TWO PUZZLES OF JUDICIAL WAGERS

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• Introduction
• A model of judicial wagers
  • Wagers without signaling
  • Wagers with signalling
• Conclusion
Yājñavalkya Smṛti:

\[ sapaṇaś ced vivādaḥ syāt tatra hīnaṁ tu dāpayet \]
\[ daṇḍaṁ ca svapaṇaṁ caiva dhanine dhanam eva ca \]

If the dispute should be with a wager, then he should make the defeated party pay the fine and his own wager as well, but only the contested amount to its owner.

Inconclusive findings by Lariviere. But

- The wager may have been placed by one or by both parties.
- The recipient might have been the king (the court), the opponent, or even both.
- The size of the wager seems not to have been fixed and was probably up to each party.
Lariviere: “The paṇa seems … not to be a factor at all in deciding the case … .”

A Lariviere king would simply ignore the wagers placed by the parties and decide on the evidence available to him. In that case, the parties do not have any incentive to offer a non-zero wager.

- If the ruling goes in their favor, they do not have to pay the wager.
- If the ruling goes against them, they lose the case and have to pay the wager as an additional fine.

But, matters are even worse: The king is tempted to rule against a party that has placed a wager. Hence, the wager-placing party doubly loses.

- First, it increases the possibility of a negative ruling.
- Second, it loses the wager.
INTRODUCTION: THE SCARCE-EVIDENCE PUZZLE

Lariviere:

- the texts “point out what should be an important point in the general description of legal procedure
- two categories
- just the sort of thing which one would expect to find often repeated (or at least alluded to) in other basic smṛtis
- but these three verses are the only ones that we find in the whole corpus of dharma-śāstra
- This is unusual.
- It might not be so unusual if the verses gave a thorough and complete description of the pana, but that is hardly the case.
- In both texts, ... the verses ... are found with a hodge-podge of more or less unconnected and general statements about legal procedure.
WAGERS WITHOUT SIGNALING: THE TRIAL

- person $d$ (defendant)
- accused by some other person $a$ (accuser)
- of not paying back a loan $x$.
- wagers $w_d$ and $w_a$ from $\{0, w\}$ with wager combination $(w_d, w_a)$
- If party $i$ ($i \in \{d, a\}$) loses his case, he has to pay $w_i$ (which might be zero) to the king.
- payoff functions of the parties

\[
(p_d, p_a) = \begin{cases} 
(0, -w_a), & \text{verdict in favour of defendant} \\
(-x - w_d, x), & \text{verdict in favour of accuser}
\end{cases}
\]
WAGERS WITHOUT SIGNALING: EVIDENCE AND VERDICT

- Binary evidence $e$:
  - innocent defendant: $e = d$
  - truthtelling accuser: $e = a$
  
  $e = d$ is the same as $e = -a$, $e = a$ is the same as $e = -d$

- Binary verdict $v$:
  - verdict clears the defendant: $v = d$
  - Verdict pronounces the defendant guilty: $v = a$

- Verdict and evidence:
  - verdict in line with the evidence: $v = e$ (correct with probability $q > 1/2$)
  - verdict against the evidence $v = -e$ (correct with probability $1 - q$)
WAGERS WITHOUT SIGNALING: QUALITY OF EVIDENCE

- king’s justice payoff
  - justice payoff $J$ if his ruling is correct
  - represent the loyalty that the just king enjoys among his people
- Arthaśāstra: *viraktā yānty amitraṃ vā bhartāraṃ ghnanti vā svayam*
  - when they are disloyal, they either go over to the enemy or kill their lord themselves.
- Expected justice payoff $qJ$ for a verdict following the evidence, but only $(1 - q)J$ otherwise
- kings’s payoff $p_K(v) = \begin{cases} 
qJ + w_{-v}, & v = e \\
(1 - q)J + w_{-v}, & v = -e 
\end{cases}$
WAGERS WITHOUT SIGNALING: TWO STAGES

1. Parties choose wagers.
2. King chooses verdict.

Solution concept: **backward induction**

Assumption: defendant innocent.
WAGERS WITHOUT SIGNALING: KING’S VERDICT

Two conflicting motives:

- evidence following \( v = e \) by \( q > 1/2 \)
- ruling against a wager-risking party \( v = -e \)

Example, \( w_D = w, w_A = 0 \).

