Assignment #12: IRS and competitive equilibrium

- **IRS and competitive equilibrium: basic setup**

  **Firms.** Consider a one sector economy. There is continuum of length one of identical firms. Each firm has access to the following technology

  \[ Y(i) = A K(i)^\alpha L(i)^\beta \quad A > 0; \quad \alpha + \beta > 1; \quad i \in [0, \ldots, 1] \]  

  where \( Y(i) \) denotes output of firm \( i \), \( K(i) \) physical capital employed by firm \( i \), and \( L(i) \) is labor employed by firm \( i \). Notice that there are increasing returns to scale (IRS) in \( Y \)-production at the level of the individual firm.

  **Households.** On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market.

  **Question.** Does total household income equal total output? Stated differently, is the assumption of IRS compatible with a competitive equilibrium? Provide a concise economic reasoning.

- **Solution (basic setup)**

  Firms maximize profits by taking the interest rate \( r \) and the wage rate \( w \) as given. This implies

  \[ r = p_Y \alpha \frac{Y}{K} \quad \text{and} \quad w = p_Y \beta \frac{Y}{L} \]  

  Capital and labor earn their value marginal products, respectively. This factor payment scheme would, however, represent a logically inconsistency since total factor earnings would exceed the value of overall production

  \[ p_Y \alpha \frac{Y}{K} + p_Y \beta \frac{Y}{L} = (\alpha + \beta) p_Y Y > p_Y Y \]

  \( \Rightarrow \) The assumption of IRS is not compatible with a competitive equilibrium.

- **IRS and competitive equilibrium: Marshallian externalities**

  **Firms.** Consider a perfectly competitive, one sector economy. There is a continuum of length one of identical firms. The output technology of the individual firm reads as follows

  \[ Y(i) = K(i)^\alpha L(i)^\beta \quad \alpha + \beta = 1; \quad \alpha, \beta > 0; \quad i \in [0, \ldots, 1] \]  

  where \( \bar{K} = \int_0^1 K(i) \, di \) and \( \bar{L} = \int_0^1 L(i) \, di \) denote the average (across firms) levels of capital and labor, respectively. (Notice that \( \bar{K} \) doesn’t change if \( K(i) \) changes by one unit since the individual is of mass zero. The discrete analogue of this statement is as follows. Imagine an economy which comprises, say, 10 000 firms. If one firm would increase its \( K(i) \) by one unit, then the average stock of capital, \( \frac{1}{10000} \sum_{i=1}^{10000} K(i) = \frac{1}{10000} K(1) + \ldots + \frac{1}{10000} K(10000) \), remains approximately constant because the weight of any individual firm, \( \frac{1}{10000} \), is small. More specifically, if an individual firm increases its capital stock by one unit, then the average capital stock increases by \( \frac{1}{10000} \) units. The same reasoning applies to \( \bar{L} \) and \( L(i) \).) The implicit assumption is that there are positive spill-over effects in the production sphere. Total factor productivity of the individual firm is higher the higher is the overall input of capital and labor. Notice that there are constant returns to scale (CRS) at the level of the individual firm but IRS at the aggregate level.

  **Households.** On the household side there is mass one of identical households who own the capital stock \( K \) and are endowed with \( L \) units of labor, which are supplied inelastically to the labor market.

  **Question.** Does total household income equal total output? Stated differently, is the assumption of IRS compatible with a competitive equilibrium? Provide a concise economic reasoning.

- **Solution (Marshallian externalities)**

- **IRS and competitive equilibrium: Monopolistic competition**

  **Firms.** The economy comprises two sectors. In the final output sector (CRS, perfectly competitive) there is mass one of identical firms. The output technology reads

  \[ Y(j) = \left( \int_0^1 x(i)^4 \, di \right)^{1/\lambda} \quad 0 < \lambda < 1; \quad j \in [0, \ldots, 1] \]  

