Harald Wiese¹

A game-theoretic analysis of judicial wagers

¹ University of Leipzig, Postfach 920, D-04009 Leipzig, Germany, tel.: 49 341 97 33 771, e-mail: wiese@wifa.uni-leipzig.de
The author is thankful for calculations and many helpful suggestions by Hendrik Kohrs and Katharina Lotzen. He has also benefitted from insightful discussions with Alexander Fink, Richard Lariviere, and Tim Scholz.
Abstract:
This paper is about an Old Indian judicial institution called *pana* ("wager"). It is briefly considered in two *mūlasmṛtis*, only. We explain the rationale of judicial wagers by showing that high wagers may indicate truthfulness. We argue that wagers are an institution inferior to that of ordeals which explains why the latter have been used much more extensively than the former.
Consider a defendant who is accused of a misdeed. Since defendant and accuser are not able sort out this disagreement between themselves, they resort to the king for a judgement. The usual procedure is this: The king considers the evidence presented to him and decides in favor of the defendant or of the accuser.

Apart from the “objective” evidence, the parties to a legal conflict may try to underline the trueness of their respective assertions by other means. In particular, and with special relevance for Old Indian law, they may resort to ordeals. Ordeals are a manner of saying: “I am speaking the truth; this will be revealed by God.”2 Apparently, a second manner to insist on one’s truthfulness is the “judicial wager” called pana in the Old Indian law literature. Basically, a judicial wager amounts to proclaiming: “I am speaking the truth; if found otherwise by the king, I will pay the appropriate fine, and, on top, make a payment of size $x$.” In the conclusion, we will briefly comment on wager-type institutions in other legal traditions.

Lariviere (1981b) presents the scarce mūlasmrti evidence and analyzes the texts offered by commentators and the nibandhakāras. Here let is suffice to present Yājñavalkya 2.18 together with Lariviere’s (1981b, p. 135) translation:

\[
\text{sapaṇaś ced vivādaḥ syāt tatra hīnaṃ tu dāpayet} / \\
\text{daṇḍaṃ ca svapaṇaṃ caiva dhanine dhanam eva ca} //
\]

If the dispute should be with a wager, then he should make the defeated party pay the fine and his own wager as well, but only the contested amount to its owner.

There is no need to repeat Lariviere’s inconclusive findings in detail. We can summarize them (for our purposes) in the following manner:

- The wager may have been placed by one or by both parties.
- The recipient might have been the king (the court), the opponent, or even both.
- The size of the wager seems not to have been fixed and was probably up to each party.

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2 According to Manu 8.114 (in Olivelle 2005, p. 173), a defendant is to “carry fire, stay submerged in water, or touch separately the heads of his sons and wife. When the blazing fire does not burn a man, the water does not push him up to the surface, and no misfortune quickly strikes him, he should be judged innocent by reason of his oath.” Ordeals would have been carried out in the context of formal trials but also as so-called restorative ordeals (see Brick 2010). For economic analyses of ordeals, see Leeson (2012) and Wiese (undated).
In this paper, we try an economic analysis of judicial wagers. We assume that the amounts placed by the opponents were decided by themselves individually and that the king or a third party was the recipient, as seems to be the case for Yājñavalkya. The third party may have been the Brahmmins who tend to benefit from many transactions (see the conclusions).

Furthermore, we assume that the king decides cases on the basis of both (a) the evidence available to him and (b) the wagers offered by the agents. With respect to (b), the king might think that an accuser who files a correct complaint or an innocent defendant tend to decide on a higher wager than dishonest accusers or defendants. Then, the king tends to rule in favor of the agent with the highest wager. Lariviere (1981b, p. 143) does not entertain this possibility when he writes: “The pana seems … not to be a factor at all in deciding the case … .” For further comments on the king’s incentives, see the conclusion.

