Universität Leipzig
Wirtschaftswissenschaftliche Fakultät

MASTER VWL – PRÜFUNG (MIDTERM EXAM)

<table>
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<th>DATUM:</th>
<th>24.05.2017</th>
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<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
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<tr>
<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
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ERLÄUTERUNGEN (Explanation)

1) Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden Aufgabe 1. und 2. gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, exercises 1. and 2. will be scored.)

2) Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3) Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4) Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

ZUGELASSENE HILFSMITTEL: keine

Punkte:

| DATUM, UNTERSCHRIFT DES PRÜFERS: |                  |
Exercise 1: Small Open Economy under Capital Adjustment Costs (20 points)

Consider a small open economy, such that the interest rate \( r \) is constant. The size of the population is also constant and time is continuous. The firm sector consists of mass one of perfectly competitive firms producing a homogenous output good according to

\[
Y(t) = AK(t)^{\alpha}L^{1-\alpha},
\]

where \( K \) is capital, \( L \) is labor, \( A > 0 \) is constant total factor productivity (TFP), and \( \alpha \in (0,1) \). The firms’ planning horizon is infinity. There are investment costs, denoted as \( IC(t) \). Installing the amount of \( I(t) \) capital goods (gross investment) requires the following amount of final output

\[
IC(t) = I(t) \left( 1 + \frac{\theta}{2} \frac{I(t)}{K(t)} \right),
\]

with \( \theta > 0 \).

The firm maximizes the present value of its cash flow subject to a capital accumulation equation, \( \dot{K}(t) = I(t) - \delta K(t) \), where \( \delta > 0 \) denotes the depreciation rate. The firm hence solves the following dynamic problem

\[
\max_{\{I, L\}} \int_0^\infty \left[ Y(t) - w(t)L - IC(t) \right] e^{-\alpha} dt
\]

s.t. \( \dot{K}(t) = I(t) - \delta K(t) \), \( K(0) = K_0 \)

Assignments

1. Set up the first-order conditions and derive the investment demand as a function of the costate variable, denoted by \( q(t) \), and \( K(t) \).

2. Provide a short and concise economic interpretation of i) the costate variable, ii) its effect on the investment demand, and iii) the effect of \( K(t) \) on the investment demand.

3. Derive the steady state value of i) the investment ratio, \( I/K \), and ii) the costate variable. Provide a brief economic explanation for why the costate variable is greater than one.

Reminder: To set up the first-order conditions for an optimal solution requires to form the (current-value) Hamiltonian function, which may be stated as

\[
H(x(t), u(t), q(t)) := I[x(t),u(t)] + q(t)f[x(t),u(t)],
\]

where \( I[x(t),u(t)] \) denotes the instantaneous objective function, \( f[x(t),u(t)] \) the right-hand side of the dynamic constraint \( \dot{x}(t) = f[x(t),u(t)] \), \( x(t) \) the vector of state variables, \( u(t) \) the vector of control variables, and \( q(t) \) the vector of costate variables. The necessary first-order conditions may then be expressed as:

\[
\frac{\partial H}{\partial u(t)} = 0 \quad \forall \ t \in [0,\infty] \quad \text{and} \quad \dot{q}(t) = -\frac{\partial H}{\partial x(t)} + r q(t) \quad \forall \ t \in [0,\infty].
\]
Exercise 2: Capital income taxation in the Ramsey model (20 points)

Consider a neoclassical model economy of the Ramsey-type. Time is continuous, the time index $t$ is often suppressed to simplify the notation.

On the household side, there is mass one of identical households. Each household is endowed with $L > 0$ units of labor per period of time, which are supplied inelastically to the labor market, and possess $a(0) > 0$ units of initial wealth. Households can invest their wealth by purchasing bonds, which are issued by firms. The dynamic budget constraint of the individual household reads as follows

$$\dot{a} = ra + wL - c,$$

where $\dot{a} = \frac{da(t)}{dt}$, $r$ denotes the real interest rate, $w$ the wage rate per unit of labor, and $c$ consumption, respectively. Moreover, optimal household consumption, assuming logarithmic preferences, follows the well-known Keynes-Ramsey rule: $\dot{c} = c(r - \rho)$, where $\dot{c} = \frac{dc(t)}{dt}$.

On the firm side, there is mass one of identical firms. Each firm has access to a standard Cobb-Douglas production technology: $Y = K^\alpha L^{1-\alpha}$, where $Y$ denotes output, $K$ the stock of physical capital, $L$ the amount of employed labor, and $0 < \alpha < 1$ a constant parameter, respectively.

Thought experiment: Assume that the government introduces a linear capital income tax, which must be paid by households. This newly introduced tax rate is assumed to satisfy: $0 < \tau < 1$. Assume further that the tax revenues are repaid to the households in a lump sum manner.

Assignments

1. Analyze the long run effect of this tax policy on output $Y$ by calculating the steady state.

2. Provide a qualitative analysis of the dynamic effect resulting from this tax policy by employing an appropriate phase diagram.

3. What are the consequences of introducing a linear wage income tax? (Assume again that tax revenues are repaid to the households in a lump sum manner.) A brief verbal description together with a concise economic reasoning suffices.
Exercise 3: Miscellaneous (20 points)

(1) Intertemporal elasticity of substitution (10 points)

Consider the following intertemporal utility function

\[ U = \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + \omega \ln l_t \right], \]

where \( \beta \) is the subjective discount factor, \( c_t \) is consumption, \( l_t \) is leisure, \( \omega \) is a preference parameter for leisure, and \( t \) denotes the (discrete) time index, respectively.

(a) Derive the intertemporal elasticity of substitution in leisure.

(b) How does the growth factor of optimal leisure, \( \frac{l_{t+1}}{l_t} \), respond to a one percent increase in the current wage \( w_t \), assuming that \( w_{t+1} \) and \( r \) do not change?

Hints: The intertemporal elasticity of substitution in leisure is defined as

\[ IES_j = \frac{d \left( \frac{l_t}{l_k} \right) / \left( \frac{l_t}{l_k} \right)}{d \left( \frac{U_k}{U_j} \right) / \left( \frac{U_k}{U_j} \right)}, \]

where \( U_j = \frac{\partial U}{\partial l_j} \) for \( i \in \{j, k\} \), and \( k \neq i \) refer to two different periods of time. In the household optimum it holds that \( \frac{U_j}{U_{t+1}} = (1 + r) \frac{w_t}{w_{t+1}} \), where \( r \) is the interest rate.

(2) Capital market equilibrium (10 points)

Consider a model economy with two assets. First, there is an equity share, its value is denoted as \( v(t) \), which pays a dividend of \( \pi(t) \) each period. Second, there is a bond which pays a (constant) rate of return of \( r \). Capital market equilibrium requires that the following condition holds: \( \hat{v}(t) + \pi(t) = r \hat{v}(t) \).

(a) Provide a concise economic interpretation of this capital market equilibrium condition.

(b) Explain the relationship between the capital market equilibrium condition \( \hat{v}(t) + \pi(t) = r \hat{v}(t) \) and the following forward-looking solution: \( \hat{v}(0) = \int_{t=0}^{\infty} e^{-rt} \pi(t)dt \).

(c) Consider \( \hat{v}(0) = \int_{t=0}^{\infty} e^{-rt} \pi(t)dt \). How does the asset price \( \hat{v}(0) \) change, everything else the same, if \( r \) declines? Provide a concise economic interpretation.

(d) How does the solution \( \hat{v}(0) = \int_{t=0}^{\infty} e^{-rt} \pi(t)dt \) simplify provided that \( \pi(t) \) is time invariant?