1 Different shocks in the RBC model

1.1 TFP shocks

Consider the following baseline RBC model:

$$\max_{\{C_t, L_t\}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + \eta(1 - L_t)]$$

s.t. $K_{t+1} = B_t K_t^\alpha L_t^{1-\alpha} + (1 - \delta) K_t - C_t$

$$B_t = B_{t-1} e^{\epsilon_t}$$

$K_0$ is given, $B_0 = 1$

$K_t \geq 0$, $\lim_{t \to \infty} \beta^t E_0 (\mu_t K_t) = 0$

a) Derive the first-order conditions and provide an economic interpretation.

One of the main problems of the standard RBC approach lies in the idea that business cycle fluctuations are driven predominantly by shocks to productivity, implying that there are both periods of technical progress ($\epsilon_t > 0$) as well as periods of technical regress ($\epsilon_t < 0$) (King and Rebelo, 1995, p. 930). Therefore, the next two exercises will introduce two alternative sources of economic fluctuations within the RBC setup.

1.2 Fiscal policy shocks

Instead of TFP-shocks consider fiscal policy shocks in the form of an exogenous and stochastic tax rate:

$$\max_{\{C_t, L_t\}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + \eta(1 - L_t)]$$
\[ K_{t+1} = (1 - \tau_t)K_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t - C_t + T_t \]

\[ (1 + \tau_t - \bar{\tau}) = (1 + \tau_{t-1} - \bar{\tau})e^{\epsilon_t} \]

\[ K_0 = \text{given} \]

\[ K_t \geq 0, \lim_{t \to \infty} \beta^t E_0(\mu_t K_t) = 0, \]

where \( \tau_t \) is a linear income tax rate and \( T_t = \tau_t K_t^\alpha L_t^{1-\alpha} \) is a lump sum transfer, viewed as being exogenous from the perspective of the individual agent. The innovation term is specified as \( \epsilon_t \sim N(0, \sigma) \). It is assumed that the realization of the innovation term is always sufficiently close to zero such that it always holds that \( 0 \leq \tau_t \leq 1 \). It can be verified that the steady state tax rate is equal to the model parameter \( 0 \leq \bar{\tau} \leq 1 \).

a) Derive the first-order conditions and provide an economic interpretation.

### 1.3 Oil price shocks

Consider now a small-open, producer-consumer economy described by the following economic structure:

\[
\max_{\{C_t, L_t\}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t + \eta(1 - L_t)]
\]

s.t. \( K_{t+1} = K_t^{\alpha_1} L_t^{\alpha_2} M_t^{1-\alpha_1-\alpha_2} + (1 - \delta)K_t - C_t - P_t M_t \)

\[ P_t = P_{t-1} e^{\epsilon_t} \]

\[ K_0 = \text{given}, \quad P_0 = \text{given} \]

\[ K_t \geq 0, \lim_{t \to \infty} \beta^t E_0(\mu_t K_t) = 0, \]

where \( M_t \) is crude oil, \( P_t \) the exogenously given price of oil, \( 0 < \gamma < 1, \epsilon_t \sim N(0, \sigma) \), and \( \alpha_1, \alpha_2 \in (0, 1), \alpha_1 + \alpha_2 < 1 \).

a) Derive the first-order conditions and provide an economic interpretation.

### 2 Convex Price Adjustment Costs (Rotemberg, 1982)

Consider a firm that produces a differentiated good \( X_{jt} \) under monopolistic competition. If the firm adjusts its supply price, it must bear convex price adjustment cost \( (PAC_{jt}) \) of
the form

\[ PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2 \]

where \( \bar{\pi} \) denotes the steady state inflation factor \( \frac{P_t}{P_{t-1}} \) (which is unity in case of zero inflation) and \( \phi > 0 \) a constant technology parameter. Each firm solves the following dynamic optimization problem

\[
\max_{\{p_{jt}\}} \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{p_{jt}}{P_t} - \lambda_t \right) X_{jt}^D - PAC_{jt} \right] \\
\text{s.t. } X_{jt}^D = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t \\
PAC_{jt} = \frac{\phi}{2} \left( \frac{p_{jt}}{p_{jt-1}} - \bar{\pi} \right)^2
\]

where \( p_{jt} \) denotes the price of good \( X_{jt} \) at time \( t \), \( P_t \) a price index, \( \lambda \) the real marginal cost, \( X_{jt}^D \) the demand for good \( j \), \( \epsilon > 1 \) constant parameter, and \( Y_t \) is aggregate income.

a) Sketch the optimal solution \( \{p_{jt}\} \) of this dynamic optimization problem by setting up the difference equation that governs the evolution of the sequence of optimal prices.

b) Determine the price in the steady state.

c) Discuss the similarities and the differences between this "convex price adjustment setup" and the Calvo (1983) approach.