Our paper addresses three questions. (i) Can Lariviere’s assessment given above (“wager no factor in deciding the case”) be plausible? (ii) What is the rationale behind using wagers? (iii) Why were wagers much less successful than ordeals? While the last question is addressed in the conclusion, we build game-theoretic models to answer the questions (i) and (ii).

Methodologically, we consider strategic games (see, for example, Gibbons 1992, pp. 1-12). Strategic games consist of players, their strategies, and the payoffs the players obtain for all strategy combinations. In the present case, the players are the two parties, the defendant and the accuser. The strategies are the wagers placed by the parties. A strategy combination consists of a wager placed by the defendant and a wager placed by the accuser. In the following section, we derive the payoffs. We then check for dominant strategies and Nash equilibria. A dominant strategy is a best strategy irrespective of the other player’s strategy. A Nash equilibrium (also simply called equilibrium) is a condition for stability. A strategy combination is called an equilibrium if no player can profit from deviating unilaterally (i.e., by choosing another strategy while the other players stick to their strategies in the strategy combination).

Our theoretical models yield these findings:

- If the king disregards the wagers for his decision, the players will choose zero wagers. To our mind, this result is an answer to (i), against Lariviere.

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3 For reasons of simplicity, we do not model a sequential game where the accuser announces his wager before the defendant does (or the other way around). In view of Asahāya’s procedural description (see Lariviere 1981b, p. 141), this is certainly deplorable. As is often the case in economic models, the weak excuse is tractability.
• If the king disregards the evidence or if the quality of evidence is very poor, the parties’ wager decisions are independent of whether the defendant is guilty or not.
• Wagers are a positive function of the probability of wager-based judgements.
• Wagers tend to be higher for the innocent defendant (the honest accuser) than for the dishonest accuser (the guilty defendant). This result is a vital ingredient for answering (ii).
• A party with a small amount of money to spend on wagers will be deemed guilty more often. This result may well be an important part of an answer to (iii).

II. A model of judicial wagers

A. Setup

We assume a person D (defendant) who is accused by some other person A (accuser) of not paying back a loan $x$. Both D and A are free to place wagers $w_D$ and $w_A$, respectively. If party $i$ ($i \in \{D, A\}$) loses his case, he has to pay $w_i$ to the king.

If the king rules in favor of the defendant, D obtains zero payoff. However, if the defendant is found guilty, he has to pay (back) both the loan $x$ and his wager $w_D$. In this latter case, the accuser obtains the amount $x$ that he claims. We denote party $i$’s payoff by $p_i$ and write the payoffs for the defendant and the accuser (in that order) by

$$\begin{cases} (0, -w_A), & \text{verdict in favor of defendant} \\ (-x - w_D, x), & \text{verdict in favor of accuser} \end{cases}$$

The king decides a percentage $\eta$ of cases on the basis of (objective) evidence, i.e., on the basis of all evidence except the wagers. For these cases, he manages to arrive at a correct verdict with probability $q \geq 1/2$. Thus, $q$ can be understood as the quality of evidence-based judgement. Note that $q = 1/2$ means that the king rules correctly every second case. This amounts to throwing a fair coin.

The other cases (the portion $1 - \eta$) are decided on the basis of the agents’ wagers. Let

$$\pi_i = \frac{w_i}{w_D + w_A}, i = D, A$$
be the conditional probability for deciding in favor of agent $i$ given that the king does not form an evidence-based judgement. In case of $w_D = w_A = 0$ (where $\pi_i$ is not defined), the king uses the conditional probabilities $\pi_D = \pi_A = 1/2$. Equation (2) makes clear that the king (who does not use any non-wager evidence) tends to rule in favor of the party with the highest wager. Note $\pi_D + \pi_A = 1$. (2) is known in the literature as Tullock’s contest success function (see Tullock 1975). However, in contrast to Tullock’s model, the wager has to be paid by the losing party, only.

