Universität Leipzig  
Wirtschaftswissenschaftliche Fakultät

MASTER VWL – PRÜFUNG (MIDTERM EXAM)

<table>
<thead>
<tr>
<th>DATUM:</th>
<th>17.05.2016</th>
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</thead>
<tbody>
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<td>MODUL:</td>
<td>ADVANCED MACROECONOMICS</td>
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<td>PRÜFER:</td>
<td>PROF. DR. THOMAS STEGER</td>
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</tbody>
</table>

ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

ZUGELASSENE HILFSMITTEL: **keine**

Punkte:

| DATUM, UNTERSCHRIFT DES PRÜFERS: |          |
Exercise 1: Ramsey growth model (20 points)

Consider an infinitely lived consumer-producer agent (“Robinson Crusoe”) with intertemporal preferences over consumption as given by

\[ U(t) = \int_{t=0}^{\infty} \ln[c(t)] \cdot e^{-\rho t} dt, \]

where \( c(t) \) denotes consumption at time \( t \in \mathbb{R} \) and \( \sigma, \rho > 0 \) constant preference parameters.

The available production technology has the following shape

\[ y(t) = A k(t)^{\alpha}, \]

where \( y(t) \) denotes final output, \( k(t) \) the stock of physical capital, and \( A > 0 \) and \( 0 < \alpha < 1 \) constant technology parameters, respectively. Final output can either be consumed or used to build up the stock of capital. Capital depreciates at constant rate \( \delta > 0 \). Hence, the stock of capital evolves according to

\[ \dot{k}(t) = A k(t)^{\alpha} - \delta k(t) - c(t) \quad \text{with} \quad k(0) = \text{given}. \]

Assignments

1. Determine the Keynes-Ramsey rule of optimal consumption growth and provide a concise economic interpretation.

2. Determine the (unique) interior steady state \((\tilde{k}_1 > 0, \tilde{c}_1 > 0)\) and the boundary steady state \((\tilde{k}_2 > 0, \tilde{c}_2 = 0)\) analytically.

3. Draw a phase diagram that displays the interior steady state \((\tilde{k}_1 > 0, \tilde{c}_1 > 0)\) and the boundary steady state \((\tilde{k}_2 > 0, \tilde{c}_2 = 0)\). Show that \((\tilde{k}_2 > 0, \tilde{c}_2 = 0)\) cannot represent the endpoint of an optimal consumption path.

Reminder: To set up the first-order conditions for an optimal solution requires to form the (current-value) Hamiltonian function, which may be stated as

\[ H[x(t), u(t), \mu(t)] := I[x(t), u(t)] + \mu(t) f[x(t), u(t)], \]

where \( I[x(t), u(t)] \) denotes the instantaneous objective function, \( f[x(t), u(t)] \) the right-hand side of the dynamic constraint \( \dot{x}(t) = f[x(t), u(t)] \) and \( \mu(t) \) the current-value shadow price. The necessary first-order conditions may then be expressed as:

\[ \frac{\partial H(\cdot)}{\partial u(t)} = 0 \quad \forall \ t \in [0, \infty], \]

\[ \dot{\mu}(t) = -\frac{\partial H(\cdot)}{\partial x(t)} + \rho \mu(t) \quad \forall \ t \in [0, \infty], \quad \text{and} \quad \lim_{t \to \infty} e^{-\rho t} \mu(t) k(t) = 0. \]
Exercise 2: Dixit-Stiglitz (1977) model (20 points)

Consider a static economy with imperfect product market competition. There are two sectors. The final output sector produces a final output, denoted as $X$, under perfect competition by employing a set of intermediate goods, denoted as $x_i$ with $i \in \{1, \ldots, N\}$. The intermediate goods sector employs labor $L_i$ to produce intermediate goods $x_i$ under monopolistic competition. Production technologies are as follows

\[
\text{Final Output: } X = \sum_{i=1}^{N} x_i^\alpha \text{ with } 0 < \alpha < 1
\]

\[
\text{Intermediate Goods: } x_i = L_i \text{ for all } i \in \{1, \ldots, N\}
\]

On the household side there is a continuum of mass one of identical households. Every household is endowed with $L$ units of time which are supplied inelastically to the labor market. Total labor supply is hence given by $L$. Total labor demand may be expressed as $\sum_{i=1}^{N} L_i$.

Assume further that there is an infinite number of possible varieties $x_i$. Setting up a new intermediate goods firm is associated with fixed costs $F > 0$ in units of $X$, implying that profits of the intermediate goods firm read: $\pi_i = p_i x_i - w L_i - F$.

Assignments

1. Determine the number of $x_i$ firms, denoted as $N$, in equilibrium. Discuss how $N$ depends on $F$. Provide a concise economic interpretation.

2. How does production of the final output good $X$ depend on the number of firms $N$? Provide a concise economic interpretation.
Exercise 3: Miscellaneous (20 points)

(1) Optimal saving and dynamic inefficiency (10 points)

The reduced form of the well-known Solow model may be expressed as follows

$$\dot{k}(t) = sk(t)^{\alpha} - (n+x)k(t) \quad \text{with} \quad k(0) = k_0,$$

where $0 < \alpha < 1$ denotes a constant technology parameter, $0 < s < 1$ the (time invariant) saving rate, $n \geq 0$ the population growth rate, $x \geq 0$ the rate of technical progress, $t$ a continuous time index, $\dot{k}(t) := dk(t)/dt$, and $k(t)$ the stock of physical capital per effective units of labor (i.e. $k := K/AL$), respectively.

(a) For which range of the saving rate $s$ does dynamic inefficiency occur?

(b) Discuss concisely whether dynamic inefficiency can occur (i) in the Ramsey growth model and (ii) in the overlapping generations (OLG) model? Provide a concise economic reasoning.

(2) The OLG model (10 points)

Consider a dynamic model of a perfectly competitive, closed economy. On the household side there are overlapping generations. Time is discrete. Each individual lives for two periods ("youth" and "retirement") only. In the first period an individual supplies inelastically one unit of labor earning labor income $w_t$. A share of that income is consumed. The rest is saved in order to finance consumption in the second period. Optimal savings $s_t$ are given by

$$s_t = \frac{\beta}{1+\beta} w_t,$$

where $0 < \beta < 1$ is the discount factor.

The perfectly competitive firm sector is described by the production function

$$Y_t = A[\alpha K_t + (1-\alpha)L],$$

where $K_t$ is the capital stock, $L > 0$ is labor, $A > 0$ is a constant TFP parameter, and $0 < \alpha < 1$. Production factors are rewarded with their marginal products.

(a) In equilibrium it must hold that $s_t = k_{t+1}$. Given this information, solve the model further such that it reduces to one difference equation in only one variable, $k_t$.

(b) Calculate the steady state, denoted as $\bar{k}$.

(c) Assume that the initial capital stock $k_0$ is smaller than the steady state capital stock, i.e. $k_0 < \bar{k}$. Illustrate the dynamics of the model in a graph with $k_t$ on the horizontal and $k_{t+1}$ on the vertical axis.