1. Lecture (2 SWS): weakly

2. Problem session/Tutorial (1 SWS): to be announced

3. Grading:
   - Exam (90 min.)
   - Essay (20 pages) probably in collaboration with GIZ + presentation
   - Exam and essay have equal weights

4. Language is English - take this a an opportunity and not as an obstacle!! Moreover, the language is formal - don’t be scared!

5. My office hour: I am there - in I209 - for you on Tuesdays 5-6 pm and whenever I am available.
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1.1 Introduction - Some facts

- World population: around 7 billion

- 1 billion don’t have access to clean drinking water; 10,000 children die every day from diseases caused by contaminated water.

- 2.4 billion don't have access to sanitation

- Life expectancy at birth varies between 53 years to 77 years in very poor and very rich countries
1.1 Introduction - Some facts

World population growth, 1750-2150


Copyright © 2001 Population Reference Bureau
1.1 Introduction - Some facts

World Population Distribution by Region, 2003 and 2050 (Source: Todaro and Smith, 2009)

(a) Total population 2003: 6.313 billion
(b) Total population 2050: 9.198 billion

Source: Data from Population Reference Bureau.
1.1 Introduction - Some facts

The Parade of World Income (Source: D. Weil, 2005)

1.1 Introduction - Some facts

Evolution of world GDP
1.1 Introduction - Some facts

- Western Europe
- Western Offshoots
- Latin America
- East Europe
- Asian countries
- Africa - Afrique

GDP per Capita (int. $)

- 1820
- 1870
- 1913
- 1950
- 2000

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1.1 Introduction - Some facts

GDP per Capita: Selected OECD Countries
1.1 Introduction - Some facts

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1.1 Introduction - Some facts

GDP per Capita in Africa

int. $
1.1 Introduction - Some facts
1.1 Introduction - Some facts

Divergence among East German regions (Schäfer and Steger, 2010)

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1.1 Introduction - Some facts

→ stratification and the possible shift towards a bi-modal world income distribution has been emphasized first by Quah = ”Twin Peaks” phenomenon (Durlauf and Quah, 1994)

⇒ multiple equilibria ?

Why do societies choose bad equilibria???
1.1 Introduction - Some facts

Comparing wealthy countries to their own history

- Life expectancy at birth rose in Japan from 35 to 81 within 1 century
- Due to better nutrition, average height of British men rose 9.1 cm between 1775 and 1975
- South-Korea and the Philippines were nearly identical economies during the sixties

What about (relatively) poor countries?

- Egypt, Indonesia, and Brazil face higher life expectancies than members of the British nobility class at the beginning of the 20th century
- The fraction of people with income of less than 1$ per day fell by one third between 1980 and 1998
- The average African household consumed 20% less in 1998 than it did 25 years before
- Argentina was one of the wealthiest countries at the beginning of the 20th century but failed to close up
1.1 Introduction - Some facts

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→ relative ranking of countries have changed little between 1960 and 2000

→ origins of the very large income differences across nations are not found in the post-war era. There are striking growth differences during this period but the world income distribution has been more or less stable, with a slight tendency towards becoming more unequal.

→ the "big divergence" has taken place over the last 200 years.
1.2 Introduction - What is economic development?

The puzzle of why economies perform differently and the theory of economic growth are as old as the theory of economic thought:


After WWII the research question did not disappear, but was divided among different research fields:

- Macroeconomics
- Economic Development
- Industrial Organization
- Economic History

Since two decades Economic Growth is an independent research field whereby others claim that Development Economics was an amalgamation of traditional fields.
What distinguishes Growth Theory from Development Economics?
1.2 Introduction - What is economic development?

Distinction between economic growth and economic development?

