

\[
demand\,\,schedule:\,\,c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} Y_t
\]

production technology: $c_{jt} = A_t L_{jt}$

where $c_{jt}$ denotes the amount of consumption good $j$ at time $t \in \{0, 1\}$ (i.e. time is discrete), $p_{jt}$ denotes the price of $c_{jt}$, $P_t$ a price index (and the price of the numeraire good $Y_t$), $\theta > 1$ a constant parameter, $Y_t$ household income, $A_t$ a stochastic technology parameter, and $L_{jt}$ the amount of labor employed by firm $j$.

We assume that there are nominal price rigidities according to Calvo (1983). That is, each period there is a constant probability $0 \leq \omega \leq 1$ that the firm is not allowed to adjust its goods price. Assuming that the firm can reset its goods price at $t = 0$, the firm’s problem may then be written as

\[
\max_{p_{j0}} \mathbb{E}\left[\frac{p_{j0} c_{j0}}{P_0} - MC_0 c_{j0} + \beta \omega \left(\frac{p_{j0} c_{j1}}{P_1} - MC_1 c_{j1}\right)\right]
\]

s.t. (1) and (2)

where $0 < \beta < 1$ denotes the discount factor, $MC_t$ the real marginal (equal to average) costs at time $t$, and $E_0$ expectations conditional on information available at time $t = 0$.

**Assignments**

1. Let $\frac{W_t}{P_t}$ denote the real wage. Describe the real marginal costs of $c_{jt}$ in terms of the numeraire good $Y_t$.

2. Assume the firm is allowed to set its goods price at $t = 0$. Determine this optimal goods price $p_{j0}$.

3. Discuss the similarities and the differences between the Calvo (1983) approach and the "convex price adjustment cost setup".

**Solution sketch**

**Question (a).** Producing one additional unit of $c_{jt}$ requires $1 = A_t L_{jt}$ or $L_{jt} = 1/A_t$ units of labor. The real marginal costs in terms of the numeraire good are then given by

\[
MC_{jt} = \frac{W_t}{P_t} \frac{1}{A_t}
\]

**Question (b).** The firm’s objective function (the expected present discounted value of profits) may be written as

\[
\text{obj} := E_0\left[\frac{p_{j0} \left(\frac{p_{j0}}{P_0}\right)^{-\theta}}{P_0} Y_0 + \frac{W_0}{A_0} \frac{1}{P_0} (\frac{p_{j0}}{P_0})^{-\theta} Y_0 + \beta \omega \left(\frac{W_1}{P_1} \frac{1}{A_1} \frac{p_{j0}}{P_1}^{-\theta} Y_1\right)\right]
\]

\[
\text{obj} := E_0\left[\frac{\left(\frac{p_{j0}}{P_0}\right)^{1-\theta}}{P_0} Y_0 - \frac{W_0}{A_0} \frac{1}{P_0} (\frac{p_{j0}}{P_0})^{1-\theta} Y_0 + \beta \omega \left(\frac{W_1}{P_1} \frac{1}{A_1} \frac{p_{j0}}{P_1}^{1-\theta} Y_1\right)\right]
\]

Forming the necessary first-order condition $\frac{\partial \text{obj}}{\partial p_{j0}} = 0$ for a profit maximum gives

\[
\frac{\partial \text{obj}}{\partial p_{j0}} = E_0\left[\left(1 - \theta\right) \left(\frac{p_{j0}}{P_0}\right)^{-\theta} Y_0 + \theta \frac{W_0}{P_0} A_0 (\frac{p_{j0}}{P_0})^{-\theta+1} Y_0 + \theta \frac{W_1}{P_1} A_1 (\frac{p_{j0}}{P_1})^{-\theta+1} Y_1\right] = 0
\]

Recall that the goal is to solve for $p_{j0}$. Using $E(x_t) = x_0$ and collecting marginal profit and marginal cost terms gives

\[
(1 - \theta) (p_{j0})^{-\theta} Y_0 (P_0)^{1-\theta} + \theta (1 - \theta) (p_{j0})^{-\theta+1} W_0 Y_0 (P_0)^{1-\theta} + \beta \omega (p_{j0})^{-\theta+1} E_0 \left[\frac{W_1}{A_1} Y_1 (P_1)^{1-\theta}\right] = 0
\]

Bringing the marginal cost terms to the RHS and dividing through by $(p_{j0})^{-\theta}$ gives
Finally, solving for $p_{j,0}$ gives

$$p_{j,0} = \frac{\theta}{\theta - 1} \frac{W_{0}}{A_{0}} \frac{y_{0}}{(P_{0})^{1-\theta}} + \beta \omega E_{0} \left( \frac{W_{1}}{A_{1}} - \frac{y_{1}}{(P_{1})^{1-\theta}} \right)$$

(10)

Notice that $\omega = 0$ (regime of flexible prices) implies an optimal goods price according to

$$p_{j,0} = \frac{\theta}{\theta - 1} \frac{W_{0}}{A_{0}}$$

(11)

which is the standard mark-up pricing result.

**Illustration: interior, unique solution for $p$**

Solution sketch (as simple as possible)