Interest rate rules and monetary targeting:

What are the links?*

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Abstract:

The paper derives the monetary policy reaction function implied by money growth targeting. It consists of an interest rate response to deviations of the inflation rate from target, to the change in the output gap, to money demand shocks and to the lagged interest rate. We show that this type of inertial interest rate rule characterises the Bundesbank’s monetary policy from 1979 to 1998 quite well. This result is robust to the use of real-time or ex post data. The main lesson is that, in addition to anchoring long-term inflation expectations, monetary targeting introduces inertia and history-dependence into the monetary policy rule. This is advantageous when private agents have forward-looking expectations and when the level of the output gap is subject to persistent measurement errors.

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Interest rate rules and monetary targeting: What are the links?

1 Introduction

There is an extensive literature on optimal and estimated monetary policy reaction functions. These range from the "classic" Taylor rule (Taylor, 1993) and numerous variants of it (e.g. Clarida et al., 1998; Mehra, 2001; Christiano and Rostagno, 2001; Gerlach-Kristen, 2003; Chadha et al., 2004) to nominal income rules (e.g. McCallum and Nelson, 1999; Rudebusch, 2002) and different specifications of speed limit policies (Orphanides, 2003b; Walsh, 2004; Stracca, 2007). In the last decade, the most prominent monetary policy rules were those in the spirit of Taylor (1993). According to these rules, the short-term real interest rate should be raised if inflation increases above target and/or if the level of real output rises above trend. The popularity of such rules stems from their simplicity and their alleged robustness across a wide array of macroeconomic models. In addition, the case for Taylor rules has been strengthened by the claim made by Clarida et al. (1998) and others that the monetary policy of many central banks, especially the Fed’s monetary policy under Paul Volcker and Alan Greenspan and the Bundesbank’s monetary policy during the era of monetary targeting (1979-1998) can well be captured by a forward-looking variant of the Taylor rule.

However, one shortcoming of these studies is that they abstract from the measurement problems which policymakers face with respect to key variables entering the Taylor rule like the equilibrium level of the real interest rate and the level of the output gap. For the US, Orphanides (2001, 2003b) has demonstrated that the use of real-time information can considerably change the outcome of an analysis of past monetary policy decisions. In particular, he finds that a Taylor rule based on real-time data tracks the Fed’s monetary policy in the 1970s quite closely and thus would not have been helpful in avoiding the policy mistakes of that era which can be identified today with the advantage of hindsight. In a similar vein, Gerberding et al. (2004, 2005) have shown that the use of real-time data for Germany considerably changes the assessment of the Bundesbank’s monetary policy reaction function. According to their analysis, the Bundesbank did not respond to the level of the output gap as suggested by the Taylor
rule, but rather to the change in the output gap as well as to deviations of (expected) inflation and money growth from their respective target values. Furthermore, their results suggest that the monetary policy of the Bundesbank was characterised by a high degree of interest rate inertia.

Interestingly, targeting the rate of change rather than the level of the output gap has recently been advocated by a number of authors, such as Orphanides (2003a) and Walsh (2003, 2004). They point out that output growth targeting is advantageous if estimates of the level of the output gap are subject to much greater uncertainty than estimates of its change (as has historically been the case). Another advantage is that targeting the change in the output gap makes monetary policy more history-dependent, which is an important element of the optimal commitment policy in forward-looking models (Woodford, 1999). However, the latter argument has been put forward only recently, and thus does not answer the question why the Bundesbank might have looked more at changes than at the level of the output gap.

In the present paper, we take up this question and argue that the Bundesbank’s focus on inflation and output growth - and the resulting robustness against misperceptions of the output gap - was a direct consequence of its use of money growth as an intermediate target variable. To shed further light on this issue, we develop an analytical framework which allows us to derive the interest rate reaction function implied by monetary targeting (part 2 of the paper). We do this for the simple case of strict monetary targeting, but we also consider several modifications. In particular, we allow for the possibility that the central bank accommodates shocks to money demand, and we take into account that the central bank’s objective function may include other targets besides the money growth target. In our model, money-based interest rate rules feature a response to the lagged interest rate, to deviations of inflation from target, to the change in the output gap and possibly, but not necessarily, an additional response to short-run movements of money. In the third part of the paper, we show that this type of inertial interest rate rule characterises the Bundesbank’s monetary policy from 1979 to 1998 quite well. Furthermore, we demonstrate that this result is robust to the use of ex post or real-time data. In section 4, we discuss the economic reasoning and consequences of all the arguments incorporated in the interest rate rule. Section 5 summarises and concludes.
2 Mapping monetary targeting into an interest rate reaction function

From 1975 to 1998, the Bundesbank announced annual targets for monetary growth. According to the Bundesbank’s own descriptions, the money growth targets were used as intermediate targets which served to attain the ultimate objective of safeguarding the value of the currency.¹ In this section, we develop an analytical framework that allows us to derive the interest rate rule implied by the Bundesbank’s version of monetary targeting (MT). As a starting point, we outline the method used for the derivation of the target values. In a second step, we derive the interest rate rule for the simple textbook case of strict monetary targeting, defined as a strategy where the central bank only cares about achieving the money growth target each period. In the third section, we show how this interest rate rule has to be modified if the central bank takes a medium-term perspective and follows a policy of accommodating shocks to money demand (as the Bundesbank did). Finally, we extend the analysis to the case where the central bank’s objective function includes other targets besides the (medium-term) money growth target.²

2.1 Determinants of the target values

The analytical background for the derivation of monetary targets is provided by the quantity theory of money.³ The quantity theory states that for given long-run rates of change in velocity and real output, trend inflation can be pinned down by controlling trend money growth:

\[ \Delta p_t = \Delta m_t - \Delta v_t + \Delta v_t \]  