3 Price setting a la Calvo (1983) - 2-Period Example

Consider a firm that exists for two periods only (period \( t = 0 \) and \( t = 1 \)) and acts under monopolistic competition. It produces a differentiated consumption good \( c_{j,t} \). The demand schedule and the production technology are as follows

\[
\text{Demand Schedule: } \quad c_{j,t} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} Y_t 
\]

\[
\text{Production technology: } \quad c_{j,t} = A_t L_{j,t}
\]
where \( c_{j,t} \) denotes the amount of consumption good \( j \) at time \( t \in \{0, 1\} \) (i.e. time is discrete), \( p_{j,t} \) denotes the price of \( c_{j,t} \), \( P_t \) a price index (and the price of the numeraire good \( Y_t \)), \( \theta > 1 \) a constant parameter, \( Y_t \) household income, \( A_t \) a stochastic technology parameter, and \( L_{j,t} \) the amount of labor employed by firm \( j \).

We assume that there are nominal price rigidities according to Calvo (1983). That is, each period there is a constant probability \( 0 \leq \omega \leq 1 \) that the firm is not allowed to adjust its goods price. Assuming that the firm can reset its goods price at \( t = 0 \), the firm’s problem may then be written as

\[
\max_{\{p_{j,0}\}} \mathbb{E}_0 \left[ \frac{p_{j,0} c_{j,0}}{P_0} - MC_{0} c_{j,0} + \beta \omega \left( \frac{p_{j,0} c_{j,1}}{P_1} - MC_{1} c_{j,1} \right) \right]
\]

s.t. eq. (1), eq. (2),

where \( 0 < \beta < 1 \) denotes the discount factor, \( MC_t \) the real marginal (equal to average) costs at time \( t \), and \( E_0 \) expectations conditional on information available at time \( t = 0 \).

a) Let \( \frac{W_t}{P_t} \) denote the real wage. Describe the real marginal costs of \( c_{j,t} \)-production in terms of the numeraire good \( Y_t \).

b) Assume the firm is allowed to set its goods price at \( t = 0 \). Determine this optimal goods price \( p_{j,0} \).

c) Discuss the similarities and the differences between the Calvo (1983) approach and the "convex price adjustment cost setup”.

4 A baseline New-Keynesian model

Consider a closed economy with mass one of identical households. Households supply labor, purchase goods for consumption, and hold bonds. Intertemporal utility is given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \frac{\chi}{2} N_t^2 \right],
\]

where \( \beta \) is the subjective discount factor, \( C_t \) is consumption, and \( \chi \) is a utility parameter reflecting the degree of disutility derived from working \( N_t \). Households maximize eq. (3)
subject to the budget constraint

\[ C_t + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + (1 + i_t) \frac{B_{t-1}}{P_{t-1}} + \Pi_t, \]

where \( B_t \) is the nominal holding of bonds, \( P_t \) is the aggregate price level, \( W_t \) is the nominal wage, \( i_t \) is the nominal interest rate, and \( \Pi_t \) are real profits from the intermediate goods sector. The composite consumption good \( C_t \) is given by a CES index

\[ C_t = \left( \int_0^1 c_{jt} \theta \frac{d\theta}{\theta} \right)^{\frac{\theta}{\theta-1}}, \]

where \( \theta > 1 \).

a) Derive the demand for \( c_{jt} \).

b) Derive the Euler equation and the intra-temporal optimality condition for leisure and consumption.

The firm sector consists of mass one of identical firms. Each firm hires labor to produce a differentiated good that is sold in monopolistically competitive markets under the production function \( c_{jt} = A_t N_{jt} \), where \( N_{jt} \) is the labor demanded by firm \( j \) in period \( t \) and \( A_t \) is a stochastic TFP variable with \( E_t(A_{t+1}) = 1 \). Each firm sets the price of the good it produces, but that is not possible in every period. The probability that a firm cannot adjust its price in the next period is given by \( \omega \) (Calvo pricing).

c) Derive the optimal price \( p^*_{jt} \).

d) Derive the New-Keynesian Phillips Curve.


Consider a closed economy with two groups of agents, gatherers and farmers. Both groups are of size one. The gatherers solve:

\[ \max U_0 = \sum_{t=0}^{\infty} \beta^t c_t, \]
subject to
\[ q_t(k_t - k_{t-1}) + R_{t-1}b_{t-1} + c_t = y_t + b_t, \]
where \( \beta \) is the subjective discount factor, \( q_t \) is the price of a non-depreciating production factor (e.g., “land”) in units of the consumption good, \( k_t \) is the amount of land owned in period \( t \), \( b_t \) are bonds (a loan contract signed in \( t \)), \( R_t \) is the real gross interest rate on the bonds, and \( y_t \) is income from production. Gatherers can produce a homogeneous final good in period \( t \) by purchasing land in period \( t - 1 \) according to the production function

\[ y_t = k_t^\alpha. \]

a) Derive the FOCs of the gatherer’s problem (consider only the interior solution).

Show that one FOC implies that the real interest rate \( R_t \) is constant and provide an economic interpretation for the other FOC.