- \( e = a \): king follows evidence by \( qJ + w > (1 - q)J + 0 \)
- \( e = a \): king will ignore evidence if \( (1 - q)J + w > qJ + 0 \) holds

Proposition:

\[
v = \begin{cases} 
  d, & (w_d, w_a) \in \{(0,0), (0,w), (w,w)\} \\
  d, & (w_d, w_a) = (w,0) \text{ and } w < (2q - 1)J \\
  a, & (w_d, w_a) = (w,0) \text{ and } w > (2q - 1)J
\end{cases}
\]
Example

- \( w_D = w, w_A = 0, e = a, (1 - q)J + w > qJ + 0 \)
- \( v = d = -e \) ! The (innocent!) defendant would have been cleared if he had chosen the zero wager as did his opponent.

A zero wager is always better than the positive wager in the present case.

The king likes to obtain the wager, but does not attribute any signal quality to the wager.

**Proposition:** Foreseeing the verdicts, each of the two parties has the dominant strategy 0 equilibrium \( (w_d^*, w_a^*) = (0,0) \).
Different wagers: The party with positive wager is honest/truthful with probability 1.

Equal wagers: Evidence $e$ and quality of evidence $q$ relevant as of the evidence in the case of wagers without signaling.

The king tries to use the wagers as a signal for truthtelling. He may
- succeed in a separating equilibrium (different wagers)
- fail in a pooling equilibrium (identical wagers)

kings’s payoff $p_K(v, e, w_d, w_a) = \begin{cases} 
0 \cdot J + w, & w_v = 0, w_{-v} = w \\
J + 0, & w_v = w, w_{-v} = 0 \\
qJ + w_{-v}, & v = e, w_d = w_a \\
(1 - q)J + w_{-v}, & v = -e, w_d = w_a 
\end{cases}$
WAGERS WITH SIGNALING: THREE STAGES

1. King chooses wager amount.
2. Parties choose wagers.
3. King chooses verdict.

Solution concept: backward induction
WAGERS WITH SIGNALING: KING CHOSES VERDICT

- \( w_d = w_a \)

  The king’s wager payoff does not depend on whether he rules in line with the evidence or not. By \( q > 1/2 \) the king’s payoff is maximal if he follows the evidence.

- \( w_d = w, w_a = 0 \)

  The king who rules against the accusant, obtains the justice payoff \( J \), but no wager payoff.

  In contrast, if he rules against the defendant, he obtains the justice payoff zero, but the wager payoff \( w \).

  Thus, the king follows the wager signal and rules in favour of the defendant if \( J > w \) holds. Such a king will be called a **just king.** For him, his justice payment is larger than the positive wager payment.

- \( w_d = 0, w_a = w \)

  *vice versa*
**WAGERS WITH SIGNALING: PARTIES CHOOSE WAGERS (UNJUST KING)**

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<td>$[0, -w]$</td>
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**Proposition:** Assume that the defendant is innocent. Assume an unjust king ($w > J$). The action combination $(w_d^*, w_a^*) = (0,0)$ is a pooling equilibrium (in dominant actions).

The unjust king then obtains the payoff $qJ$. 
**WAGERS WITH SIGNALING: PARTIES CHOOSE WAGERS (JUST KING)**

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**Proposition:** Three constellations:

P $\frac{x}{w} \leq \frac{1-q}{q}$ (small stake $x$, large wager $w$, and/or low quality of evidence $q$)

Two equilibria. $(w, 0)$ risk-dominates $(0, w)$. The king’s payoff in the first equilibrium: $J$

Q $\frac{1-q}{q} < \frac{x}{w} < \frac{q}{1-q}$ (medium stake and/or wager)

One equilibrium only: $(w, 0)$. Again, the king’s payoff is $J$.