We first turn to the case of an innocent defendant. He may be cleared for two different reasons. Either the king uses the evidence (with probability $\eta$) and draws the correct conclusion (the latter occurring with probability $q$). Thus, the innocent defendant is cleared by evidence with probability $\eta \cdot q$. Alternatively, the king disregards the evidence (with probability $1 - \eta$) and clears the defendant with probability $\pi_D$ (see equation (2)).

Summarizing, the innocent defendant obtains a favorable ruling with probability

$$\eta q + (1 - \eta) \frac{w_D}{w_D + w_A}$$

Therefore, the innocent defendant’s expected payoff is given by

$$p_D(w_D, w_A) = [\eta q + (1 - \eta)\pi_D] \cdot 0 + [1 - \eta q - (1 - \eta)\pi_D](-x - w_D)$$

The dishonest accuser obtains the payoff

$$p_A(w_D, w_A) = [\eta q + (1 - \eta)\pi_D](-w_A) + [1 - \eta q - (1 - \eta)\pi_D]x$$

If, on the other hand, the defendant is guilty and the accuser honest, we obtain the payoff functions

$$p_D(w_D, w_A) = [\eta(1 - q) + (1 - \eta)\pi_D] \cdot 0$$

$$+ [1 - \eta(1 - q) - (1 - \eta)\pi_D](-x - w_D)$$

and

$$p_A(w_D, w_A) = [\eta(1 - q) + (1 - \eta)\pi_D] \cdot (-w_A)$$

$$+ [1 - \eta(1 - q) - (1 - \eta)\pi_D]x$$
Note that (3) and (5) differ with respect to $q$, only. If we replace $(1 - q)$ by $\bar{q}$, the formulas look the same, apart from the bar. $\bar{q}$ is the probability that the king arrives at a wrong evidence-based judgement. By $q \geq 1/2$, we have $\bar{q} \leq 1/2$.

**B. Evidence-based judgement**

Let us assume that the defendant is innocent. We start our analysis with two extreme cases, $\eta = 1$ (evidence-based judgement, only) and $\eta = 0$ (wager-based judgement, only). If the king disregards the wagers, the parties do not have any reason to choose a non-zero wager. Indeed, this can clearly be seen from the payoff functions (3) and (4) for the special case of $\eta = 1$:

\[(3B) \quad p_D(w_D, w_A) = q \cdot 0 + [1 - q](-x - w_D)\]

\[(4B) \quad p_A(w_D, w_A) = q(-w_A) + [1 - q]x\]

In (3B) and (4B), D and A never benefit from a positive wager. Thus, the defendant has $w_D = 0$ as dominant strategy. Similarly, $w_A = 0$ is a dominant strategy for the accuser. Finally, $(w_D^*, w_A^*) = (0,0)$ is a Nash equilibrium and, in fact, the only Nash equilibrium for $\eta = 1$.

The same equilibrium is obtained if the defendant is guilty. To see this, just interchange $q$ and $1 - q$ in equations (3B) and (4B). We summarize our results:

**Proposition B:**

If the king were to decide on the basis of evidence, only, both players place zero wagers.

In light of proposition B, Lariviere’s position (“wager no factor in deciding the case”) seems difficult to defend.

**C. Wager-based judgement**

The other extreme case is $\eta = 0$ where the king’s judgement is based on the wagers, only. We obtain the payoff functions

\[(3C) \quad p_D(w_D, w_A) = \frac{w_D}{w_D + w_A} \cdot 0 + \left[1 - \frac{w_D}{w_D + w_A}\right](-x - w_D)\]

\[(4C) \quad p_A(w_D, w_A) = \frac{w_D}{w_D + w_A}(-w_A) + \left[1 - \frac{w_D}{w_D + w_A}\right]x\]
Note that the quality of evidence is unimportant and so is the question whether the defendant is guilty or not.