(1)

where \( p, m, y \) and \( v \) are the (logs of the) price level, the money stock, real income and the income velocity of money, respectively, and the bars denote trend values. Starting from Eq. (1), the target value for money growth in year \( t \), \( \Delta m_t^T \), are derived from three macroeconomic benchmark figures: (1) a price assumption or price norm reflecting the maximum price increase the central bank is willing to tolerate, \( \Delta p_t^T \), (2) the growth rate

¹ See, for instance, Deutsche Bundesbank, Report for the Year 1980, p. 32.
² This last extension is quite natural against the background that money growth targets are only means (an indicator variable or intermediate target) to reach the final goal of monetary policy, e.g. price stability.
³ For the following considerations, see also Issing (1992) and Neumann (1997).
of production potential, $\Delta y^*_i$, and (3) the trend rate of change in the velocity of circulation $\Delta v^*_i$. While the first had to be set normatively, the latter two were unknown and hence had to be estimated:

$$\Delta m^*_i = \Delta p^*_i + \Delta y^*_i - \Delta v^*_i$$  \hspace{1cm} (2)

Despite its quantity-theoretic foundations, the implementation of monetary targeting in general differs from conventional monetarist thinking in a number of ways. First, the targets are usually not formulated in terms of the monetary base, but in terms of a broadly defined monetary aggregate.\(^4\) Second, central banks do not attempt to control the money stock directly, but follow an indirect approach of influencing money demand by varying key money market rates and bank reserves. Third, even though the actual targets usually have a one-year horizon, the Bundesbank Council for example stressed their medium-term nature and was prepared to tolerate short-term deviations of money growth from the target path if that seemed justified with respect to the ultimate objective of price stability.

### 2.2 Deriving the interest rate rule implied by strict monetary targeting

Despite this built-in flexibility (to which we will come back in section 2.3), it is useful to first derive the interest rate rule implied by the simple textbook case of “strict” monetary targeting.\(^5\) Hence, for the time being, let us assume that the central bank sets interest rates with a view of minimising deviations of money growth from target only ($L$ indicates the loss):

$$L_i = (\Delta m_t - \Delta m^*_i)^2$$  \hspace{1cm} (3)

Furthermore, to keep matters as simple as possible, let us assume that the link between money and interest rates can be adequately described by a standard money demand

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\(^4\) From 1975 to 1987, the Bundesbank, for example, targeted the central bank money stock, defined as currency in circulation plus the required minimum reserves on domestic deposits calculated at constant reserve ratios with base January 1974. The ratios were 16.6% for sight deposits, 12.4% for time deposits and 8.1% for savings deposits. After the mid-eighties, the heavy weight on currency increasingly proved to be a disadvantage, and when setting the target for 1988, the Bundesbank switched to the money stock M3. See Deutsche Bundesbank (1995), p. 81f.

\(^5\) Taylor (1999), Orphanides (2003b) as well as Kilponen and Leitemo (2007, 2008) also consider this case.
function that relates real money holdings to output $y$ (which proxies the transactions volume) and the interest rate $i$ (which proxies opportunity costs):\(^6\)

$$
(m_t - p_t) = \gamma_1 \cdot y_t - \gamma_2 \cdot i_t + \epsilon_t
$$

(4)

The parameters $\gamma_1$ and $\gamma_2$ denote the income elasticity and the (semi) interest elasticity of money demand, respectively. In (4), $\epsilon_t$ captures short-run dynamics and shocks to money demand. Hence, money growth $\Delta m$ is related to the inflation rate $\Delta p$, the change in the nominal interest rate $\Delta i$ and the growth rate of output $\Delta y$ through

$$
\Delta m_t = \Delta p_t + \gamma_1 \cdot \Delta y_t - \gamma_2 \cdot \Delta i_t + \Delta \epsilon_t
$$

(4a)

Eq. (4a) allows us to relate the Bundesbank’s estimate of the trend change in velocity in Eq. (2) to the determinants of long-run money demand. Inserting (4a) into the first difference of the quantity identity, $\Delta m_t + \Delta v_t = \Delta y_t + \Delta p_t$, and solving for $\Delta v_t$ yields:

$$
\Delta v_t = (1 - \gamma_1) \cdot \Delta y_t + \gamma_2 \cdot \Delta i_t - \Delta \epsilon_t.
$$

(5)

Hence, the trend change in velocity is a function of potential output growth and of the change in the steady-state level of the nominal interest rate (if there is any):\(^7\)

$$
\Delta v_t^* = (1 - \gamma_1) \cdot \Delta y_t^* + \gamma_2 \Delta i_t^*.
$$

(6)

Eq. (6) can now be used to replace $\Delta v_t^{*,ext}$ in Eq. (2). Abstracting from potential changes in the steady-state level of the nominal interest rate (as the Bundesbank did),\(^8\) the formula for the money growth target simplifies to

$$
\Delta m_t^T = \Delta p_t^T + \Delta y_t^{*,ext} - (1 - \hat{\gamma}_1) \Delta y_t^{*,ext} = \Delta p_t^T + \hat{\gamma}_1 \Delta y_t^{*,ext},
$$

(7)

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\(^6\) Such a money demand equation may be derived from first principles as in Woodford (2003).

\(^7\) According to the Fisher equation, the long-run nominal interest rate can be decomposed into the long-run (natural) real rate of interest and the long-run rate of inflation (and possibly risk premia $r_p$), that is $i^*_t = r^* + \Delta p_t^* (\text{or } r_p)$, so that any trend in $i_t^*$ must be due to an upward or downward trend in the real rate of interest and/or trend inflation (or risk premia).