- Developed countries of today are those that have grown steadily over the past 200 years
- Economic growth and economic development may therefore be part of the same process
- Economic development is a dynamic process over time, i.e. it makes good sense to employ tools of dynamic macroeconomics
- However, models of economic growth emphasize different aspects:
  - Focus on balanced growth or transitional dynamics leading to balanced growth
  - Behavior along or near a balanced growth path may be a good approximation for developed countries (Kaldor facts)
  - Economists (f.e. Simon Kuznets) have argued that many aspects of economic growth are far from the balanced growth path benchmark
1.2 Introduction - What is economic development?

Simon Kuznets, *Modern Economic Growth*, 1966:

We identify the economic growth of nations as a sustained increase in per capita or per worker product, most often accompanied by an increase in population and usually by sweeping structural changes. ... industrial structure,... the process of industrialization,... the distribution of population between the country side and the cities, ... urbanization,... relative economic position of groups ... distinguished by employment status, ... the distribution of product by use - among household consumption, capital formation, and the government consumption...

→ in the absence of a unified theory it may be necessary or useful to analyze economic development and economic growth separately.
In recent years research has done substantial progress in developing such unified theories.

Nevertheless, current growth literature is far from a satisfactory framework that can achieve this objective (Acemoglu, 2009, pp. 694).

Moreover, the distinction between the theory of economic growth and economic development is rather artificial and due to different strands of literature on the two topics.

From a scientific point of view the literature on development is rather non-theoretical, i.e. narrative and documents market failures and economic relationships in less-developed economies.

From the point of view of modern macro economic theory, we would like to get a deeper understanding: Why are some countries developed while the majority is not?
Introduction - What is economic development?

The "growth" view:

By the problem of economic development I mean simply the problem of accounting for the observed pattern, across countries and across time, in levels and rates of growth of per capita income. This may seem too narrow a definition, and perhaps it is, but thinking about income patterns will necessarily involve us in thinking about many other aspects of societies too, so I would suggest that we withhold judgement on the scope of this definition until we have a clearer idea of where it leads us. [R. E. Lucas (1988)]
D. Ray, Development Economics, 1998
1.2 Introduction - What is economic development?

The "development" view:

We should never lose sight of the ultimate purpose of the exercise, to treat men and women as ends, to improve the human conditions, to enlarge people’s choices... . A unity of interests would exist if there were rigid links between economic production (as measured by income per head) and human development (reflected by human indicators such as life expectancy or literacy, or achievements such as self-respect, not easily measured). But these two sets of indicators are not very closely related. [P. P. Streeten (1994]
1.3 Introduction - The "development" view

Todaro and Smith: ...[development economics] is the economics of contemporary poor, underdeveloped nations with varying ideological orientations, diverse cultural backgrounds, and very complex yet similar economic problems...

Traditional economic measure(s): increase in GDP or in the real per capita GDP

"New Economic View of Development": development is a multidimensional process involving major changes in social structures, popular attitudes, and national institutions, acceleration of economic growth, the reduction of inequality, eradication of poverty. → this is precisely what (parts) of modern growth theory tries to do!
Sen’s ”Capabilities” Approach: *Economic growth cannot be sensibly treated as an end in itself. Development has to be more concerned with enhancing the lives we lead and the freedoms we enjoy.*

→ Todaro and Smith’s Three Objectives of Development:

1. To increase the availability and widen the distribution of basic life-sustaining goods

2. To raise levels of living

3. To expand the range of economic and social choices
In September 2000 the United Nations adopted eight Millennium Development Goals:

1. Eradicate extreme poverty and hunger
2. Achieve universal primary education
3. Promote gender equality and empower women
4. Reduce child mortality
5. Improve maternal health
6. Combat HIV/AIDS, malaria, and other diseases
7. Ensure environment sustainability
8. Develop a global partnership for development
1.4 Introduction - The "growth" view

The importance of economic growth

- The real per capita GDP in the US grew on average by 1.75% per year between 1870 and 1990.
- An annual growth rate of one percent less would match the long-run growth experience of India (0.64%), Pakistan (0.88%) or the Phillipines (0.86%) between 1900 and 1987.
- An annual growth rate of 0.75% would have generated an US real per capita GDP in 1990 close to that in Mexico and Hungary and less than that of Portugal and Greece.
- An annual growth rate of 2.75% instead would have generated an US real per capita GDP in 1990 which it will not reach until 2059.
- Japan’s real per capita GDP in 1990 was about 20 times that in 1890.
- The US real per capita GDP is 39 time the value of Ethiopia.
The importance of economic growth

- If Ethiopia would grow at a rate of 1.75% per year, it would take 239 years to catch up with the 1990 level of the US and still 152 years if it would start to grow at Japan's long-term growth rate of 2.75% per year.