We now assume that the parties have limited funds to spend on the wager. Let $B_D > 0$ be the maximal amount that D can spend on the wager and define $B_A > 0$ correspondingly. We disregard the unlikely cases of $B_D = x$ or $B_A = x$. As shown in the appendix, the richer party tends to win the lawsuit. We divide the results in two groups. The first group deals with “best responses”, i.e., it gives answers to the question of how a player should act in the presence of a given strategy of the other player. The second group concerns Nash equilibria:

**Proposition C:** Assume that the king decides on the basis of wagers, only. We find these results on best responses:

- The quality of evidence is irrelevant and the players’ strategies do not depend on whether the accuser or the defendant make correct assertions.
- Each party chooses $w_i = 0$ if the other party chooses more than $x$. $x$ can be called the limit wager, i.e., the wager that makes the other party give up.
- Each party chooses $w_i = B_i$ if the other party chooses less than $x$. Thus, a party chooses the maximally possible wager if the other chooses a wager below the amount in dispute.

And here are the results on Nash equilibria:

- If both players are poor (relative to the contested amount) both choose the maximal amount available for the wager in equilibrium. Thus, the richer (in terms of money to spend for wagers) player chooses a higher wager than the poorer one. Therefore, the richer player will have a higher chance of winning the lawsuit.
- If a player’s budget is smaller than the contested amount, while the other player’s budget is below, the richer chooses to wager the contested amount or any larger amount, and forces the other player to choose zero. If both players’ budgets are sufficiently large, either of them can drive the other’s wager down to zero in equilibrium.

**D. Judgements based on both very poor evidence and wagers**

We now turn to the case where the king bases his judgement on both objective evidence ($\eta > 0$) and on the wagers placed by the contestants ($1 - \eta > 0$). We
even restrict attention to $\eta > 1/3$. However, we assume that the evidence is very poor ($q = \bar{q} = 1/2$) so that the king just tosses a coin when making non-wager decisions.

Irrespective of the defendant’s innocence or guilt, we obtain the following results (see the appendix for mathematic details):

- For sufficient wager budgets, the limit wagers (i.e., the wagers sufficient to make the opponent choose the zero wager) are identical and equal to $2 \frac{1-\eta}{2-\eta} x$.
- For sufficient wager budgets, the equilibrium wagers are $(w_D^*, w_A^*) = \left( \frac{1-\eta}{2-\eta} x, \frac{1-\eta}{2-\eta} x \right)$. Figure 1 depicts the two reaction functions for $\eta = 1/2$. Where both are fulfilled (crossing point), we have the Nash equilibrium.
- If the accuser has a limited budget below the one in equilibrium, he chooses a lower wager and the defendant a higher one than in the unrestrained equilibrium. This case is depicted in figure 2.

**Proposition D:**
Assume that the king bases his judgement on very poor evidence and on wagers.

- The parties’ wager decisions are independent of whether the defendant is guilty or not.
- The wager necessary to make the other player refrain from placing a positive wager depends positively on the probability that the king uses the wagers for his decision.
- The equilibrium wagers are also a positive function of the probability of wager-based judgements. Inversely, the equilibrium wagers tend towards zero as the probability of using objective evidence tends towards 1. (Compare proposition B.)
- A limited budget of one player affects the equilibrium wagers for both players.

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4 For $\eta \geq 1/3$, we can be sure of a unique Nash equilibrium in sections D and E. For $\eta < 1/3$, we may obtain three equilibria. See the appendix for section IID.
Figure 1: Best response functions and equilibrium

Figure 2: The accuser has limited funds

E. **Judgements based on imperfect evidence and on wagers**
We now turn to the most general case where the king considers both the wagers and the evidence (where we continue to assume $\eta > 1/3$). The evidence is not useless, but still imperfect: $1/2 < q < 1$.