\(^8\) While it can be argued that the successive lowering of the Bundesbank’s price norm from 5 % in 1975 to 2 % in 1985 did in fact lead to a decrease in the trend rate of inflation, the Bundesbank did not take this into account when deriving its money growth targets but assumed that the nominal interest rate is constant (or at least stationary) in the long-run, see Deutsche Bundesbank (1992, p. 27f). One reason for ignoring an expected (short-run) downward trend in the nominal interest rate due to a trend decline in inflation is that it would imply an upward correction of the money growth target which would in turn decrease the speed at which the trend rate of inflation is brought down.
where $\hat{\gamma}_1$ denotes the central bank’s estimate of the parameter $\gamma_1$. Combining Eq. (4a) and Eq. (7) yields the following formula for the money growth gap:

$$
\Delta m_t - \Delta m_t^* = \left( \Delta p_t - \Delta p_t^* \right) + \gamma_1 \left( \Delta y_t - \Delta y_t^{*,ext} \right) + \left( \gamma_1 - \hat{\gamma}_1 \right) \Delta y_t^{*,ext} - \gamma_2 \Delta i_t + \Delta c_t
$$

(8)

Setting the money growth gap equal to zero (as implied by the minimisation of Eq. (3)) and solving for the nominal interest rate, we arrive at the interest rate rule implied by strict monetary targeting:

$$
i_t = i_{t-1} + \frac{1}{\gamma_2} \left( \Delta p_t - \Delta p_t^* \right) + \frac{\gamma_1}{\gamma_2} \left( \Delta y_t - \Delta y_t^{*,ext} \right) + \frac{\left( \gamma_1 - \hat{\gamma}_1 \right)}{\gamma_2} \Delta y_t^{*,ext} + \frac{1}{\gamma_2} \Delta c_t
$$

(9)

Provided that the central bank’s estimate of $\hat{\gamma}_1$ is unbiased, the term $(\gamma_1 - \hat{\gamma}_1) \Delta y_t^{*,ext}$ can be subsumed into an error term, $u_t$, which leaves us with:

$$
i_t = i_{t-1} + \frac{1}{\gamma_2} \left( \Delta p_t - \Delta p_t^* \right) + \frac{\gamma_1}{\gamma_2} \left( \Delta y_t - \Delta y_t^{*,ext} \right) + \frac{1}{\gamma_2} \Delta c_t + u_t
$$

(9a)

According to Eq. (9a), strict monetary targeting implies interest rate inertia (due to the presence of the lagged interest rate among the feedback variables) and further interest rate reactions to deviations of inflation from target, to deviations of actual output growth from potential output growth (which is equivalent to the change in the output gap), and to $\Delta c_t$ which captures (changes in) short-run dynamics and fluctuations of money demand.

2.3 Modelling medium-term monetary targeting

The implied response to money demand shocks is usually viewed as a major drawback of monetary targeting. To avoid the associated inefficiency, the Bundesbank from the outset interpreted its money growth targets as medium-term rather than short-term targets. In line with this interpretation, it practised a policy of accommodating short-term fluctuations in money demand which were judged to be irrelevant for trend money growth and thus for trend inflation (see, e.g. Baltensperger, 1998; Deutsche Bundesbank, 1998, 36f.). To capture this element of the Bundesbank’s strategy, we

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9 Long-run money demand for M3 in Germany showed a stable pattern over the whole monetary targeting period, even after German unification (see, inter alia, Hubrich, 1999; Scharnagl, 1998; Wolters et al., 1998).
replace Eq. (3) by the assumption that the Bundesbank Council targeted an adjusted money growth variable, \( \Delta m_{t}^{adj} \), which was supposed to capture trend money growth:

\[
L_{t} = (\Delta m_{t}^{adj} - \Delta m_{t}^{T})^{2}
\]

(3a)

\( \Delta m_{t}^{adj} \) is defined as actual money growth minus the central bank’s estimate of the money demand shock, \( \Delta \varepsilon_{t}^{est} \):

\[
\Delta m_{t}^{adj} = \Delta m_{t} - \delta \cdot \Delta \varepsilon_{t}^{est}
\]

(10)

In order to allow for less than full accommodation of shocks, the term \( \Delta \varepsilon_{t}^{est} \) is multiplied by a parameter \( \delta \) which measures the degree of accommodation \((0 \leq \delta \leq 1)\). Combining (10) with (8), we get the following formula for the adjusted money growth gap

\[
\Delta m_{t}^{adj} - \Delta m_{t}^{T} = \left( \Delta p_{t} - \Delta p_{t}^{T} \right) + \gamma_{1} \left( \Delta y_{t} - \Delta y_{t}^{*,est} \right) + (\gamma_{1} - \hat{\gamma}_{1}) \Delta y_{t}^{*,est} - \gamma_{2} \cdot \Delta i_{t} + (1 - \delta) \Delta \varepsilon_{t} + \delta (\Delta \varepsilon_{t} - \Delta \varepsilon_{t}^{est})
\]

(11)

Setting the adjusted money growth gap once more equal to zero and solving for the interest rate yields the interest rate rule implied by medium-term monetary targeting:

\[
i_{t} = i_{t-1} + \frac{1}{\gamma_{2}} \left( \Delta p_{t} - \Delta p_{t}^{T} \right) + \gamma_{1} \left( \Delta y_{t} - \Delta y_{t}^{*,est} \right) + \frac{(1 - \delta)}{\gamma_{2}} \Delta \varepsilon_{t} + u'_{t}
\]

(12)

where all estimation errors have once again been subsumed into the error term \( u' \).

Eq. (12) encompasses the polar cases of strict short-term monetary targeting (with \( \delta = 0 \)) and of “optimal” medium-term monetary targeting where short-term fluctuations of money growth around trend are fully accommodated (with \( \delta = 1 \)).

