### ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden Aufgabe 1. und 2. gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, exercises 1. and 2. will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

### ZUGELASSENE HILFSMITTEL:

Keine

### Punkte:
Exercise 1: Ricardian Equivalence (20 points)

Private sector. Consider a neoclassical economy that is populated by mass one of identical households and lasts for two periods. Time is discrete and indexed by \( t \in \{1, 2\} \). The capital market is perfect and households have perfect foresight. Life-time utility of each household is given by

\[
U = \ln C_1 + \ln G_1 + \frac{1}{1 + \rho} (\ln C_2 + \ln G_2),
\]

where \( C_1 (C_2) \) denotes private consumption in period \( t = 1 (t = 2) \), \( G_1 (G_2) \) a public commodity in period \( t = 1 (t = 2) \) and \( \rho > 0 \) the discount rate, respectively. Each household earns a constant labor income in both periods \( (Y_1, Y_2) \). A linear tax rate is levied on total income, including capital income. The periodic budget constraints in period 1 and 2 may be expressed as

\[
S_1 + C_1 = (1 - \tau_1) Y_1
\]

\[
C_2 = S_1 + (1 - \tau_2) (Y_2 + rS_1),
\]

where \( S_1 \) denotes household savings in the first period, \( Y_1 (Y_2) \) labor income in period \( t = 1 (t = 2) \), \( 0 \leq \tau_1 \leq 1 \) \((0 \leq \tau_2 \leq 1)\) a linear income tax in period \( t = 1 (t = 2) \), and \( r > 0 \) the exogenous interest rate, respectively.

Government. There are two scenarios with regard to the financing of government expenditures \((G_1 \text{ and } G_2)\):

First, in the baseline scenario (balanced budget), the government finances its expenditures entirely by tax revenues, such that its periodic budget constraints read

\[
G_1 = \tau_1^BY_1
\]

\[
G_2 = \tau_2^B (Y_2 + rS_1),
\]

where \( 0 \leq \tau_1^B \leq 1 \) \((0 \leq \tau_2^B \leq 1)\) is the income tax rate in period \( t = 1 (t = 2) \).

Second, in the alternative scenario (mixed tax and debt finance), the government levies a lower income tax rate in the first period \( (\tau_1^A < \tau_1^B) \). Government expenditures \((G_1)\) are financed by tax revenues \((\tau_1^AY_1)\) and by issuing bonds \(B_1\). In the second period, the government levies an income tax rate \((\tau_2^A)\) such that it can finance \(G_2\) plus bond repayment according to \((1 + r)B_1\).

Assignments

(a) Assume that the household maximizes life-time utility. Determine the level of optimal private consumption in both periods \((C_1 \text{ and } C_2)\). (10 points)

(b) Write down the government’s periodic budget constraints that describe the alternative scenario (mixed tax and debt finance). (5 points)

(c) How does the sequence of optimal private consumption \(\{C_1, C_2\}\) differ between the baseline scenario – described by budget rules (1) and (2) – and the alternative scenario? A qualitative economic reasoning suffices. (5 points)
Exercise 2: A Baseline New-Keynesian Model (20 points)

**Households.** Consider a closed economy with mass one of identical households. Time is discrete and indexed by \( t \in \mathbb{N} \). Households derive utility from the consumption of a composite good \( C_t \) and disutility from labor \( N_t \) according to

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t),
\]

where \( \beta \in (0, 1) \) is the subjective discount factor. The function \( u(C_t, N_t) \) is increasing in \( C_t \) and decreasing in \( N_t \). The composite consumption good \( C_t \) is given by a CES index

\[
C_t = \left( \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},
\]

where \( \theta > 1 \).

**Firms.** The firm sector consists of mass one of identical firms producing differentiated goods that are sold in monopolistically competitive markets. The firms’ price-setting behavior is modeled by the Calvo (1983) approach. The problem of a typical firm is hence

\[
\max_{p_{jt}} E_0 \sum_{t=0}^{\infty} \omega^t \beta^t \left[ \frac{C_t}{C_0} \right]^{-1} \left[ \frac{p_{jt}}{P_t} c_{jt} - \lambda_t c_{jt} \right],
\]

where \( \lambda_t \) are marginal costs, \( p_{jt} \) is the price set by firm \( j \) in period 0, \( P_t \) is the aggregate price level, and \( \omega \) is the probability that the firm cannot adjust its price in the next period.

**Assignments**

(a) Derive the optimal demand for \( c_{jt} \) as a function of prices and the composite consumption good \( C_t \). (Hints: Minimize total consumption expenditures subject to a constant level of the composite consumption good \( C_t \). You can set the Lagrangian multiplier equal to the aggregate price level \( P_t \) without deriving it.) (7 points)

(b) Derive the optimal price, \( p^*_jt \), a firm chooses when it can adjust its price in period 0. (9 points)

(c) The central bank controls the nominal interest rate according to a Taylor rule. Explain briefly how an expansionary monetary policy shock is propagated within the model economy under reasonable parameter values. (4 points)
Exercise 3: Miscellaneous (20 points)

(1) Search and Matching Model (10 points)

Consider a closed economy where labor markets are frictional and employment relationships are long-lasting. Firms post vacancies to employ labor and rent capital in order to produce a numeraire good. The representative firm maximizes its profits by solving the following problem

$$\max_{N_0, K_0, v_0} \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[ Z_t K_t^\alpha N_t^{1-\alpha} - W_t N_t - R_t K_t - \kappa v_t \right]$$

s.t. \( N_t = (1 - \delta N) N_{t-1} + q(\theta_t) v_t \) for \( t = 0, 1, 2, \ldots \)

Notation: \( \beta, \lambda_t, \lambda_0 \): Stochastic discount factor; \( Z_t \): Total factor productivity; \( K_t \): Capital; \( N_t \): Employment; \( 0 < \alpha < 1 \): Production elasticity w.r.t. capital; \( W_t \): Wage rate; \( R_t \): Gross interest rate; \( \kappa > 0 \): Vacancy posting cost; \( v_t \): Vacancies; \( \delta N \): Labor separation rate; \( q(\theta_t) \): Vacancy filling rate; \( \theta_t \): Labor market tightness.

(a) Set up the Lagrangian of the above stated optimization problem and derive the first order conditions.

(b) Give a short and precise economic interpretation of the first order conditions.

(2) Optimal Saving and Dynamic Inefficiency (10 points)

The reduced form of the well-known Solow model may be expressed as follows

$$\dot{k}(t) = sk(t)^\alpha - \delta k(t) \quad \text{with} \quad k(0) = k_0,$$

where \( 0 < \alpha < 1 \) denotes a constant technology parameter, \( 0 \leq s \leq 1 \) the (time invariant) saving rate, \( \delta > 0 \) the depreciation rate, \( t \) a continuous time index, and \( k(t) \) the stock of physical capital per units of labor, respectively.

(a) Set up the optimization problem that allows to determine the saving rate according to the golden rule.

(b) For which range of the saving rate does dynamic inefficiency occur?

(c) Discuss very briefly whether dynamic inefficiency can occur (i) in the Ramsey growth model and (ii) in the overlapping generations (OLG) model?