Assume an innocent defendant. Then (see the appendix for subsection D), the limit wagers are

$$w_D^L = \frac{1 - \eta}{1 - \eta(1 - q)} x < \frac{1 - \eta}{1 - \eta q} x = w_A^L$$

While we cannot express the Nash equilibrium explicitly in terms of the parameters (the model is too complex), the appendix for subsection E shows that an increase in the quality of evidence $q$, for the case of an innocent defendant, will increase $w_D^*$ and decrease $w_A^*$ for a sufficiently high proportion $\eta$ of evidence-based judgements.

**Proposition E:**
Assume that the king bases his judgement on imperfect evidence and on wagers. Then, the honest party tends to choose a larger wager than the dishonest one. In particular, we find:

- The honest party can deter the dishonest party from choosing a positive wager with a relatively small limit wager (see inequality above).
- An increase in the quality of evidence makes the dishonest party choose a lower wager in equilibrium.
- An increase in the quality of evidence makes the honest party choose a higher wager in equilibrium for $\eta > 1/2$.$^5$

### III. Conclusion

While this paper deals with judicial wagers in India, Matthiass (1888, pp. 5-18, and 1912, pp. 341-347) argues that they were present in other Indo-European judicial traditions, also. The author understands wagers as central to the transition from “self-help, that is, physical combat” (Matthiass 1912, p. 342) to increasingly formalized third-party involvement: “If the contending parties appealed to a trusted person they merely gave up the personal encounter, in the place of which there now appeared the assertion by each party that he was right; in other words, contending opinions took the place of personal conflict. Each of the parties had to show evidence of the earnestness of his opinion and of his firm belief

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$^5$ For $\frac{1}{3} < \eta < \frac{1}{2}$, see appendix for subsection IID.
in his contention, else there would have been no inducement why he should surrender his right to self-help. Substitution of personal opinion was appropriately followed by a deposit made with the trusted party; this deposit, which the trusted party was to surrender to the victor, constituted the penalty that the defeated party incurred. Thus, we see that the parties made a wager and that the oldest form of arbitral court was the wager-court” (Matthiass 1912, p. 342). Later on, argues Matthiass (1912, p. 343), wagers became increasingly important as fees for these third parties.

Matthiass links wagers to deposits and court fees. Both connections are plausible steps in the evolution of law. Note, however, that it would be unusual to let the conflicting parties individually decide on these fees. In any case, the motives for placing lower or higher amounts are still left unexplained. And this is where the present paper tries to make a contribution. We find that wagers can be rationalized in the following manner: The honest party to a conflict tends to place a higher wager than the dishonest one and thus obtains a favorable judgement with higher probability. This does not necessarily mean (and we do not claim) that the conflicting parties or the law-text writers had this in mind when they applied or described wagers.

We now turn to question (iii) raised in the introduction. Judicial wagers have serious drawbacks. First, a cash-stripped party may just not be able to place high wagers. Second, a better quality of evidence leads to better evidence-based judgements and also to better wager-based judgements. Thus, it may not help matters much if we suppose that the king decides on the basis of evidence if the evidence is of good quality, and on the basis of wagers, otherwise.

The third point refers to the king who might have been the recipient of the wagers. Then, the parties may suspect that the king has financial reasons when using the wagers as a basis for his judgement. Doing so and/or the suspicion that he might do so, will certainly undermine any confidence in the justice system. Also, the king may then be torn between two motives. On the one hand, he takes high wagers as an indication for truthful behavior and tends to rule in favor of the high-wager agent. On the other hand, ruling against the agent with high wagers is financially profitable for him. For these mixed motives, one may conjecture that a third party like the Brahmans, rather than the king himself, was the recipient. However, the nibandhakāra evidence collected by Lariviere (1981b) does not provide any support (see introduction).

These three drawbacks may be the reason why, in India, judicial wagers seem to have gone out of fashion many centuries before ordeals did (compare Lariviere (1981b, 144) and Lariviere (1981a)).