Direct estimation of (12) would require information about policymakers’ real-time perceptions of \( \Delta \varepsilon_{t} \). However, Eq. (8) allows us to circumvent this problem by expressing \( \Delta \varepsilon_{t} \) in terms of observable variables only

\[
\Delta \varepsilon_{t} = \Delta m_{t} - \Delta m_{t}^{T} - \left( \Delta p_{t} - \Delta p_{t}^{T} \right) - \gamma_{1} \left( \Delta y_{t} - \Delta y_{t}^{*,est} \right) - (\gamma_{1} - \hat{\gamma}_{1}) \Delta y_{t}^{*,est} + \gamma_{2} \cdot \Delta i_{t}
\]

(8a)

Substituting (8a) into (12) yields:

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10 The estimates of the Bundesbank’s reaction function presented by Neumann (1997) suggest that money demand shocks were not fully accommodated (see p. 187f). However, as pointed out by Neumann (FN 15), this result may also be due to the fact that the variable which he uses to proxy money demand shocks is likely to mix in an unknown fashion shocks to money demand and innovations in money supply.
\[ i_t = i_{t-1} + \frac{1}{\gamma_2} (\Delta p^*_t - \Delta p^T_t) + \frac{\gamma_1}{\gamma_2} (\Delta y^*_t - \Delta y^{*,ext}_t) + \frac{(1 - \delta)}{\delta \gamma_2} (\Delta m^*_t - \Delta m^T_t) + u'_t, \quad (13) \]

where the estimation errors have again been subsumed into the error term \( u' \). Note that in this specification of the interest rate rule, the coefficient of the money growth gap captures the degree to which the interest rate setting by the central bank responds to money demand shocks. As a consequence, if the central bank fully accommodates shocks to money demand (\( \delta = 1 \)), one would only observe a response to inflation and the change in the output gap, but no response to the money growth gap at all. By contrast, if there is no accommodation at all (\( \delta = 0 \)), the feedback coefficient of the money gap would go to infinity.

### 2.4 Admitting additional short-term objectives

Up to now, we have assumed that the central bank's objective function is one-dimensional in the sense that it only cares about achieving the (intermediate) monetary target. However, and realistically, we now take into account that the central bank pursues further goals. Potential candidates are the standard objectives of minimising deviations of inflation, output and the interest rate from their respective target/natural rate levels.\(^{11}\) The corresponding intertemporal loss function is

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ (\Delta p^*_t - \Delta p^T_t)^2 + \lambda_y (y^*_t - y^*_t)^2 + \lambda_i (i^*_t - i^*_t)^2 + \lambda_m (\Delta m^{*adj}_t - \Delta m^{T}_t)^2 \right], \quad (14)
\]

where \( \beta \) is the discount factor, \( E \) is the expectations operator and \( \lambda_y, \lambda_i \) and \( \lambda_m \) are the relative weights attached to the output, interest rate and money growth stabilisation objectives. The implications of including a money growth target in an otherwise standard central bank objective function have been analysed by Söderström (2005) and Beyer et al. (2008). In their setup, the money growth target acts as a commitment device which helps the central bank to get closer to the optimal, but infeasible commitment solution. As shown by Beyer et al. (2008), under certain reasonable assumptions, the targeting rule characterising optimal discretionary policy under this type of objective function can be transformed into an implicit interest rate rule of the following form:

\(^{11}\) For a welfare-theoretic justification of these objectives, see Woodford, 2003, Chapter 6.
\[ i_t = (1 - \rho) \left[ \left( \frac{\partial i_t^*}{\partial \Delta p_t} \right) + \frac{\partial i_t^*}{\partial \Delta p_t^T} \right] + \rho \cdot i_{t-1} \]  

where the response coefficients \( \phi_p, \phi_x, \phi_m \) and \( \rho \) are functions of the model parameters. Eq. (15) differs from the interest rate rules we have considered so far – that is Eqs (9a), (12) and (13) - by introducing an additional response to the level of the output gap and by allowing the degree of interest rate smoothing to differ from one. Setting \( \Delta m_{\text{adj}} = \Delta m \), and thus ignoring the medium-term nature of money growth targets, would allow us to estimate (15) directly. This is in fact done by many empirical studies, such as Clarida et al. (1998). However, the framework developed in Section 2.3 allows us to go one step further and gain additional insights into the degree of medium-term orientation of a monetary targeting strategy. By substituting (8a) into (11), we are able to express the adjusted money growth gap in terms of observable variables and forecast errors:

\[ \Delta m_{\text{adj}} - \Delta m_t^T = \delta(\Delta p_t - \Delta p_t^T) + \gamma(\Delta y_t - \Delta y_t^*) + \delta(\gamma - \gamma_t^*)(\Delta y_t^*) + \delta(\gamma - \gamma_t^*)(\Delta y_t^*) 
- \delta(\gamma - \gamma_t^*)(\Delta y_t^*) + (1 - \delta)(\Delta m_t - \Delta m_t^T) + \delta(\Delta e_t - \Delta e_t^*) \]  

(11a)

Using Eq. (11a) to replace the adjusted money growth gap in Eq. (15), and subsuming the estimation errors into the error term, \( \xi_t \), yields:

\[ i_t = (1 - \rho') \left[ \left( \frac{\partial i_t^*}{\partial \Delta p_t} \right) + \frac{\partial i_t^*}{\partial \Delta p_t^T} \right] + \rho \cdot i_{t-1} + \xi_t \]  

(16)

with \( \rho' = \frac{\rho + (1 - \rho) \cdot \phi_m \cdot \delta \cdot \gamma_t}{1 + (1 - \rho) \cdot \phi_m \cdot \delta \cdot \gamma_t} \geq \rho \)

In our view, Eq. (16) encompasses all potentially important elements of flexible monetary targeters such as the Bundesbank. First of all, it takes into account that the central bank may have at least partially accommodated shocks to money demand (that is, \( \delta > 0 \)). As a consequence, the weight that policymakers attach to the money growth targets does not only show up in the interest rate response to actual money growth, but also in the response to inflation and to output growth. Note that in the limit, if the central bank fully accommodates all shocks to money demand, Eq. (16) will not feature a response to money growth at all, despite the fact that it follows a strategy of monetary targeting. Second, Eq. (16) encompasses the feedback variables implied by monetary
targeting as well as the ingredients of more standard interest rate rules such as the popular Taylor rule. Hence, this specification of the policy rule enables us to test various hypotheses about a central bank’s monetary policy strategy. For instance, if the estimated coefficients of the inflation gap and the output gap turn out to be significantly positive, whereas the coefficients of the change in the output gap and the money growth gap are insignificant, we would regard this as evidence in favour of the claim made by Clarida et al. (1998) that the Bundesbank preached monetary targeting, but in fact followed a Taylor rule. If, however, we find the estimated coefficients of the output growth gap and/or the money growth gap to be significantly positive, we will interpret this as evidence that the money growth targets played an important role in the policy decisions.