  In the intermediate goods sector (IRS, monopolistic competition) there is mass one of identical firms. Each firm has access to the following technology

  \[ x(i) = K(i)^\alpha L(i)^\beta \quad \alpha, \beta > 0; \quad \alpha + \beta > 1 \]
**Households.** On the household side there is mass one of identical households who own the capital stock $K$ and are endowed with $L$ units of labor, which is supplied inelastically to the labor market. Households are the owners of the firms. Hence, total earnings of the representative household is given by

$$\text{Earnings} = \frac{\pi_Y}{\text{profit of typical firm}} + \frac{\pi(i)}{\text{profit of typical s-firm}} + rK + wL$$

(11)

**Market structure.** Factor markets are perfectly competitive. The final output sector is perfectly competitive. The intermediate good sector is monopsonistically competitive.

**Question.** Does total household income equal total output? Stated differently, is the assumption of IRS compatible with a competitive equilibrium? Provide a concise economic reasoning.

- **Solution (Monopolistic competition)**

  **Final output sector.** Final output firms maximize profits by taking $p(i)$ as given ($p_Y = 1$)

$$\max_{x(i)} \left\{ \int_0^q x(i)^{\lambda} \, di - \int_0^q p(i) \, x(i) \, di \right\}$$

(12)

FOC : \( \frac{1}{\lambda} \left( \int_0^q x(i)^{\lambda} \, di \right)^{1-1} \lambda x(i)^{1-1} - p(i) = 0 \quad \text{for all } i \in [0, ..., 1] \)

(13)

\( \left( Y^1 \right)^{1-1} x(i)^{1-1} - p(i) = 0 \)

(14)

\( p(i) = Y^1 \lambda x(i)^{1-1} \) \hspace{1cm} \text{(inverse demand)}

**Intermediate goods sector.** Intermediate goods firm maximize profits by taking $r$ and $w$ as given

$$\max_{K,L} \left\{ Y^{1-\lambda} K(i)^{\lambda a} L(i)^{\lambda b} - rK(i) - wL(i) \right\}$$

(16)

$$\max_{K,L} \left\{ Y^{1-\lambda} K(i)^{\lambda a} L(i)^{\lambda b} - rK(i) - wL(i) \right\}$$

(17)

FOC1 : \( Y^{1-\lambda} \lambda a K(i)^{\lambda a-1} L(i)^{\lambda b} - r = 0 \) \hspace{1cm} \( r = \lambda a \frac{Y^{1-\lambda} x(i)^{1-1}}{K(i)} \)

(18)

FOC2 : \( Y^{1-\lambda} \lambda b L(i)^{\lambda b-1} - w = 0 \) \hspace{1cm} \( w = \lambda b \frac{Y^{1-\lambda} x(i)^{1-1}}{L(i)} \)

(19)

**Remark.** Notice that $K$ and $L$ are underpaid relative to the perfect-competition equilibrium. This can be seen by writing, say, \( r = \lambda a \frac{Y^{1-\lambda} x(i)^{1-1}}{K(i)} \) (due to, in equilibrium, \( Y = p(i) \, x(i) \). The reason is that when deciding upon the optimal amount of, say, capital, the monopolist takes into account that an increased employment of capital leads to more output and hence deteriorates the price $p(i)$.

**Equilibrium.** Since the final output sector is perfectly competitive, profits $\pi_Y$ must vanish in equilibrium

$$\pi_Y = Y - \int_0^q p(i) \, x(i) \, di = 0 \quad \Rightarrow \quad Y = \int_0^q p(i) \, x(i) \, di \quad \text{in symmetric equ.}$$

(20)

Profits of the typical intermediate good firm $\pi(i)$ may then be expressed as follows

$$\pi(i) = p(i) \, x(i) - \frac{\lambda \alpha Y}{K(i)} = Y - \lambda \alpha Y - \lambda \beta Y = (1 - \lambda (\alpha + \beta)) Y$$

(21)

Total earnings of the representative household may accordingly be expressed as follows

$$\left( 1 - \lambda (\alpha + \beta) \right) Y + \lambda \alpha \frac{Y}{K} + \lambda \beta \frac{Y}{L} = Y$$

(22)

\( \Rightarrow \) The assumption of IRS is compatible with a competitive equilibrium.