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6 A In this context, note the “Varuna rule”. It stipulates that the king is to throw confiscated property into water. This apparent waste of resources calls out for an explanation (attempted by Wiese 2015).
If “objective” evidence is not used by a judge, ordeals or wagers may be used. Related to both ordeals and wagers is the nearly 1000 years old English institution of “trial by battle” used to settle unclear land disputes. Here, representatives of the opponents fought against each other and the winning party obtained (or kept) the contested land. Now, trial by battle can be understood as a particular form of ordeal (God makes the honest party win).

However, neither God nor ordeal-officiating priests were involved, at least as far as the economic analysis by Leeson (2011) goes. The opponents hire champions to fight for them and the outcome is mainly dependent on the money spent to hire a champion (or even several, in order to dry out the champions market for the opponent). Leeson’s mathematical analysis is then similar (but simpler) to the one carried out here in subsection II C. Indeed, Leeson also uses Tullock’s contest success function to describe the probability that a proponent’s champion will win. The similarity between wagers and trial for battle is that opponents put forward money amounts. In the Indian case, the paṇa is wagered and has to be paid only if the king’s ruling is adverse. In the English trials by battle, the money spent for champions is lost for both good or bad outcomes.

Appendix for subsection IIC

The first derivative of $p_D(w_D, w_A)$ (see (3C)) with respect to $w_D$ is

$$\frac{\partial p_D(w_D, w_A)}{\partial w_D} = \frac{w_A(x - w_A)}{(w_A + w_D)^2} = \begin{cases} 
= 0, & (w_A = 0 \text{ and } w_D > 0) \text{ or } w_A = x \\
> 0, & 0 < w_A < x \\
< 0, & w_A > x 
\end{cases}$$

which allows to derive the defendant’s best response function given by

$$w_D^R(w_A) = \begin{cases} 
(0, B_D], & w_A = 0 \\
B_D, & 0 < w_A < x \\
[0, B_D], & w_A = x \\
0, & w_A > x 
\end{cases}$$

Similarly, the accuser’s best response function is given by

$$w_A^R(w_D) = \begin{cases} 
(0, B_A], & w_D = 0 \\
B_A, & 0 < w_D < x \\
[0, B_A], & w_D = x \\
0, & w_D > x 
\end{cases}$$
In a Nash equilibrium, both reaction functions need to be fulfilled. If both \( B_D \) and \( B_A \) are smaller than \( x \), the third and fourth lines in the best response functions are excluded. The first lines cannot be involved either. Note that \( w_A = 0 \) (or \( w_D = 0 \)) cannot be part of an equilibrium. The reason is \( w_D^R(0) > 0 \) and hence \( w_A^R(w_D^R(0)) = B_A > 0 \). Indeed, the remaining second lines yield a unique equilibrium \((w_D^*, w_A^*) = (B_D, B_A)\).

Assume \( B_A > x \) and \( B_D < x \). Again, \( w_A = 0 \) cannot be part of an equilibrium. However, \((w_D^*, w_A^*) = (0, x)\) is an equilibrium, which might be called limit equilibrium. Equilibria of this type are also given by \( w_D^* = 0 \) and \( w_A^* > x \), i.e., \( w_A^* \) is from the non-empty interval \((x, B_A]\). No other equilibria exist in this case. It is easy to see that the inverse case \((B_A < x \text{ and } B_D > x)\) leads to similar equilibria.

Finally, consider \( B_A > x \) and \( B_D > x \). Here, we have two types of equilibria. First, \((w_D^*, w_A^*) = (x, x)\) is an equilibrium. Second, we have limit equilibria with \( w_D^* = 0 \) and also limit equilibria with \( w_A^* = 0 \).