3 Estimating the reaction function of the Bundesbank

In this section, we provide empirical evidence on the Bundesbank’s monetary policy reaction function during the era of monetary targeting. In line with other studies, we neglect the first turbulent and volatile years (1975-1978) and focus on the more stable period after the inception of the EMS (1979-1998). The specification of the reaction function is based on the interest rate rule derived in the previous section. However, the theoretical model imposes some complications which need to be dealt with. One difficulty is that, obviously, not all the parameters can be identified by estimating Eq. (16). One way to solve this problem would be to estimate the structural version of Eq. (16) for given values of the parameters of the money demand function, $\gamma_1$ and $\gamma_2$. Note, however, that in order to test the hypotheses we are interested in, it suffices to pin down the Bundesbank’s overall response to each of the feedback variables included in Eq. (16). Hence, we restrict ourselves to estimating the following reduced-form version of (16):

$$i_t = (1 - \rho) \left( i_t^* + \phi_{\Delta y} \cdot (\Delta y_t - \Delta y_t^*) + \phi_y \cdot (y_t - y_t^*) + \phi_{\Delta m} \cdot (\Delta m_t - \Delta m_t^*) \right) + \rho \cdot i_{t-1} + \eta_t \quad (17)$$

where $\eta_t$ is a linear combination of the forecast errors included in $\zeta_t$ and an exogenous disturbance term. Another difficulty is that in general, the contemporaneous values of the rate of inflation, (the change in) the output gap and money growth will not
be known to policymakers at the time the decisions are made and hence have to be estimated. Unfortunately, for the period in question, real-time data on policymakers’ or Bundesbank staff forecasts of the variables in question are not available. On the other hand, the RHS variables of Eq. (17) are determined simultaneously with the policy instrument, and hence may not be uncorrelated with the error term. To avoid the resulting endogeneity problems, we use instrumental variables estimation and instrument the RHS-variables of Eq. (17) by a vector of variables $I_t$ which are (sufficiently closely) correlated with the explanatory variables but orthogonal to $\eta_t$ (for details on the instrument sets, see notes below Tables 1a-2b).

In any empirical work on monetary policy reaction functions, an important question is to which extent one is able to reconstruct policymakers’ real-time information sets. The first generation of empirical studies on monetary policy reaction functions in the spirit of Taylor (1993) was based on ex post revised data. Influential examples include Clarida and Gertler (1997) or Clarida et al. (1998, 2000). However, Orphanides (2001, 2003c) has pointed out that ex post data on key macro variables may differ considerably from the information available to policymakers at the time the decisions are made. This so-called real-time data problem stems from the fact that some potential determinants of monetary policy suffer from considerable measurement problems and are often substantially revised over time. Indeed, with the advantage of hindsight we now know that measurement problems are particularly pronounced for the level of the output gap, which plays a prominent role in interest rate rules of the Taylor type. Interestingly, this is not specific to the US but seems to be an international phenomenon (see Gerberding et al. (2005) for Germany, Gerdesmeier and Roffia (2005) for the Euro Area, Kamada (2004) for Japan, Nelson and Nikolov (2003) for the UK and Orphanides (2001) for the USA). For the purpose of practical monetary policy, estimating reaction functions on revised data is hence inappropriate \textit{a priori} since it introduces measurement errors into the estimated equations, leading to biased estimates (and test statistics).

On the other hand, more recently, the argument has been put forward that the available real-time data sets do not fully reflect the information set available to policymakers at the time the decisions are taken. For instance, the analysis of a broad set of indicators may enable policymakers to implicitly circumvent the measurement
problems underlying real-time estimates. If this were true, policymakers’ own (implicit) estimates of key macro variables may differ from those contained in real-time data sets (which are usually based on published data and staff estimates).

As the outcome of this debate is still open, our approach is to use ex post data as well as real-time data to estimate the Bundesbank’s reaction function. Looking at both sets of results seems particularly appropriate in the context of the present paper since money growth targeting implies a response to the “true” rate of inflation and the “true” rate of output growth which determine the observed change in money demand.\textsuperscript{12} As our benchmark ex post series, we match the last available vintage of official Bundesbank estimates of the production potential (dating from Jan. 1999) with the March 1999 vintages of all other data.\textsuperscript{13}

Table 1a summarizes the results of estimating Eq. (17) on ex post data. Note that in the estimations, the natural rate of interest, $i_t^*$, was proxied by the sum of a constant and the (time-varying) price assumption, $\Delta p_t^\pi$. Furthermore, in line with the one-year horizon of the money growth targets, we have estimated policymakers’ response to annual (four-quarter) rather than quarterly rates of inflation, output growth and money growth. Finally, in order to generalize our analysis to contemporary and forward-looking specifications of the policy rule, we have replaced the inflation variable in Eq. (17) by $(\Delta p_{r,n} - \Delta p_{r,n}^\pi)$ and allowed $n$ to vary between 0 and 6.\textsuperscript{14} Turning to the results, note first that in all cases, the J-statistic confirms the validity of the over-identifying restrictions. Second, the coefficient of the inflation gap, $\phi_{\Delta p}$, is significantly positive for all values of $n$. Third, the coefficient of the level of the output gap, $\phi_y$, is significant only for $n=0$, suggesting that in this case, the output gap acts as an indicator of future inflation rather than as an independent feedback variable. Fourth, the coefficients of the output growth gap, $\phi_{\Delta y}$, and of the money growth gap, $\phi_{\Delta m}$, are significantly positive for all values of $n$. However, for the specification with the lowest standard error, $n=3$, the interest rate response to the money growth gap is significant only at the 10% level, while the response to both the inflation gap and the output growth gap are significant at

\textsuperscript{12} Coenen et al. (2005) analyse the role of money as an indicator of current real output. See also Scharnagl et al. (2007).
\textsuperscript{14} The interpretation of the results for $n>0$ is discussed in more detail in Section 4.3.
the 1% level. Fifth, with estimated values of \( \rho' \) between 0.80 and 0.91, the rule exhibits a high degree of interest rate smoothing. Finally, dropping the insignificant output gap leaves the results unchanged with the only exception that \( \phi_{\Delta p} \) increases somewhat for most forecast horizons (see Table 1b). As a consequence, the response to inflation becomes significantly larger than one for all values of \( n \).

