(1) Die Klausur besteht aus sechs Aufgaben. Hiervon sind vier Aufgaben zu bearbeiten! Sollten Sie alle sechs Aufgaben bearbeiten, werden die ersten vier Aufgaben gewertet. (The exam consists of six exercises. Of these three exercises four exercises have to be edited. If you have edited all six exercises, the first four exercises will be scored.)

(2) Zur Bearbeitung stehen insgesamt 120 Minuten zur Verfügung. (To process the exam you have 120 minutes available.)

(3) Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

(4) Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore, take care of a neat writing in your own interest.)

ZUGELASSENE HILFSMITTEL: keine

Punkte:
Exercise 1: A baseline New-Keynesian Model (20 points)

Consider a closed economy with mass one of identical households. Households supply labor, purchase goods for consumption and hold bonds. Intertemporal utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \frac{\chi}{2} N_t^2 \right],$$

where $\beta$ is the subjective discount factor, $C_t$ is consumption, and $\chi$ is a utility parameter reflecting the degree of disutility derived from labor $N_t$. Households maximize intertemporal utility subject to the budget constraint

$$C_t + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t,$$

where $B_t$ is the nominal holding of bonds, $P_t$ is the aggregate price level, $W_t$ is the nominal wage, $i_t$ is the nominal interest rate, and $\Pi_t$ are real profits from the intermediate goods sector. The composite consumption good $C_t$ is given by a CES index

$$C_t = \left( \int_0^1 \frac{\theta-1}{\mu(t)} c^{\\theta-1} \mu(t) \theta dt \right)^{\frac{1}{\theta-1}},$$

where $\theta > 1$.

The firm sector consists of mass one of identical firms producing differentiated goods that are sold in monopolistically competitive markets. The price-setting behavior of firms is modelled by the Calvo (1983) approach. The problem of a typical firm is hence

$$\max_{p_{tj}} E_0 \sum_{t=0}^{\infty} \omega^t \beta^t \left[ C_t \right]^{-1} \left[ \frac{P_{j0}}{P_t} c_{\mu} - \lambda_t c_{\mu} \right],$$

where $\lambda_t$ are marginal costs, $p_{j0}$ is the price set by firm $j$ in period 0, and $\omega$ is the probability that the firm cannot adjust its price in the next period.

Assignments

1. Derive the optimal demand for $c_{\mu}^*$ as a function of prices and the composite consumption good $C_t$. (7 points)

2. Derive the optimal price $p_{j0}^*$. (7 points)

3. Assume the central bank controls the nominal interest rate according to a Taylor rule. Explain briefly how an expansionary monetary policy shock is propagated within the model under reasonable parameter values. How would your results change if $\omega \rightarrow 0$? (6 points)
Exercise 2: The Tirole (1985) model (20 points)

Consider a dynamic model of a closed economy with asset price bubbles. Time is discrete. On
the household side there are overlapping generations. Each individual lives for two periods.
There is no population and technological growth. Individuals can purchase also a second asset
besides capital $k_t$. This second asset has a fundamental value of zero. A bubble is present
whenever the second asset’s price is strictly positive. Therefore, this asset is just labeled
“bubble” and denoted by $m_t > 0$.

Households solve the following problem

$$
\max_{c_{y,t}, c_{o,t+1}, a_t, m_t} \ln(c_{y,t}) + \beta \ln(c_{o,t+1})
$$

s.t. $a_t + p_t m_t + c_{y,t} = w_t$

$$
c_{o,t+1} = (1 + r_{t+1})a_t + p_{t+1}m_t
$$

$$
a_t, m_t \geq 0,
$$

where $c_{y,t}$ is consumption of a young individual in period $t$, $c_{o,t+1}$ is consumption of an retired
individual in period $t+1$, $0 < \beta < 1$ is the discount factor, $a_t$ is the amount of savings supplied
to firms (capital), $p_t \geq 0$ is the price of the bubble, $m_t$ is the amount of bubbles demanded by
the household, $w_t$ denotes the wage rate, and $r_t$ is the interest rate.