**Appendix for subsection IID**

We first deal with a general \( 1/2 \leq q < 1 \). We also assume \( 0 < \eta < 1 \). The first derivative of \( p_D(w_D, w_A) \) (see (3) in subsection A) with respect to \( w_D \) is

\[
\frac{\partial p_D(w_D, w_A)}{\partial w_D} = (1 - \eta) \frac{w_A}{(w_A + w_D)^2} (x - w_A) - \eta (1 - q)
\]

By \( \eta > 0 \) and \( q < 1 \), the first order condition yields two real-valued solutions, but one is non-positive and can be disregarded (we deal with zero wagers otherwise). The other solution is

\[
w_D^{fo}(w_A) = \frac{1}{1 - q} \frac{1 - \eta}{\eta} w_A(x - w_A) - w_A
\]

By \( w_D^R(w_A) \geq 0 \) we find A’s limit wager \( w_A^L = \frac{1 - \eta}{1 - q \eta} x \).

The second derivative of \( p_D(w_D, w_A) \) (see (3) in subsection A) with respect to \( w_D \) is
\[
\frac{\partial^2 p_D(w_D, w_A)}{(\partial w_D)^2} = -2(1 - \eta) \frac{w_A(x - w_A)}{(w_A + w_D)^3} \begin{cases} = 0, & w_A = 0 \text{ or } w_A = x \\ > 0, & w_A > x \\ < 0, & 0 < w_A < x \end{cases}
\]

Therefore, we obtain the defendant’s best response function

\[
w^R_D(w_A) = \begin{cases} \varepsilon, & w_A = 0 \\ w^f_0(w_A), & 0 < w_A < w^L_A \text{ and } B_D \geq w^f_0(w_A) \\ B_A, & 0 < w_A < w^L_A \text{ and } B_A < w^f_0(w_A) \\ 0, & w_A \geq w^L_A \end{cases}
\]

In the first line, \(\varepsilon\) is to be understood as a very small real number that is larger than 0. Although this is mathematically incorrect, we think of \(\varepsilon\) as the smallest positive number. With respect to the second line, note that the second derivative is negative for \(0 < w_A < x\) and hence for \(0 < w_A < w^L_A < x\). In the third line, the defendant is restricted by his limited funds.

For the accuser, we employ a similar procedure and find

\[
w^f_0(w_D) = \frac{11 - \eta}{q \eta} w_D(x - w_D) - w_D
\]

\[
w^L_D = \frac{1 - \eta}{1 - \eta(1 - q)} x
\]

and

\[
w^R_A(w_D) = \begin{cases} \varepsilon, & w_D = 0 \\ w^f_0(w_D), & 0 < w_D < w^L_D \text{ and } B_A \geq w^f_0(w_D) \\ B_A, & 0 < w_D < w^L_D \text{ and } B_A < w^f_0(w_D) \\ 0, & w_D \geq w^L_D \end{cases}
\]

With respect to the first-order conditions, we discuss in detail: (i) the conditions for a unique equilibrium, (ii) the effect of \(q\) on \(w^*_D\) and \(w^*_A\), and (iii) the maximum of \(w^f_0\) as a function of \(w_D\) in the special case of \(q = 1/2\).

(i) With the help of the very powerful algebraic software Mathematica\textsuperscript{©} 9, we examine the equilibrium (or the equilibria) resulting from the above first-order conditions. We proceed as follows: We substitute the reaction function \(w^f_0(w_D)\) into the reaction function \(w^f_0(w_A)\). The resulting equation \(w_D = w^f_0\left(w^f_0(w_D)\right)\) is now solved for \(w_D\) with command “Reduce[\(w_D =\)
\( w_{D}^{f,\alpha} \left( w_{A}^{f,\alpha} (w_{D}) \right) \) \&\& 0 < w_{D} < x \&\& 0 < \eta < 1 \&\& 1/2 \leq q < 1, w_{D}, \text{Reals}” and we obtain an explicit expression for \( w_{D}^{\ast} \). Then, we also obtain \( w_{A}^{\ast} = w_{A}^{f,\alpha} (w_{D}^{\ast}) \).