- One digit differences in growth rates over thirty years have enormous consequences for standards of living:
  South Korea (6.8% per year between 1960 and 1990) versus Iraq (2.1%) → while South-Korea raised its real per capita GDP by factor 8, Iraq lowered by 0.5

- Singapore, Taiwan, Botswana, Malta, and Japan made similar growth experiences between 1960 and 1990 and raised their levels of real per capita GDP by a factor 5 (over one generation)
The importance of economic growth

- In order to understand differences in living standards it is important to understand the reasons for different growth experiences in different countries, since even small differences in growth rates lead cumulated over time to huge differences in the standard of living we may enjoy.

→ ...if we can learn about government policy options that have even small effects on the long-term growth rate, then we can contribute much more to the improvements in standards of living than has been provided by the entire history of macroeconomic analysis of counter-cyclical policy and fine tuning.

(Barro and Sala-i-Martin 1995)
1.4 Introduction - The "growth" view

Why should we care about cross-country income differences?

- High income levels reflect high standards of living
- However: economic growth might increase pollution and/or individual aspirations, such that the bundle of consumption no longer generates the same level of utility
- On the other hand: there are striking differences between standards of living when one compares a rich country with a less-developed one
1.4 Introduction - The "growth" view
1.4 Introduction - The "growth" view
1.5 Introduction - The pontifex between ”growth” and ”development”

Proximate versus fundamental causes of economic growth

- Ideally, one would like to answer the question why countries grow at different rates at a casual level
- This refers to the following thought experiment: *if, all else equal, a particular characteristic of the country where changed exogenously, what would be the effect on equilibrium growth?*
- Answering this question requires the isolation of endogenous variables
- For this reason it is fair to start with correlates = proximate cause of economic growth
1.5 Introduction - The pontifex between "growth" and "development"
1.5 Introduction - The pontifex between ”growth” and ”development”
1.5 Introduction - The pontifex between ”growth” and ”development”

Proximate versus fundamental causes of economic growth

- If proximate causes were important in generating large cross-country differences, why do certain economies fail to improve their technologies, invest more in physical capital, and accumulate more human capital?

⇒ Fundamental causes (Acemoglu, 2009)
  
  1. (history dependent) multiple equilibria → ”luck”

  2. geographic differences

  3. institutional differences

  4. cultural differences
Models you know consider labor as an exogenous factor of production.

BUT: labor input differs considerably among individuals with respect to its quantity and quality.

We may differentiate between:

- physical strength (= health)
- level of education

Note: economic development improves health and productivity of the working force.

→ Causality? Does increasing income per capita improve health or vice versa?
Unified Growth Theory seeks to capture the process of development over the entire course of human history. That is, capturing the epoch of Malthusian stagnation that characterized most of human history, the contemporary era of modern growth and the forces that triggered the transition between the two regimes.

Focus is the endogenous switch from one regime to another.

The insights are not generated by parametrically pre-determined multiple steady states but temporary stages - epochs - of growth.

Hypothesis: Hurdles faced by less developed countries remain obscure until the forces that triggered economic development in the developed world are thoroughly explored.
North (1990, p. 3):
- Institutions are the rules of the game in a society or, more formally, are the humanly devised constraints that shape human interaction.
- In consequence [institutions] structure incentives in human exchange, whether political, social, or economic.

