## ERLÄUTERUNGEN (Explanation)

1. Die Klausur besteht aus drei Aufgaben. Hiervon sind zwei Aufgaben zu bearbeiten! Sollten Sie alle drei Aufgaben bearbeiten, werden die ersten zwei Aufgaben gewertet. (The exam consists of three exercises. Of these three exercises two exercises have to be edited. If you have edited all three exercises, the first two exercises will be scored.)

2. Zur Bearbeitung stehen insgesamt 60 Minuten zur Verfügung. (To process the exam you have 60 minutes available.)

3. Sie können die Klausur entweder in deutscher oder englischer Sprache beantworten. (You can answer the exam either in German or English.)

4. Gewertet werden kann nur jener Teil der Antworten, der in angemessener Zeit entzifferbar ist. Achten Sie daher in eigenem Interesse auf eine klare Schrift. (Only the part of the answers, which is legible in a reasonable time, can be considered. Therefore take care of a neat writing in your own interest.)

## ZUGELASSENE HILFSMITTEL: keine

## Punkte:

| DATUM, UNTERSCHRIFT DES PRÜFERS: |  |  |  |
Exercise 1: The Kiyotaki-Moore (1997) model (20 points)

Consider a closed economy with two groups of agents, gatherers and farmers. Both groups are of measure one. Gatherers solve:

\[
\max_{\{c_t\}} U_0 = \sum_{t=0}^{\infty} \beta^t c_t,
\]

s.t. \( q_t (k_t - k_{t-1}) + R_t^c b_{t-1} + c_t = y_t + b_t \)

where \( \beta \) denotes the subjective discount factor, \( q_t \) the price of land, \( k_t \) the amount of land owned in period \( t \), \( b_t \) bonds, \( R_t \) the real gross interest rate on bonds, and \( y_t \) the income from production. Gatherers can produce a homogeneous final good in period \( t \) by purchasing land in period \( t-1 \) according to the production function:

\[ y_t = k_{t-1}^\alpha \quad \text{with} \quad 0 < \alpha < 1. \]

Farmers can produce the same good but face a different production technology:

\[ y_t^f = (a + c)k_t^\alpha, \]

where \( y_t^f \) denotes goods produced by farmers and \( k_t^\alpha \) land owned by farmers. Only \( a k_{t-1}^\alpha \) units of the output are tradable. Farmers solve:

\[
\max_{\{c_t\}} U_0^f = \sum_{t=0}^{\infty} (\beta^t)^f c_t^f,
\]

s.t. \( q_t (k_t^f - k_{t-1}^f) + R b_{t-1}^f + c_t^f = y_t^f + b_t^f, \)

\[ R b_{t}^f \leq q_{t+1}^f k_{t-1}^f \]

\[ c_t^f \geq c k_{t-1}^f \]

where \( \beta^f < \beta \) denotes farmer’s subjective discount factor and superscript \( f \) indicates that a variable refers to farmers. Assume \( c > \left( \frac{1}{\beta^f} - 1 \right) a \) such that both inequality constraints are always binding.

Assignments

1. Derive the first-order conditions (FOC) of the gatherer's problem (consider only the interior solution). Show that one FOC implies that the real interest rate \( R_t \) is constant and provide an economic interpretation for the other FOC.
2. Explain briefly the assumptions in Kiyotaki and Moore (1997) that give rise to the farmer’s borrowing constraint.
3. Write down the farmer's land demand equation. (Hint: Just claim – without proving – that both inequality constraints are binding.)
4. Land is normalized to one. Write down the equilibrium conditions for the land market and the bond market. Who is a borrower and who is a lender in this economy?
Exercise 2: R&D-based growth - the Romer (1990) model (20 points)

Consider a dynamic macroeconomic model with R&D. On the production side this economy comprises three sectors: (i) a perfectly competitive R&D sector; (ii) a monopolistically competitive intermediate goods sector; and (iii) a perfectly competitive final output sector. The respective output technologies are as follows:

\[ \dot{A} = \eta L_A A \quad \text{with} \quad A(0) = A_0 \]

Intermediate goods \[ x(i) = k(i) \quad \text{for all} \quad i \in [0, \ldots, A] \]

Final output \[ Y = L_Y^{-a} \int_{i=0}^{A} x(i)^{\alpha} \, di \]

Assignments

1. Consider the production technology \( \dot{A} = \eta L_A A \). Assume that \( L_A \) increases by 1 percent. By how much does, ceteris paribus, R&D output \( \dot{A} \) change in proportional terms?

2. Determine the steady state growth rate of final output \( Y \), assuming that (i) the Keynes-Ramsey rule holds, i.e. \( r = g + \rho \) (logarithmic preferences); (ii) equilibrium profits of the typical intermediate goods producer are \( \pi_x = (1 - \alpha) a L_Y^{-a} x^\sigma \); and (iii) there is an R&D subsidy such that profits of the typical R&D firm read \( \pi_{\text{R&D}} = p_A \dot{A} - (1 - s_A) w_{\text{R&D}} L_A \).

3. The government increases \( s_A \) from \( s_A = 0 \) to some \( 0 < s_A < 1 \). What happens to the steady state growth rate and intertemporal welfare? (A verbal discussion is sufficient.) Provide a concise economic reasoning.

Notation: \( \eta \): constant technology parameter; \( L_A \): amount of labor devoted to R&D; \( L_Y \): amount of labor devoted to \( Y \)-production; \( x(i) \): number of intermediate goods of type \( i \); \( k(i) \): capital used in \( x(i) \)-production; \( \pi_x \): profit of the typical \( x \)-producer; \( \pi_{\text{R&D}} \): profit of the R&D firm; \( g \): steady state growth rate of \( Y, C, K \), and \( A \); where \( C \): consumption, \( K \): stock of capital; \( \sigma, \rho > 0 \): constant preference parameters; \( 0 \leq s_A < 1 \): R&D subsidy parameter.
Exercise 3: Miscellaneous (20 points)

(1) Oil price shocks in the RBC model (10 points)

Consider a small open producer-consumer economy where a final output good is produced by employing the production factors capital and crude oil. The only source of uncertainty is the oil price which is modelled as an exogenous stochastic variable. The representative agent maximizes expected intertemporal utility

$$\max_{\{C_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \ln C_t$$

s.t.  

$$K_{t+1} = K_t^\alpha M_{t+1}^{1-\alpha} + (1-\delta)K_t - C_t - P_t M_t, \quad K_0 = \text{given}.$$ 

**Notation:** 

- $C_t$ denotes consumption of the output good, 
- $\beta$ the subjective discount factor, 
- $K_t$ capital, 
- $M_t$ crude oil, 
- $\alpha < 1$ a constant technology parameter, 
- $\delta \geq 0$ the capital depreciation rate, and 
- $P_t$ the oil price.

(a) Derive the stochastic Euler equation and the demand for crude oil.

(b) Explain briefly how an increase in the oil price shock induces a recession.

(2) The Fisher Equation (10 points)

Consider a model economy with two assets. First, there is an equity share, its value is denoted as $v(t)$, which pays a dividend of $\pi(t)$ each period. Second, there is a bond which pays a (constant) rate of return of $r$.

(a) Set up the capital market equilibrium condition or, equivalently, the condition which describes the absence of arbitrage opportunities. Provide a concise economic interpretation of this capital market equilibrium condition.

(b) Explain the relationship between the capital market equilibrium condition and the forward-looking solution: 

$$v(0) = \int_{-\infty}^{0} e^{-r} \pi(t) dt.$$ 

(You are not expected to solve the differential equation which describes capital market equilibrium.)