Before turning to the interpretation of these results, we check whether using real-time data instead of ex post data makes any difference. For that purpose, we re-estimate Eq. (17) using the real-time data set compiled by Gerberding et al. (2004). We find that all real-time estimates reveal a significant reaction to the inflation gap, to the change of the output gap and to the money growth gap, while the feedback from the level of the output gap turns out to be insignificant. Again, the response to the money growth gap is weakest for an inflation forecast horizon \( n \) of three quarters which is the specification with the lowest standard error. The rule also exhibits a high degree of interest rate smoothing.\(^{15}\) Moreover, the parameters of the change in the output gap and the inflation gap are not too far apart from each other.\(^{16}\) In fact, for \( n=0 \), the point estimate of \( \phi_{\Delta y} \) is even slightly above \( \phi_{\Delta p} \), which is perfectly in line with the parameter restrictions implied by monetary targeting in the case of \( \eta > 1 \) (see Eqs (13) and (16)).\(^{17}\)

These results prove to be quite robust to changes in the forecast horizon \( n \) (1\( \leq n \leq 6 \)), to the exact timing of the inflation and output variables, to the concrete specification of the money gap (annual growth rates, annualised 6-month growth rates, level specifications), and to the choice of alternative proxies for the unobserved forecasts of inflation (consumer prices, output deflator, consensus forecasts).\(^{18}\)

However, what is perhaps most surprising is that the results based on real-time data differ only slightly from the results in the ex-post setting. One explanation for this congruence is that (in contrast to other central banks) policymakers at the Bundesbank focussed their attention on indicator variables which were exposed to measurement

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\(^{15}\) In Gerberding et al. (2007), section 4.2, it is shown that the significance of the lagged interest rate reflects "true" interest rate smoothing and is not caused by measurement errors in the target interest rate or by the omission of important RHS variables (serially correlated errors).

\(^{16}\) Rudebusch (2002a) shows that nominal income targeting performs well when inflation is forward-looking.

\(^{17}\) The income elasticity of broad money demand is generally estimated to be greater than one in the case of the euro area and in Germany, see e.g., Bruggemann et al. (2003) and Scharnagl (1998).

\(^{18}\) See Table 2 in Gerberding et al. (2004) and further calculations which are available upon request.
errors only to a comparatively small extent. Figure 1 illustrates that this is indeed the case. First of all, as shown in Figure 1(a), the measurement errors regarding the change in the output gap are much smaller and much less persistent than the measurement errors regarding the level of the output gap. Second, when splitting up the change in the output gap into actual output growth and potential output growth (Figure 1(b)), we find that the measurement errors in output growth and the change in the output gap follow very similar patterns, while the measurement errors regarding potential output growth are smaller, but more persistent. Finally, as illustrated by Figure 1(c), revisions in consumer prices and in money growth are even smaller in size throughout the sample period, with money growth figures being hardly ever revised at all. While this may not be true for other countries over different sample periods, Coenen et al. (2005) reach very similar conclusions with respect to euro-area data since 1999.

4 Interpretation of the results

Several of the results presented in the previous section deserve further discussion. In this section, we first interpret the estimated responses to inflation, output growth and money growth in light of the theoretical model developed in Section 2. Second, we outline potential advantages of responding to the change rather than to the level of the output gap. Third, we discuss the results for forward-looking specifications of the policy rule ($n>0$).

4.1 Interpreting the estimated response to the money growth gap

Taken literally, our theoretical model of monetary targeting derived in Section 2 implies an interest rate response to (policymakers’ estimates of) contemporaneous inflation, output growth and money growth (see Eqs (13) and (16)). And in fact, for $n=0$, our estimates of the feedback coefficients correspond well with the predictions of the theoretical model, particularly in the real-time setup. Recalling the “structural” version of the interest rate rule:

$$i_t = (1 - \rho_1) \left[ i_t^- + (\phi_p + \phi_m \delta) \cdot (\Delta p_t - \Delta p_t^-) + \phi_y \cdot (y_t - y_t^-) + \phi_m \cdot (\Delta y_t - \Delta y_t^-) + \phi_m \cdot (\Delta y_t - \Delta y_t^-) + \phi_m \cdot (\Delta y_t - \Delta y_t^-) + \phi_m \cdot (\Delta y_t - \Delta y_t^-) + \phi_m \cdot (\Delta y_t - \Delta y_t^-) \right] + \rho_1 \cdot i_{t-1} + \xi_t$$

it is even possible to use the estimates of the reduced-form coefficients (from Table 1a and 2b) to extract some information on $\delta$, the degree of accommodation of money
demand shocks. To illustrate the linkages, Table 3 shows the values of $\delta$ which result for given values of the reduced-form coefficients taken from Table 1a ($\hat{\phi}_p=1.03; \hat{\phi}_y=1.25; \hat{\phi}_m=0.54$) and Table 2b ($\hat{\phi}_p=2.30; \hat{\phi}_y=2.57; \hat{\phi}_m=1.05$), respectively, and two alternative values of $\gamma_1$, namely 1 and 1.3. Interestingly, with values of $\delta$ between 0.64 and 0.71, this simple exercise suggests that the Bundesbank Council either accommodated most, but not all shocks to money demand or that it accommodated (some) shocks only partly. The remaining influence of money demand disturbances on the Bundesbank’s interest rate decisions (which, in our model, is reflected in the estimated values of $\phi_{\text{mn}}$) may simply reflect policy mistakes, possibly due to difficulties in identifying the shocks in real time. It may also reflect a conscious decision by policymakers to show some response to deviations of money growth from target, even if they were believed to be caused by shocks and therefore not to feed into prices in the medium to long run (e.g. for credibility reasons).\footnote{Additional reasons why it might be helpful for policymakers to look at money are discussed in Gerberding et al. (2004), Section 5.}