Why are there inefficient institutions as a result of a social choice and what is the point of reference?
Source: Acemoglu 2009
Source: Acemoglu 2009
Growth theory isolates several factors as being responsible for sustained economic growth, for example: human capital, physical capital, research and development...

As has been argued already these factor stress the importance of correlates to economic growth, i.e. they are endogenous with respect to the economic environment.
institutional differences = differences in a broad cluster of social arrangements:
1. security of property rights
2. contracting institutions
3. entry barriers
4. incentives to provide public goods
5. ...

A tractable definition differentiates between
1. political institutions = rules affecting political decision process
2. economic institutions = economic arrangements (taxes, property rights...)

Institutions differ across countries and do matter for economic growth
→ Again: why do some societies choose institutions which are harmful for economic growth?
Since there are economic events that would benefit all members of society, the main ingredient of the political economy approach is social conflict.

Hence, individuals have conflicting preferences over economic institutions.

In general, societies consist of different groups with differing economic and political power.

The implementation of distortionary policies is therefore due to

1. Revenue extraction: attempt of a powerful group to transfer rents from other groups to themselves.
2. Factor price manipulation: enrichment of other groups may constitute a risk (replacement motive) to the powerful and/or decrease their profits.

Further source of inefficiencies: lag of commitment to future policies induce so-called hold-up problems, i.e. a range of policies comes into action after investments are undertaken.
Bad institutions may also be the reason why capital does not flow automatically to the poor countries.

But if there are bad institutions is it then a good idea to provide foreign aid at large?

Are Charter Cities a way to follow?
1.7 Introduction - Solow growth model

- Production function in per capita terms: \( y = f(k) \), where \( f(k) \) satisfies the neoclassical properties.
- Per capita savings: \( sf(k) \), with \( 0 < s < 1 \).
- Population grows with the rate \( n > 0 \) and capital depreciates with \( 0 < \delta < 1 \).
- Since \( I = S = \dot{K} + \delta K \), the capital intensity evolves according to
  \[
  \dot{k} = sf(k) - (n + \delta)k \tag{1}
  \]
  or
  \[
  \frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n + \delta). \tag{2}
  \]
  → poor economies should grow faster than rich economies.
Hypothesis that poor economies tend to grow faster per capita than rich ones, without controlling for any other characteristics is called *absolute convergence*.

Usually the growth rates are uncorrelated with the initial position the more heterogenous the set of countries is.

The theory works for homogenous countries or regions.

If steady states differ: *conditional convergence*.
1.7 Introduction - Solow growth model

\[ \frac{sf(k)}{k} \]

\[ k_p \quad k_r \quad k^* \quad k \]

\[ n+\delta \]

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1.7 Introduction - Solow growth model

Apart from the well known Ramsey model, the overlapping generations (OLG) model of Allais (1947), Samuelson (1958), and Diamond (1965) is the second basic micro-based approach in macro-economics.

OLG:
- at each point in time different generations are alive and may be interacting with each other through markets
- special feature: preferences of generations who are not alive may not be incorporated in current market transactions
- advantage: it is possible to study the aggregate effects of life-cycle saving
- The competitive equilibrium does in general not coincide with the planer solution (contrary to the Ramsey model).
- The competitive equilibrium may even not be Pareto optimal due to overaccumulation of life-cycle savers.
Individuals at different stages of their life-cycles interact on markets.

Simplest form of an OLG model is sufficient for many applications:

- Individuals live for two periods.
- At any point in time, the economy is composed of two cohorts (=generations): young and old
- Individuals work in their first period of life supplying inelastically one unit of labor earning a real wage of $w_t$.
- Individuals in their first period of life consume and save a part of their first-period income for second-period consumption.
- Savings of the young in period $t$ constitutes the capital stock that is used to produce output in $t + 1$.
- Old agents retire and consume their savings plus accrued interests.
Individuals live for two periods and derive utility out of first and second-period consumption $c_{1t}$ and $c_{2t+1}$, where lifetime utility is captured by

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \frac{1}{1+\rho} u(c_{2t+1}). \quad (3)$$

$\rho > 0$ captures the notion of pure time preference and $\beta = \frac{1}{1+\rho} < 1$ is referred to as the discount factor of future consumption, such that $\rho = \frac{1-\beta}{\beta}$.