R $\frac{x}{w} > \frac{q}{1-q}$ (large stake, small wager, and/or low quality of evidence)

One equilibrium only: $(w, w)$. The king’s payoff is $qJ + w$. 
The diagram illustrates the payoff structure for different constellations involving a king and parties. The axes are labeled as follows:

- **x** (horizontal axis) ranging from 0 to 1 for **w** and from 0 to 1 - q for **J**
- **y** (vertical axis) ranging from 0 to 1 - q for **w** and from 0 to q for **J**

The constellations are differentiated by the payoff conditions:

- **Constellation R**
  - King payoff: \( qJ + w > J \)
  - Parties: \( (w, w) \)
  - King: \( qJ \)
  - Particles: \( eq. (w, w) \)

- **Constellation Q**
  - King payoff: \( qJ + w < J \)
  - Parties: \( (w, w) \)
  - King: \( J \)
  - Particles: \( eq. (w, w) \)

- **Constellation P**
  - King payoff: \( J \)
  - Parties: \( eq. (w, 0) \)
  - King: \( J \)
  - Particles: \( eq. (w, 0) \)

The diagram also includes regions labeled as "unjust king," "just king," and "superjust king."
**Proposition:** Assume that the defendant is innocent.

The king will never choose a wager amount that would make him unjust. In case of $qJ < x$, the payoff maximizing king chooses the wager $w^* = \min \left( J, \frac{1-q}{q} x \right)$ and makes the pooling equilibrium $(w, w)$ happen.

If, on the contrary, $qJ > x$ holds, the king chooses a sufficiently high wager so as to guarantee a separating equilibrium.
Related to both ordeals and wagers is the nearly 1000 years old English institution of “trial by battle” used to settle unclear land disputes.

- Representatives of the opponents fought against each other with clubs, and the winning party obtained (or kept) the contested land. The opponents hire champions to fight for them and the outcome is mainly dependent on the money spent to hire a champion (or even several, in order to dry out the champions market for the opponent).

- There are important differences between a trial by battle and a trial with a wager. The important similarity consists in the fact that the opponents need to risk money.
  - In the Indian case, the paña is wagered and has to be paid only if the king’s ruling is adverse.
  - In the English trials by battle, the money spent for champions is lost for both good or bad outcomes.

Significantly, this English institution did not survive for long.
Wagers can be rationalised in the following manner:

- The honest party to a conflict is more willing to risk a wager than the dishonest party. Indeed, if both parties have placed a positive wager, the innocent one can hope to win if the quality of evidence is sufficiently large.

- Having the possibility of differing signals in mind, the king may be happy to choose relatively high wagers that make the honest party risk the wager and make the dishonest one choose the zero wager. This holds if the stakes are small in relation to the expected justice payment.

- Inversely, however, the king may choose to be just, but not superjust, and to determine a wager amount that is relatively large, but sufficiently small so that both parties choose the positive wager.
CONCLUSION III: THE SCARCE-EVIDENCE PUZZLE

- Leeson (2012) and Wiese (2016) show why ordeals might have been quite sensible institutions. The theoretical ideas put forward in these papers go together well with the fact that ordeals were quite successful and in use for many centuries.

- In contrast, judicial wagers have serious drawbacks. A cash-striped party may just not be able to place the wager amount required by the king. Then, separation is not driven by
  - the honesty or truthfulness of the parties, but by
  - their more or less deep pockets.

  This fact will surely make a king’s subjects suspicious of that institution.

- The subjects will sometimes observe that the king obtains the wager amount. That, also, will not contribute to the king’s reputation. The parties may suspect that the king has financial reasons when using the wagers as a basis for his judgement. Doing so and/or the suspicion that he might do so, will certainly undermine any confidence in the justice system.

- Of course, dharmaśāstra authors may not find good reason to write extensively about an institution long gone extinct. This is probably the solution to the scarce-evidence puzzle.