Assignments

1. Derive the FOC of the household problem and determine optimal total savings $s_t = a_t + p_t m_t$. Provide a concise economic interpretation of your results. (6 points)

2. In a market equilibrium it holds that i) the supply of capital equals the demand of
capital, i.e. $k_{t+1} = a_t$, ii) bubbles are freely disposable, i.e. $p_t \geq 0$, and iii) bubbles are
not too large, i.e. $p_t m_t < s_t$. What would a violation of condition iii) imply? Write
down the resulting no-arbitrage condition and explain briefly why it has to be
satisfied. (4 points)

3. The perfectly competitive firm sector is described by the production function $y_t = k_t^\alpha$, where $k_t$ is capital in per-worker units. It further holds that $m_t = \delta = 1$, where $\delta$
denotes the depreciation rate of capital. Write down the reduced system of equations
which represents the whole economy (hint: two difference equations in two variables).
(4 points)

4. Determine the bubble-less steady state ($p_t = p_{t+1} = 0$) and the bubbly steady state
($p_t = p_{t+1} > 0$). State the necessary condition for the existence of a bubbly steady state
in a market equilibrium. (6 points)
Exercise 3: Productive government expenditures - Barro (1990) model (20 points)

Consider a perfectly competitive economy. There is a continuum of length one of identical firms, indexed by \( i \in [0,1] \). Output technology of firm \( i \) is given by

\[
Y_i = G^\beta K_i^\alpha L_i^{1-\alpha},
\]

where \( Y_i \) denotes final output produced by firm \( i \), \( G \) a public input (productive government expenditures), \( K_i \) the stock of physical capital employed by firm \( i \), \( L_i \) the amount of labor employed by firm \( i \), and \( 0<\alpha,\beta<1 \).

It is assumed that the government runs a balanced budget, i.e.

\[
G = \tau Y,
\]

where \( 0<\tau<1 \) denotes a linear tax rate and \( Y = \frac{1}{1} \int_0^1 Y_i \, di \) denotes aggregate income, respectively.

(a) Set up, by eliminating \( G \), the reduced-form output technology \( Y=F(K,L) \) and determine the degree of returns to scale in the private input factors \( K \) and \( L \). (5 points)

(b) Determine the competitive wage rate and the competitive interest rate in this economy. (5 points)

(c) Assuming that the Keynes-Ramsey rule holds, how does the growth rate of consumption change in response to \( \tau \)? Provide a concise and model-based economic reasoning. (10 points)
Exercise 4: Investment Demand of Firms (20 points)

Consider a firm that produces a homogenous final output good $Y_t$ under perfect competition. The output technology, which satisfies the Inada conditions, reads

$$Y_t = F(K_t, L_t),$$

where $t$ denotes the discrete time index. The firm's planning horizon is infinity. There are investment costs (capital adjustment costs), denoted as $IC_t$. Installing the amount of $I_t$ capital goods (gross investment) requires the following amount of final output

$$I_t = I_t \left[ 1 + G \left( \frac{I_t}{K_t} \right) \right]$$

with $G \left( \frac{I_t}{K_t} \right) |_{K_t=0} = 0$, $G' \left( \frac{I_t}{K_t} \right) |_{K_t=0} > 0$, $G'' \left( \frac{I_t}{K_t} \right) > 0$.

It is assumed that the firm maximizes the present value of its cash flow (or entrepreneurial residual income) subject to a capital accumulation equation, $K_{t+1} = I_t + (1 - \delta)K_t$, i.e. the firm solves the following dynamic problem

$$\max_{\{I_t, K_t\}} \sum_{t=0}^{\infty} \frac{1}{1+r} \left( Y_t - w_tL_t - IC_t \right)$$

s.t. $K_{t+1} = I_t + (1 - \delta)K_t$

$$IC_t = I_t \left[ 1 + G \left( \frac{I_t}{K_t} \right) \right]$$

$$Y_t = F(K_t, L_t)$$

$K_0$ given.