Moreover, $u(\cdot)$ is subject to usual concavity assumptions: $u'(\cdot) > 0; u''(\cdot) < 0$.

As agents work only in their first period of live, wage income $w_t$ equals the present value of lifetime spending on consumption

$$w_t = c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}}. \quad (4)$$
The number of individuals born at time $t$ and working in period $t$ equals $N_t$.

Population grows at the exogenous rate $n$, such that $N_t = N_0 (1 + n)^t$, where $N_0$ is reasonably fixed to 1.

The production side is characterized by a set of competitive firms, and is represented by an aggregate production function satisfying the usual neoclassical properties:

$$ Y_t = F(K_t, L_t) $$

$$ y_t = \frac{Y_t}{L_t} = f(k_t), $$

where $k_t = \frac{K_t}{L_t}$.

Firms maximize profits by taking wages $w_t$ and rental prices for capital $r_t$ as given.
Savings by a member of generation $t$, $s_t$, are determined by the solution to the following maximization problem

$$\max_{c_1t; c_{2t+1}} u(c_1t) + \beta u(c_{2t+1}),$$  \hspace{1cm} (7)

subject to:

$$c_{1t} + s_t = w_t$$  \hspace{1cm} (8)

$$c_{2t+1} = (1 + r_{t+1})s_t.$$  \hspace{1cm} (9)

As a first-order condition to this problem we yield the familiar Euler equation

$$u'(c_1t) = \beta (1 + r_{t+1})u'(c_{2t+1}) = \frac{1 + r_{t+1}}{1 + \rho}u'(c_{2t+1}).$$  \hspace{1cm} (10)
Combining the Euler equation with the budget constraint yield the following implicit function that determines savings per capita in equilibrium

\[ s_t = s(w_t, r_{t+1}), \]  

(11)

where \( 0 < s_w < 1 \) and \( s_r \geq 0! \) (More details follow further below)

- Saving is an increasing function in wages (current and future consumption are normal goods).
- The effect of an increase in the interest rate is ambiguous:
  - \( r_{t+1} \uparrow \rightarrow \) price of \( c_{2t+1} \downarrow \). Hence individuals shift consumption from the first to the second period.
  - \( r_{t+1} \uparrow \rightarrow \) income effect that increase the set of feasible consumption possibilities.
  - \( \Rightarrow \) the net effect of income and substitution effects is ambiguous! If the elasticity of consumption in both periods \( > 1 \), the substitution effect dominates and \( s_r > 0 \).
1.8 Introduction - OLG model

- Competitive markets induce firms to hire labor and capital to the point where the respective marginal productivities equal the wage rate and the interest rate

\[ w_t = f(k_t) - kf'(k_t), \quad \text{(12)} \]
\[ r_t + \delta = f'(k_t). \quad \text{(13)} \]

- The supply of capital in period \( t \) is determined by the saving decision of the cohort born in \( t - 1 \).

- The factor market equilibrium is therefore determined by the last two equations.
1.8 Introduction - OLG model

- Let’s consider an individual born at time $t$. As we said earlier, agents born at $t$ work, consume, save and retire.
- Savings are necessary to finance retirement-consumption in the next period, but constitute also the aggregate capital stock in the subsequent period.
- In equilibrium, we know that $I = S$, such that

$$\Delta K_{t+1} + \delta K_t = I_t = S_t,$$

(14)

$$K_{t+1} - K_t = S_t - \delta K_t.$$  

(15)

Note that $K_{t+1} = S_t$, if $\delta = 1$, i.e. capital depreciates entirely within one period.
Let's assume without further loss of generality that $\delta = 1$ and therefore $K_{t+1} = S_t$

$$K_{t+1} = N_t s(w_t, r_{t+1}),$$

$$(1 + n)K_{t+1} = s(w_t, r_{t+1}).$$

Apparently, we yield for the evolution of capital per worker

$$k_{t+1} = \frac{s[w(k_t), r(k_{t+1})]}{1 + n},$$

$$k_{t+1} = \frac{s[f(k_t) - k_t f'(k_t), f'(k_{t+1})]}{1 + n}.$$  

The last equation is known as the fundamental law of motion of the OLG economy or the savings locus.