For \( \eta \geq 1/3 \), we obtain a unique equilibrium for all \( q \geq 1/2 \). For \( \eta < 1/3 \), a unique equilibrium is not guaranteed for all \( q \geq 1/2 \).

(ii) We now turn to the question of how \( w_{D}^{\ast} \) and \( w_{A}^{\ast} \) depend on \( q \) (this question is relevant for the next subsection). For that purpose, we form the partial derivative of \( w_{D}^{\ast} \) with respect to \( q \). Applying “Reduce[\( \frac{\partial w_{D}^{\ast}}{\partial q} > 0 \&\& x > 0 \&\& \frac{1}{3} < \eta < 1 \&\& 1/2 \leq q < 1, q, \text{Reals}” yields these conditions: For all \( q \geq 1/2 \) and all \( \eta \geq 1/2 \), we find \( w_{D}^{\ast} \) depends positively on \( q \). For \( \frac{1}{3} < \eta < \frac{1}{2} \) we find a decreasing effect of \( q \) on \( w_{D}^{\ast} \) for \( \frac{1}{\eta} \left( \frac{2}{1-\eta} - \frac{3}{2} \right) < q < 1 \). We also apply “Reduce[\( \frac{\partial w_{A}^{\ast}}{\partial q} < 0 \&\& \frac{1}{3} < \eta < 1 \&\& 1/2 \leq q < 1, q, \text{Reals}” and find that \( q \) always (\( q \geq 1/2 \) and all \( \eta \geq 1/3 \)) has a negative effect on \( w_{A}^{\ast} \).

Turning, now, to the special case of \( q = 1/2 \), we find all the results given in subsection IID. In particular, we have exactly one equilibrium in case of sufficient funds which is given by simultaneously fulfilling the two first order conditions given here. Note that \( q = 1/2 \) implies \( q = 1 - q \) so that the best response functions and the equilibrium do not depend on the defendant’s guilt.

(iii) Forming the derivative of \( w_{A}^{f,\alpha} \) with respect to \( w_{D} \) shows that \( w_{A}^{f,\alpha} \) is maximized if \( 2 \sqrt{\frac{2}{1-\eta} w_{D} (x - w_{D})} = 2 \frac{1-\eta}{\eta} (x - 2w_{D}) \) holds. Also, the second derivative of \( w_{A}^{f,\alpha} \) with respect to \( w_{D} \) is negative throughout. The above equation holds at \( w_{D}^{max} = \frac{x}{2} \left[ 1 - \sqrt{\frac{\eta}{2-\eta}} \right] \). We can also show \( w_{D}^{max} < w_{D}^{\ast} = \frac{1-\eta}{2-\eta} x \). Therefore, we obtain figure 1, where the Nash equilibrium occurs at the downward-sloping sections of the best response functions.

**Appendix for subsection IIE**

The previous appendix shows the conditions under which an increase in the quality of evidence \( q \) increases the (honest!) defendant’s equilibrium wager and decreases the (dishonest!) accuser’s equilibrium wager. For illustration, assume \( \eta = 1/2 \) and consider figure 3 where you see three intersection points. The north-western one (point E) is the Nash equilibrium for \( q = 1/2 \). The south-
eastern one (point G) is the Nash equilibrium for some slightly higher evidence quality. The one in between is not a Nash equilibrium. It serves to show that, processing form point E to point F and finally to point G, $w_D$ increases at both steps and $w_A$ decreases at both steps. For negatively sloped best response functions, this is true because (i) $w_A^{f.o.}$ shifts downwards when $q$ increases (compare points E and F) and (ii) $w_D^{f.o.}$ shifts rightwards when $q$ increases (compare points F and G).

Therefore, it remains to show that, for $q = 1/2$, the reaction functions are maximized at a wager that is smaller than the equilibrium one. This has been shown in the previous appendix.

![Diagram](image)

Figure 3: The effect of a small increase of $q$ beyond $1/2$.

References


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