Judicial wagers

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Overview

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- Setup
- Evidence-based judgement
- Wager-based judgement
- Judgements based on both very poor evidence and wagers
- Judgements based on imperfect evidence and on wagers
- Conclusions
Ordeals:
“I am speaking the truth; this will be revealed by God.”

Judicial wager:
“I am speaking the truth; if found otherwise by the king, I will pay the appropriate fine, and, on top, make a payment of size x.”

Wager procedure (according to Lariviere 1981):
- The wager may have been placed by one or by both parties.
- The recipient might have been the king (the court), the opponent, or even both.
- The size of the wager seems not to have been fixed and was probably up to each party.
King decides on the basis of both

- the evidence available to him and
- the wagers offered by the agents. Maybe, the king might think that an accuses who files a correct complaint or an innocent defendant tend to decide on a higher wager than dishonest accuses or defendants. But: “The paṇa seems . . . not to be a factor at all in deciding the case . . . .”
Findings:

- If the king disregards the wagers for his decision, the players will choose zero wagers.
- If the king disregards the evidence or if the quality of evidence is very poor, the parties’ wager decisions are independent of whether the defendant is guilty or not.
- A party with a small amount of money to spend on wagers will be deemed guilty more often.
- Wagers are a positive function of the probability of wager-based judgements.
- Wagers tend to be higher for the innocent defendant (the honest accusant) than for the dishonest accusant (the guilty defendant).
Setup I

- D (defendant = accused) with $w_D$
- A (accusant) with $w_A$
- Loosing party pays $w_i$ to the king.
- Payoffs

\[
(p_D, p_A) = \begin{cases} 
(0, -w_A) & \text{verdict in favor of defendant} \\
(-x - w_D, x) & \text{verdict in favor of accusant}
\end{cases}
\]
Setup II

- $\eta$: percentage of cases on the basis of (objective) evidence
- $q \geq \frac{1}{2}$: probability for correct evidence-based verdict
- $\pi_i = \frac{w_i}{w_D+w_A}$, $i = D, A$: conditional probability for deciding in favor of agent $i$ given that the king does not form an evidence-based judgement
  - If $w_D = w_A = 0$, then $\pi_D := \frac{1}{2}$, $\pi_A := \frac{1}{2}$
  - Party with highest wager is favored
  - Tullock 1975: contest success function, but the wager has to be paid by the losing party, only.
Innocent defendant obtains a favorable ruling with probability

\[ \eta q + (1 - \eta) \frac{w_D}{w_D + w_A} \]

Expected payoff for innocent defendant

\[ [\eta q + (1 - \eta) \pi_D] \cdot 0 + [1 - \eta q - (1 - \eta) \pi_D] (-x - w_D) \]

Expected payoff for dishonest accusant

\[ [\eta q + (1 - \eta) \pi_D] (-w_A) + [1 - \eta q - (1 - \eta) \pi_D] x \]
Evidence-based judgement

For $\eta = 1$, there is no reason to spend money on wagers. Expected payoff for innocent defendant

$$[1 - q] (-x - w_D)$$

Expected payoff for dishonest accusant

$$q (-w_A) + [1 - q] x$$

- $w_D = 0$ and $w_A = 0$ are dominant strategies
- Hence, $(0, 0)$ is a Nash equilibrium, and the unique one.
For \( \eta = 0 \), the king’s judgement is based on the wagers, only. Expected payoff for innocent defendant

\[
p_D(w_D, w_A) = \left[ 1 - \frac{w_D}{w_D + w_A} \right] (-x - w_D)
\]

Expected payoff for dishonest accusant

\[
p_A(w_D, w_A) = \frac{w_D}{w_D + w_A} (-w_A) + \left[ 1 - \frac{w_D}{w_D + w_A} \right] x
\]

Note that the quality of evidence is unimportant and so is the question whether the defendant is guilty or not.

- \( B_D > 0 \) : maximal amount that D can spend on the wager
- \( B_A > 0 \) : maximal amount that A can spend on the wager
- Disregard \( B_D = x \) or \( B_A = x \)
Wager-based judgement II

\[
\frac{\partial p_D (w_D, w_A)}{\partial w_D} = \frac{w_A (x - w_A)}{(w_A + w_D)^2} \begin{cases} 
= 0, & (w_A = 0 \land w_D > 0) \lor w_A = x \\
> 0, & 0 < w_A < x \\
< 0, & w_A > x
\end{cases}
\]

\[
w_D^R (w_A) = \begin{cases} 
(0, B_D], & w_A = 0 \\
B_D, & 0 < w_A < x \\
[0, B_D], & w_A = x \\
0, & w_A > x
\end{cases}
\]

\[
w_A^R (w_D) = \begin{cases} 
(0, B_A], & w_D = 0 \\
B_A, & 0 < w_D < x \\
[0, B_A], & w_D = x \\
0, & w_D > x
\end{cases}
\]
Wager-based judgement III