The second set of agents, the farmers, have a different technology for the production of a homogeneous final good which employs land:

\[ y^f_t = (a + c)k^f_{t-1}, \]

where \( y^f_t \) denotes the goods produced by the farmer, \( k^f_t \) the land owned by the farmer. Only \( ak_t \) units of the output are tradable on markets. The farmers solve:

\[
\max U^f_0 = \sum_{t=0}^{\infty} (\beta^f)^t c^f_t,
\]

subject to
\[ q_t(k^f_t - k^f_{t-1}) + Rb^f_{t-1} + c^f_t = y^f_t + b^f_t, \]

and two inequality constraints

\[ Rb^f_t \leq q_{t+1}k^f_t \]
\[ c^f_t \geq ck^f_{t-1}, \]

where \( \beta^f < \beta \) is the farmers’ subjective discount factor and the superscript \( f \) indicates
that a variable refers to the farmer. Assume that \( c > \left( \frac{1}{\beta} - 1 \right) a \) such that both inequality constraints are always binding.

b) Write down the farmer’s land demand equation. How does i) a higher land price in \( t \), and ii) a higher land price in \( t + 1 \) affect the demand for land?\(^1\)

c) The available land in the economy is normalized to one. Write down the equilibrium conditions for the market for land and bonds. Who is a borrower and who is a creditor in this economy?

d) Determine the land owned by farmers and the price of land in the steady state.

e) Compare the social planner’s solution to the decentralized outcome in the steady state. Explain your result.

6 Rational asset price bubbles: The Tirole (1985) model

Consider a dynamic model of a closed economy with asset price bubbles. Time is discrete. On the household side there are overlapping generations. Each individual lives for two periods. There is no population and technological growth. The model is isomorph to the Diamond/OLG model from the lecture, except that besides capital \( k_t \) there is now a second asset. This second asset has a fundamental value of zero. When the price of this asset is strictly positive a bubble is present. Therefore, this asset is just labeled “bubble” and denoted by \( m_t > 0 \).

Households solve the following problem

\[
\max_{c_{1,t}, c_{2.t+1}, a_t, m_t} \ln(c_{1,t}) + \beta \ln(c_{2,t+1})
\]

\[
\text{s.t. } a_t + p_t m_t + c_{1,t} = w_t
\]

\[
c_{2,t+1} = (1 + r_{t+1})a_t + p_{t+1}m_t
\]

\[
a_t, m_t \geq 0
\]

where \( c_{1,t} \) is consumption of a young individual in period \( t \), \( c_{2,t+1} \) is consumption of an retired individual in period \( t + 1 \), \( 0 < \beta < 1 \) is the discount factor, \( a_t \) is the amount of

\(^1\)Hint: You do not need to prove that both inequality constraints are binding.
savings supplied to firms (capital), \( p_t \geq 0 \) is the price of the bubble, \( m_t \) is the amount of bubbles the household demands, \( w_t \) denotes the wage rate, and \( r_t \) the interest rate.

a) Derive the FOC to the household problem and determine the optimal total savings
\[ s_t = a_t + p_t m_t. \]
Provide a concise economic interpretation of your results.

b) Initial capital is given and strictly positive, i.e. \( k_0 > 0 \). By the definition of an equilibrium the supply of capital equals the demand of capital, i.e. \( k_{t+1} = a_t \), bubbles are freely disposable, i.e. \( p_t \geq 0 \), and bubbles cannot become too large, i.e. \( p_t m_t < s_t \). Provide an economic intuition for the necessary feasibility conditions on bubbles. What does the feasibility condition imply with regards to the solution of \( x_t \)?

c) The perfectly competitive firm sector is described by the production function \( y = Ak^\alpha \), where \( k \) is capital in per-worker units and \( A \) is a constant TFP parameter. Assume – for the sake of simplicity – that the supply of bubbles is constant and equal to one, such that \( m_t = 1 \) and that capital completely depreciates within one period, i.e. \( \delta = 1 \). Write down the reduced system of equations.

d) Determine the bubble-less steady state \((p_t = p_{t+1} = 0)\) and the bubbly steady state \((p_t = p_{t+1} > 0)\). Under which condition can a bubbly steady state exist?

e) Assume \( \alpha \frac{\beta}{\beta + 2 \gamma} \). Analyze the dynamics of the model with the help of a phase diagram. Given the bubble-less economy is dynamically inefficient, which trajectories satisfy the equilibrium conditions?

\[ ^{2} \text{Hint: Substitute } c_{1,t} \text{ and } c_{2,t+1} \text{ in the objective function by making use of the two budget constraints. Define the portfolio share } x_t \equiv \frac{a_t}{p_t} \text{ and substitute } a_t \text{ and } m_t \text{ in the objective function by the corresponding expressions with } x_t \text{ and } s_t. \text{ Realize that the optimization problem can be separated into two optimization problems, one with } s_t \text{ as the choice variable, one with } x_t. \text{ Finding the optimal } s_t \text{ is now identical to the optimization problem in the OLG model without bubbles. Take corner solutions into account when looking for the optimal } x_t. \]

\[ ^{3} \text{Hint: It consists of two difference equations in } k_t \text{ and } p_t, \text{ an initial value for } k, \text{ and a constraint on } p. \]