A steady state is given by a stationary solution to this equation, such that $k_{t+1} = k_t = k^*$

$$k^* = \frac{s[f(k^*) - k^* f'(k^*), f'(k^*)]}{1 + n}.$$  

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The properties of the savings locus and the characteristics of the dynamic behavior of the model depend essentially on the sign of the following derivative

\[
\frac{dk_{t+1}}{dk_t} = \frac{-s_w k_t f''(k_t)}{1 + n - s_r f''(k_{t+1})}.
\]

If \( s_r > 0 \), the denominator is positive, and \( \frac{dk_{t+1}}{dk_t} > 0 \)
1.8 Introduction - OLG model

The diagram illustrates the OLG model with the following equations:

\[ k_{t+1} = k_t \]

The graph shows the relationship between the capital stock at time \( t \) and the capital stock at time \( t+1 \). The curves represent different scenarios, with curves A, B, and C indicating varying levels of capital accumulation.

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Given the set of assumptions so far, the OLG model can lead to multiple equilibria, to a unique stable equilibrium, or to no meaningful equilibrium at all.

In other words: the model makes few predictions without having put further structure into it.

Local stability requires

\[
\left. \frac{d k_{t+1}}{d k_t} \right|_* = \left| \frac{-s_w k^* f''(k^*)}{1 + n - s_r f''(k^*)} \right| < 1.
\]

Instability may arise for example due to \( s_r < 0 \).
To obtain reasonable results it is necessary to specify functional forms. In some cases it is possible to determine the sign of implicit derivatives.

In order to obtain closed form solutions, quite restrictive assumptions with respect to the functional forms of the production and/or the utility function are necessary.

The condition for a non-oscillatory and stable steady state reads as

$$0 < \frac{-s_w k^* f''(k^*)}{1 + n - s_r f''(k^*)} < 1.$$  \hspace{1cm} (23)
Suppose that utility functions take the familiar CRRA form

$$U_t(c_{1t}, c_{2t+1}) = \frac{c_{1t}^{1-\sigma} - 1}{1-\sigma} + \beta \left( \frac{c_{2t+1}^{1-\sigma} - 1}{1-\sigma} \right),$$

(24)

where $\sigma > 0$ and $\beta \in (0, 1)$.

Technology is subject to the following Cobb-Douglas specification

$$f(k_t) = k_t^\alpha.$$

(25)
Consequently, consumer optimization implies

\[
\frac{c_{t+1}}{c_t} = (\beta (1 + r_{t+1}))^{\frac{1}{\sigma}} \tag{26}
\]

Making use of the budget constraint yields

\[
\beta (1 + r_{t+1})^{1-\sigma} s_t^{-\sigma} = (w_t - s_t)^{-\sigma}, \tag{27}
\]

such that

\[
s_t = \frac{w_t}{\psi_{t+1}}, \tag{28}
\]

with \(\psi_{t+1} = [1 + \beta^{-\frac{1}{\sigma}} (1 + r_{t+1})^{-\frac{1-\sigma}{\sigma}}] > 1\).
Now we are able to state that savings are always less than earnings, since

$$s_w = \frac{1}{\psi_{t+1}} \in (0, 1), \quad (29)$$

and

$$s_r = \left(\frac{1 - \sigma}{\sigma}\right) (\beta(1 + r_{t+1})^{-\frac{1}{\sigma}} \frac{s_t}{\psi_{t+1}}). \quad (30)$$

Obviously, $0 < s_w < 1$.