(a) Consider, as an example, the parameterized production technology $Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$. Assume that $K_t$ increases by 1 percent. By how much does, ceteris paribus, $Y_t$ change in proportional terms? (2 points)

(b) Set up the first-order conditions and provide a concise economic interpretation. (12 points)

(c) Explain concisely the merits of the capital adjustment cost model as contrasted to the standard Ramsey growth model. (6 points)

Notation: $Y_t$: final output good at time $t \in \mathbb{N}$; $K_t$: stock of physical capital at $t$; $L_t$: labor employed at time $t$; $IC_t$: investment costs; $w_t$: wage rate, $r$: fixed interest rate, and $\delta \geq 0$: capital depreciation rate
Exercise 5: Miscellaneous (20 points)

(1) Intertemporal elasticity of substitution (10 points)
Consider the following intertemporal utility function

\[ U = \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + \omega \ln l_t \right], \]

where \( \beta \) is the subjective discount factor, \( c_t \) is consumption, \( l_t \) is leisure, and \( \omega \) is a preference parameter for leisure.

(a) Derive the intertemporal elasticity of substitution of consumption. How does relative consumption react to a one percent increase in the gross rate-of-return \((1 + r)\)?

\[ IES_k = \frac{\frac{d}{d\ln c_k} \left( \frac{c_i}{c_k} \right)}{\frac{d}{d\ln U} \left( \frac{U_k}{U_i} \right)} \]

where \( U_k = \frac{\partial U}{\partial c_k} \), and \( k \neq i \) refer to two different periods of time.

(2) Logistic difference equation (10 points)
Consider the following non-linear, one-dimensional, first order difference equation

\[ x_{t+1} = \lambda x_t (1 - x_t), \]

where \( \lambda > 1 \) is a constant parameter.

(a) Determine the steady state(s).

(b) Assume \( x_0 = \frac{1}{6} \) and \( \lambda = \frac{3}{2} \). Depict the transition of \( x \) with the help of a diagram with \( x_t \) on the horizontal and \( x_{t+1} \) on the vertical axis.
Exercise 6: Miscellaneous (20 points)

(1) Capital mobility in a small, open economy (10 points)

Consider a perfectly competitive, one-sector, small, open economy with (aggregate) final output technology

\[ Y = AK^\alpha L^{1-\alpha} \]

where \( A > 0 \) denotes total factor productivity (TFP), \( K \) the stock of capital, \( L \) the total amount of labor input, and \( 0 < \alpha < 1 \) a constant technology parameter.

The model describes a static economy, i.e. there is no continuous capital accumulation. However, the economy has free access to the international capital market and hence capital may flow into or out of the economy. The international real interest rate, denoted as \( r \), is constant. Labor is immobile.

(a) Assume that TFP (\( A \)) increases permanently by 10 percent. By how much does GDP (\( Y \)) increase under capital market integration, assuming that the real interest rate \( r \) remains constant?

(b) Provide a concise economic interpretation of your result.

(2) R&D-based growth without scale effects (10 points)

Consider a simple economy which produces a final output good \( Y \) according to the aggregate output technology \( Y = AL_Y^\alpha \), where \( A \) is a (potentially) time-varying technology parameter, \( 0 < \alpha < 1 \) a constant technology parameter, and \( L_Y \) the amount of labor employed in \( Y \)-production. The R&D technology reads as follows

\[ \dot{A} = \eta A^\phi L_A^\gamma \]

Where \( \dot{A} \equiv dA/dt \), \( \eta > 0 \), \( 0 < \phi < 1 \), \( 0 < \gamma \leq 1 \) and \( L_A \) denotes the amount of labor employed in the R&D sector. The labor market is assumed to clear at each point in time, i.e. \( L = L_A + L_Y \). Total labor supply grows at an exogenous growth rate, i.e. \( L = Loe^{nt} \), where \( n \geq 0 \) and \( t \in \mathbb{R} \) denotes the continuous time index.

(a) Determine the steady state growth rate of final output \( Y \).

(b) Is public policy effective with respect to controlling the long-run growth rate? Provide a brief reasoning for your answer.