4.2 Role of the Output Gap

The strong and robust influence of the change in the output gap on interest rate decisions points to an omitted variables bias in standard Taylor rule specifications of the Bundesbank reaction function like the one estimated by Clarida et al. (1998). In this sense, our results throw serious doubt on the widespread practice of using the Taylor rule as a reasonably accurate ex-post description of monetary policy which may be exploited, for instance, in the estimation of DSGE models based on ex-post data.

From a normative point of view, targeting the change rather than the level of the output gap can be advantageous for two different reasons. First, as demonstrated by Orphanides et al. (2000) and Walsh (2004), there may be a case for responding to the change in the output gap rather than to its level if the measurement errors in the level of the output gap are large and highly persistent. The measurement errors in the level of the output gap are defined as (the tilde refers to real-time values):

$$y_t - y_t^* - (\tilde{y}_t - \tilde{y}_t^*) = (y_t - \tilde{y}_t) - (y_t^* - \tilde{y}_t^*)$$

(18)
As shown in Figure 1, the measurement errors in the Bundesbank’s estimates of the output gap were sizable and quite persistent, as it was the case not only for the Bundesbank estimates and not only in Germany. This high degree of persistence implies that, e.g., high positive errors in period \( t \) usually follow high positive measurement errors in \( t-1 \). However, given this high degree of level persistence, the measurement errors of the change of the output gap

\[
(\Delta y_t - \Delta y'_t) - (\Delta \tilde{y}_t - \Delta \tilde{y}'_t) = \left[ (y_t - \tilde{y}_t) - (y'_t - \tilde{y}'_t) \right] - \left[ (y_{t-1} - \tilde{y}_{t-1}) - (y'_{t-1} - \tilde{y}'_{t-1}) \right]
\]  

(19)

are much smaller. Therefore, it may be preferable to focus on output growth (relative to trend growth) rather than on the level of the output gap. Orphanides (2003a), Orphanides et al. (2000) and Walsh (2004) show that in the presence of imperfect information about the level of potential output, difference rules or speed limit policies outperform simple Taylor-type rules.

Second, responding to the change in the output gap may be welfare-improving since it introduces history-dependence into the policy rule, thereby stabilising inflation expectations and, via the expectations channel, stabilising also actual inflation. To fully understand the argument, consider the following example.\(^\text{20}\) Assume that policymakers care about stabilising inflation, output and the interest rate around their respective target values. In this case, the central bank’s objective function takes the form:

\[
W = E_0 \sum_{t=0}^{\infty} \beta^t \left[ (\Delta p_t - \Delta p_t^*)^2 + \hat{\lambda}_y (y_t - y_t^*)^2 + \hat{\lambda}_i (i_t - i_t^*)^2 \right]
\]  

(20)

where \( \hat{\lambda}_y \) and \( \hat{\lambda}_i \) are the relative weights attached to output and interest rate stabilization.\(^\text{21}\) Assume further that the aggregate demand and supply equations are of the standard New-Keynesian type. Under these assumptions, the first order conditions which characterize optimal monetary policy under discretion can be transformed into

\[
i_t = i_t^* + \phi_{\Delta p} (\Delta p_t - \Delta p_t^*) + \phi_y (y_t - y_t^*)
\]  

(21)

Eq. (21) can easily be interpreted as a policy rule of the Taylor type. However, with forward-looking price setting and a short-run output inflation trade-off, there are


\(^{21}\) For simplicity, we abstract from the complications arising from a difference between the efficient and the natural level of output.
gains from commitment to a policy rule. Under commitment, the central bank takes the effects of its own actions on private sector expectations into account. As a consequence, optimal policy is not purely forward-looking, but history-dependent in the sense that it implies systematic responses to the lagged interest rate, to the lagged change in the interest rate and to the lagged output gap. Choosing the commitment solution that is optimal from a timeless perspective, the interest rate rule takes the form:

$$i_t = (1 - \hat{\rho}_i) i_t^* + \hat{\rho}_1 i_{t-1} + \hat{\rho}_2 \Delta i_{t-1} + \hat{\phi}_\Delta (\Delta p_t - \Delta p_t^*) + \hat{\phi}_\Delta (\Delta y_t - \Delta y_t^*)$$  \hspace{1cm} (22)$$

Comparing Eq. (22) with Eqs (9) and (13), we find that the optimal time-invariant policy rule under commitment shares many features with the interest rate representation of medium-term monetary targeting derived in Section 2. Of course, as discussed in Section 2, the performance of money growth targeting may suffer from the fact that it implies an additional response to money demand disturbances. However, as shown by Söderström (2005), augmenting society’s true loss function by an additional money growth target can be beneficial even if the central bank does not make any adjustment for money demand shocks. In fact, in the hybrid New Keynesian model considered by Söderström, augmenting the loss function by a money growth target enables the central bank to bridge about 80% of the gap between the outcome under discretion and the optimal commitment solution. Moreover, according to Scharnagl et al. (2007), extending the type of policy rule described by Eq. (23) to include an additional response to money growth is beneficial even in a standard New Keynesian framework if one accounts for a realistic degree of output gap uncertainty. The main reason for the welfare gain is that the information on current output growth contained in money growth data allows the central bank to reduce its response to current inflation, thus enabling it to avoid inefficient reactions to cost push shocks. According to Kilponen and Leitemo (2007), the case for money growth targeting is further strengthened when the underlying macro model features lags in the effects of monetary policy.