Moreover, $s_r \geq 0$, whenever $\sigma \leq 1$ and $s_r = 0$, if $\sigma = 1$.

$\sigma > 1$: income effect dominates the substitution effect.

$\sigma < 1$: substitution effect dominates the income effect.

$\sigma = 1$: substitution effect = income effect.
Exploiting the above mentioned equilibrium conditions, we yield

\[ k_{t+1} = \frac{s_t}{1+n} = \frac{w_t}{(1+n)\psi_{t+1}} \]  

(31)

Given \( \delta = 1 \) which implies \( 1 + r_{t+1} = f'(k_{t+1}) \), we yield

\[ k_{t+1} = \frac{f(k_t) - k_t f'(k_t)}{(1+n)[1 + \beta^{-\frac{1}{\sigma}} f'(k_{t+1})^{\frac{1-\sigma}{\sigma}}]} \]  

(32)

In light of \( y_t = k_t^\alpha \), we finally obtain

\[ k_{t+1} = \frac{(1 - \alpha)k_t^\alpha}{(1+n)[1 + \beta^{-\frac{1}{\sigma}} (\alpha k_{t+1}^{\alpha-1})^{\frac{1-\sigma}{\sigma}}]} \]  

(33)

The steady state solution is implicitly defined by

\[ (1 + n)[1 + \beta^{-\frac{1}{\sigma}} (\alpha (k^*)^{\alpha-1})^{\frac{1-\sigma}{\sigma}}] = (1 - \alpha)(k^*)^{\alpha-1}. \]  

(34)
Hence, we yield

\[
\left| \frac{dk_{t+1}}{dk_t} \right|_* = \left| \frac{1}{\frac{1}{\alpha} + (1 + n) \left( \frac{1-\sigma}{\sigma} [\beta \alpha(k^*)^\alpha - 1] - \frac{1}{\sigma} \right)} \right| < 1, \quad (35)
\]

Note that this expression approaches \( \frac{1}{\frac{1}{\alpha} - (1+n)} \) as \( \sigma \to \infty \), which is greater than -1, since \( \frac{1}{\alpha} > n \).

Since, the savings locus is also monotone, there is a unique and stable equilibrium, for any \( \sigma > 0 \).
Even the above CRRA-specification of the OLG model is not able to generate closed form solutions. For this reason many applications use log-preferences \((\sigma = 1)\). This specification is sometime referred to as the canonical OLG model.

Lifetime utility of a member of generation \(t\) is given by

\[
U_t(c_{1t}, c_{2t+1}) = \log c_{1t} + \beta \log c_{2t+1},
\]

where as before \(\beta \in (0, 1)\).

Production again is subject to

\[
y_t = k_t^\alpha.
\]
Since the Euler equation reduces to

\[ \frac{c_{2t+1}}{c_{1t}} = \beta(1 + r_{t+1}), \]  

(38)

savings satisfy

\[ s_t = \frac{\beta}{1 + \beta} w_t. \]  

(39)

Goods market equilibrium leads us to

\[ k_{t+1} = \frac{s_t}{1 + n} = \frac{\beta w_t}{(1 + \beta)(1 + n)}, \]  

(40)

\[ k_{t+1} = \frac{\beta(1 - \alpha)k_t^\alpha}{(1 + \beta)(1 + n)}. \]  

(41)

In light of the last expression we yield \( k^* = k_{t+1} = k_t \) as

\[ k^* = \left[ \frac{\beta(1 - \alpha)}{(1 + n)(1 + \beta)} \right]^{\frac{1}{1-\alpha}}. \]  

(42)
Using either (41) together with (42) or (35) for $\sigma = 1$, implies

$$\frac{dk_{t+1}}{dk_t} = \alpha < 1.$$  \hspace{1cm} (43)

In consequence, there is a unique and stable steady state.

Due to the existence of log-preferences, the savings rate is independent of income and substitution effects, which makes the behavior of this model identical to that of the basic Solow model.