Language competition

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Overview

- Synchronic models
  - Setup
  - Language learning
  - Literary production with one producer
  - Literary production with several producers
  - Non-option demand

- Diachronic models
  - Setup
  - Old-language learning with two periods
  - A three-period model with old-language learning
    - Setup
    - Current readership
    - Long-term readership
  - A forking model
Setup

- two periods: \( t = p \) (past) and \( t = n \) (now)
- two languages: 1 and 2
- number of speakers: \( q_1 \) and \( q_2 \)
- language production:
  - \( p_1 \) and \( p_2 \) in period \( p \)
  - \( n_1 \) and \( n_2 \) in period \( n \) with \( n = n_1 + n_2 \)

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period p
language 1 with production \( p_1 \)
language 2 with production \( p_2 \)

arrow

period n
language 1 with production \( n_1 \) and population \( q_1 \)
language 2 with production \( n_2 \) and population \( q_2 \)
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Payoff for a native speaker of language 1

\[ u_1 = \begin{cases} 
   p_1 + n_1 + (p_2 + n_2) - c, & \text{1 learns language 2} \\
   p_1 + n_1 + \alpha_2 (p_2 + n_2), & \text{1 does not learn language 2} 
\end{cases} \]

where

- option demand
- accessibility \( \alpha_2 \)

- a Spanish reader may partly understand Italian literary works (accessibility \( \alpha \) between 0.6 and 0.8), while a German reader’s accessibility to Italian writings is close to zero
- a reader can approach foreign literature by way of translations (which are of poorer quality than the original)
Learning language 2 is beneficial for a native speaker of language 1 if
\[ c < (p_2 + n_2) (1 - \alpha_2) =: \bar{c}_2 \]

Uniform distribution of language learning costs \( c \) on \([0, C_2]\) with \( C_2 > n + p_2 \).

Proportion of readers of language 1 that learn language 2 is

\[
\frac{\bar{c}_2}{C_2} = \frac{(p_2 + n_2) (1 - \alpha_2)}{C_2} < 1
\]

Analogously, we have

\[
\frac{\bar{c}_1}{C_1} = \frac{(p_1 + n_1) (1 - \alpha_1)}{C_1} < 1
\]
We say that language 1 dominates language 2 with respect to language learning if the percentage of language-1 learners $\frac{c_1}{C_1}$ is larger than the percentage of language-2 learners $\frac{c_2}{C_2}$. This tends to hold under the following conditions:

- It is relatively easy to learn language 1 ($C_1 < C_2$).
- In the past, literary production in language 1 was relatively large ($p_1 > p_2$).
- The current literary production in language 1 is relatively large ($n_1 > n_2$).
- and...
Accessibility for language-2 speakers to language 1 is relatively small ($\alpha_1 < \alpha_2$).

Why? Language-learning dominance of language 1 is furthered if there are many and good translations of language-2 literature into language 1. In this case, speakers of language 1 do not have large incentives to learn language 2.

If a country (let us say, France) wishes to make French dominant with respect to language learning, it should translate important works of foreign languages into French (or should subsidize these translations). It should not, however, further translations of French works into foreign languages. Note that the French government sponsored “Centre national du livre” (www.centrenationaldulivre.fr) subsidizes translations in both directions.
One literary producer who chooses $n_1$ and hence $n_2 = n - n_1$.

Readership for literary production $n_1$ is $n_1$ times

$$1 \cdot q_1 + q_2 \left( 1 \cdot \frac{\bar{c}_1}{C_1} + \alpha_1 \left[ 1 - \frac{\bar{c}_1}{C_1} \right] \right)$$

Overall readership is

$$R(n_1) = n_1 \left[ q_1 + q_2 \left( \frac{\bar{c}_1}{C_1} + \alpha_1 \left[ 1 - \frac{\bar{c}_1}{C_1} \right] \right) \right] + (n - n_1) \left[ q_2 + q_1 \left( \frac{\bar{c}_2}{C_2} + \alpha_2 \left[ 1 - \frac{\bar{c}_2}{C_2} \right] \right) \right]$$

**interpretation**

- one producer knowing both languages
- representative for a diverse set of producers
- “benevolent dictator”
- readership is of the option-demand type
Proposition:
Language 1 or language 2 become the exclusive literary language for production. In particular, language 1 tends to become the standard language if

- the population $q_1$ of language 1 is large (relative to the population $q_2$ of language 2),
- the cost $C_1$ of learning language 1 is small (relative to the cost $C_2$ of learning language 2), or
- the literary base $p_1$ of language 1 is large (relative to the literary base $p_2$ of language 2).
The effect of the accessibility parameters on language adoption is ambiguous. An increase in $\alpha_1$ has two opposing effects.