- $B_D < x$ and $B_A < x$
  - Nash equilibrium $(w_D^*, w_A^*) = (B_D, B_A)$
- $B_D < x$ and $B_A > x$
  - Limit Nash equilibria $(w_D^*, w_A^*)$ with $w_D^* = 0$ and $w_A^* \in [x, B_A]$
- $B_D > x$ and $B_A < x$ (similarly)
- $B_D > x$ and $B_A > x$ with
  - equilibrium $(w_D^*, w_A^*) = (x, x)$ and
  - limit equilibria as above
    - $w_D^* = 0$ and $w_A^* \in [x, B_A]$
    - $w_A^* = 0$ and $w_D^* \in [x, B_D]$
Judgements based on both very poor evidence and wagers

- $\frac{1}{3} < \eta < 1$
- $q = \frac{1}{2}$

Irrespective of the defendant’s innocence or guilt, we find:

The limit wagers (i.e., the wagers sufficient to make the opponent choose the zero wager) are identical and equal to $2^{\frac{1-\eta}{2-\eta}}x$. 
Judgements based on both very poor evidence and wagers II

For sufficient wager budgets, the equilibrium wagers are
\[
\left( \frac{1-\eta}{2-\eta} x, \frac{1-\eta}{2-\eta} x \right).
\]
See the figure for \( \eta = \frac{1}{2} \).
Now assume that the accusant has a limited budget below the one in equilibrium.

\[ x^L_w = \frac{2}{3}x \]

\[ x^R_w(L,D) = (D,R,A) \]

\[ w^L_A = \frac{2}{3}x \]

<table>
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<tr>
<th>Nash equilibrium without limited funds for wagers</th>
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<td>Nash equilibrium where accusant has limited fund</td>
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Judicial wagers
Assume that the king bases his judgement on very poor evidence and on wagers.

- The parties’ wager decisions are independent of whether the defendant is guilty or not.
- The wager necessary to make the other player refrain from placing a positive wager depends positively on the probability that the king uses the wagers for his decision.
- The equilibrium wagers are also a positive function of the probability of wager-based judgements. Inversely, the equilibrium wagers tend towards zero as the probability of using objective evidence tends towards 1.
- A limited budget of one player affects the equilibrium wagers for both players.
Assume $\frac{1}{2} < q < 1$ (imperfect, but not useless evidence)

Innocent defendant

Then,

the limit wagers are

$$w_D^L = \frac{1 - \eta}{1 - \eta (1 - q)} x < \frac{1 - \eta}{1 - \eta q} x = w_A^L$$

the honest party tends to choose a larger wager than the dishonest one,

an increase in the quality of evidence makes the dishonest party choose a lower wager in equilibrium.

an increase in the quality of evidence makes the honest party choose a higher wager in equilibrium for $\eta > \frac{1}{2}$.
Wagers also in other Indo-European traditions (Matthiass 1888):

“If the contending parties appealed to a trusted person they merely gave up the personal encounter, in the place of which there now appeared the assertion by each party that he was right; in other words, contending opinions took the place of personal conflict. Each of the parties had to show evidence of the earnestness of his opinion and of his firm belief in his contention, else there would have been no inducement why he should surrender his right to self-help. Substitution of personal opinion was appropriately followed by a deposit made with the trusted party; this deposit, which the trusted party was to surrender to the victor, constituted the penalty that the defeated party incurred. Thus, we see that the parties made a wager and that the oldest form of arbitral court was the wager-court”.
Matthiass links wagers to deposits and court fees.

- Both connections are plausible steps in the evolution of law.
- But: would the conflicting parties individually decide on these fees?
- In any case, the motives for placing lower or higher bids are still left unexplained. Our contribution: The honest party to a conflict tends to place a higher wager than the dishonest one and thus obtains a favorable judgement with higher probability.
Conclusions III

Judicial wagers have serious drawbacks:

1. A cash-stripped party may just not be able to place high wagers.
2. A better quality of evidence leads to better evidence-based judgements and also to better wager-based judgements. Thus, it may not help matters much if we suppose that the king decides on the basis of evidence if the evidence is of good quality, and on the basis of wagers, otherwise.
3. The king might have been the recipient of the wagers. Then, the king may then be torn between two motives.

- On the one hand, he takes high wagers as an indication for truthful behavior and tends to rule in favor of the high-wager agent.
- On the other hand, ruling against the agent with high wagers is financially profitable for him. For these mixed motives, one may conjecture that a third party like the Brahmins, rather than the king himself, was the recipient.

These three drawbacks may be the reason why, in India, judicial wagers seem to have gone out of fashion many centuries before ordeals did.
Compare the nearly 1000 years old English institution of “trial by battle” (Leeson) used to settle unclear land disputes. Here, representatives of the opponents fought against each other and the winning party obtained (or kept) the contested land.

The opponents hire champions to fight for them and the outcome is mainly dependent on the money spent to hire a champion. Leeson also uses Tullock’s contest success function to describe the probability that a proponent’s champion will win. The similarity between wagers and trial for battle is that opponents put forward money amounts. In the Indian case, the paṇa is wagered and has to be paid only if the king’s ruling is unfortunate. In the English trials by battle, the money spent for champions is lost for both good or bad outcomes.