- Disregarding language learning, we have

$$R(n_1) = n_1 \left[ q_1 + q_2 \left( \frac{c_1}{C_1} + \alpha_1 \left[ 1 - \frac{c_1}{C_1} \right] \right) \right] + \ldots$$

Readership of language 1 increases if accessibility to language 1 increases for non-learners of language 1. Thus, by this direct effect, the incentives to use language 1 as a medium of production increase with $\alpha_1$. 
However, there is also the **indirect effect** that works through the learning decision of language-2 speakers. They are less enthusiastic about learning language 1 if language 1 is more accessible. This can be seen from the proportion of readers with mother tongue 2 who learn language 1. This language-learning effect reduces the producers’ incentives to employ language 1 following an increase of $\alpha_1$.

$$R(n_1) = n_1 \left[ q_1 + q_2 \left( \alpha_1 + \frac{\bar{c}_1}{C_1} (1 - \alpha_1) \right) \right] + ...$$
A snowball-effect like mechanism might occur: One producer adopts a language and therefore, that language is more attractive to language learners, so that other producers also tend to adopt it.

Exactly one symmetric equilibrium where all producers choose language 2

Two symmetric equilibria. In one, all producers choose language 1, in the other, language 2.

Exactly one symmetric equilibrium where all producers choose language 1

$q_1$

$q_2$
Non-option demand 1

- Limited capacity of actual reading
- \( q \) capacity for reading in the overall population: each reader spends 10 days a year on reading
- no learning
- two producers \( A \) and \( B \)
- \( A \) decides on \( n^A_1 \) and \( n^A_2 = n^A - n^A_1 \)
- two languages 1 and 2

\[
R^A \left( n^A_1, n^B_1 \right) = q_1 \frac{n^A_1}{n^A_1 + n^B_1} + q_2 \frac{n^A_2}{n^A_2 + n^B_2}
\]

\[
R^A + R^B = q_1 + q_2
\]
Proposition:
Unique Nash equilibrium

\[
\left( n_1^A \right)^* = n^A \frac{q_1}{q_1 + q_2}
\]

\[
\left( n_1^B \right)^* = n^B \frac{q_1}{q_1 + q_2}
\]

which leads to

\[
\frac{\left( n_1^A \right)^* + \left( n_1^B \right)^*}{\left( n_2^A \right)^* + \left( n_2^B \right)^*} = \frac{q_1}{q_2} = \frac{\left( n_1^A \right)^*}{\left( n_2^A \right)^*} = \frac{\left( n_1^B \right)^*}{\left( n_2^B \right)^*}
\]
period p
language 0 with production \( p_0 = p \)

period n
language 0 with production \( n_0 \)
language 1 with production \( n_1 \)

period p (vernac. 0)
lang. 0 with prod. \( p_0 = p \)
period n (vernac. 1)
lang. 0 with prod. \( n_0 \)
lang. 1 with prod. \( n_1 \)
period f (vernac. 2)
lang. 0 with prod. \( f_0 \)
lang. 1 with prod. \( f_1 \)
lang. 2 with prod. \( f_2 \)
Old-language learning with two periods

Proposition:
In the diachronic model with two time periods p (past) and n (present), the producer in period n does not use language 0 for literary production.
Proposition:
In the diachronic model with three time periods p (past), n (present), and f (future), if producers aim to maximize current readership, only, the producers in periods n and f use only their respective vernaculars for literary production.
Proposition:
In the diachronic model with three time periods p (past), n (present), and f (future), if producers aim to maximize long-term readership, the producer in period f uses his vernacular, only. Assume $q_n = q_f$.

We find:

- The producer in period n employs language 0 if C is sufficiently small, and language 1, otherwise.
- The chances for language 0 being used are smaller with increasing literary production ($p_0 < n$) than with decreasing literary production ($p_0 > n$).
period p (vernac. 0)
lang. 0 with prod. \( p_0 = p \)

\[ \text{period n (vernaculars 1 and 2)} \]
\[ \text{language 0 with production } n_0 \]
\[ \text{language 1 with production } n_1 \text{ and population } q_1 \]
\[ \text{language 2 with production } n_2 \text{ and population } q_2 \]
A forking model II

Proposition:
In the diachronic forking model with two time periods p (past) and n (present) and no language learning, assume $\alpha < \alpha_0$ (Old French closer to Latin than to Old Spanish) and $q_2 < q_1$. Then, the producer in period n employs language 0 if

- $\alpha_0$ is relatively large,
- $\alpha$ is small in comparison with $\alpha_0$ (Old French much closer to Latin than to Old Spanish), and
- the population sizes do not differ too much.

Otherwise, he employs language 2.
Proposition: If, however, we have $\alpha > \alpha_0$ (Old French closer to Old Spanish than to Latin) and, again, $q_2 < q_1$, the producer in period $n$ employs language 0 if

- $C$ is sufficiently small
- $\alpha$ is relatively large and $\alpha_0$ is relatively small (Old French much closer to Old Spanish than to Latin), and
- production is decreasing ($p_0 > n$) or only mildly increasing (see article).

Otherwise, he employs language 2.