4.3 Interpreting the results for $n>\theta$

In line with the predictions of our theoretical model, the focus of our empirical analysis is on a policy rule which features a response to contemporaneous values of the

22 The advantages of focussing on this solution are explained in Woodford (2003, p. 464ff)
RHS variables. However, as a robustness check, we have also reported results for different horizons of the inflation variable, $n$, allowing it to increase from 0 up to 6 quarters (see Tables 1a-2b). While the key results of our analysis are robust to increases in the horizon of the inflation variable, there are still some differences which deserve further comment. In particular, note that in each case, increasing the horizon of the inflation gap lowers the standard error of the regression until it reaches its minimum at a forecast horizon of three quarters. This points to the presence of a forward-looking element in the Bundesbank’s interest rate decisions which is absent from the policy rule we have derived in Section 2.

From a theoretical perspective, responding to the inflation outlook $n$ periods ahead rather than to current inflation may be beneficial if private sector expectations are primarily backward-looking or if there are lags in the effects of monetary policy (see Leitemo, 2008). With lags in the transmission mechanism, money may be a leading indicator of inflation, a feature which is absent from the simple model underlying our theoretical derivation. The presence of a link between current money growth and future inflation may also explain why increasing the time horizon of the inflation forecast from zero up to three lowers the estimated values of the coefficient $\phi_{\Delta \phi}$, just to increase again for $n > 3$.

If policymakers are forward-looking and money growth leads inflation, the estimated response to the money growth gap in the baseline specification (with $n=0$) may pick up the response to the inflation outlook which is not (explicitly) included in the policy rule. Still, the fact that the coefficient on money growth remains significant for all values of $n$ points to an independent role of the money growth gap, beyond the one as a leading indicator of inflation.\footnote{In this respect, our results differ from those of Kamps and Pierdzioch (2002).}

5 Conclusions

In the present paper, we have developed an analytical framework which enabled us to derive the interest rate feedback rule implied by monetary targeting. We have shown that medium-term monetary targeting implies an interest rate response to deviations of inflation from target, to the change in the output gap, to the lagged interest
rate and to the money growth gap. The latter vanishes if the central bank accommodates all shocks to money demand. The results of our empirical analysis suggest that the Bundesbank followed such a strategy and to a large extent, but not fully, accommodated short-run fluctuations of money demand.

We have pointed out that, from a normative point of view, the response to the lagged interest rate and to the change in the output gap implied by monetary targeting may be beneficial because it introduces inertia and history-dependence into monetary policy. As shown by Giannoni and Woodford (2003), both features are important components of optimal monetary policy in standard New-Keynesian models with forward-looking expectations. In addition, responding to the change in the output gap rather than to its level may be advantageous when the latter is subject to large and persistent measurement errors as it has historically been the case.

Hence, the outcome of our analysis differs markedly from the results of other studies, like, e.g., Rudebusch and Svensson (2002) who conclude that the reaction function resulting from monetary targeting is quite unsuitable for stabilizing inflation and the output gap, even if there are no shocks to money demand. One reason for their negative verdict on monetary targeting is that their analysis abstracts from the problem of data uncertainty. In fact, they argue that it is not obvious that monetary targeting would be favoured under such uncertainty since money data are also subject to important revisions. While this may be true for the US (Amato and Swanson, 1999), Coenen et al. (2005) show that the ECB’s preferred measure of the broad money stock, M3, is subject to only small revisions after the first quarter and to negligible revisions in subsequent quarters.

Hence, the available empirical evidence suggests that the lessons from German data, together with the insights from recent research on optimal monetary policy under commitment, are more relevant for the euro area than the lessons from US data presented by Rudebusch and Svensson. Having said this and against the background of the increased uncertainty monetary policy makers in EMU are confronted with, the Eurosystem’s prominent role for money seems to be a sensible approach. Taken seriously, this orientation introduces the necessary ingredients of a robust and inertial

\[\text{See Rudebusch and Svensson (2002), footnote 26.}\]
monetary policy rule. However, in order to arrive at more definite conclusions, the present analysis needs to be complemented by further studies which take account of the structural relationships as well as of the degree of model and data uncertainty currently prevailing in the euro area. This is an important task for future research.
References


Figure 1: Measurement error in key monetary policy indicators, 1975-1998

1) The measurement errors are defined as the differences between the ex post figures (March 1999 vintages) and the initial figures.

* The calculation is based on Bundesbank estimates of potential output.
Table 1a: Ex-post estimates of the Bundesbank’s interest rate rule

Estimation equation:

\[ i_t = (1 - \rho' \cdot \Delta) \left[ \hat{i}_t + \phi_{\Delta p} \cdot E((\Delta p_{t+1} - \Delta p_{t+1}^*)|\Omega_t) + \phi_y \cdot E((y_t - y_{t+1}^*)|\Omega_t) + \phi_{\Delta m} \cdot E((\Delta m_t - \Delta m_{t+1}^*)|\Omega_t) \right] + \rho \cdot i_{t-1} + \mu_t \]

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***/**/ denotes significance at the 1% (5%/10%) level. Estimation period: 1979Q1 to 1998Q4; estimation method: GMM; HAC-robust standard errors in parentheses.

Variables: left-hand-side variable: 3-month money market rate (end-of-quarter); right-hand-side variables: inflation according to CPI; level and change in the output gap with Bundesbank’s own estimates of production potential, money growth measured by central bank money stock (until end of 1987) and M3 afterwards; ex-post series as of March 1999. To correct for extreme outliers in the residuals, we include a dummy variable in the estimations which is one in the first quarter of 1981 and zero otherwise. For further details on the data see Gerberding et al. (2004). The instrument set includes the contemporary values of inflation and the price assumption (which were known to policy makers at the end of each quarter) as well as two lags of each explanatory variable. Pretesting suggests that this instrument structure is sufficient.

R²: adjusted coefficient of determination; SEE: standard error of the regression; J-stat: p-value of the J-statistic on the validity of overidentifying restrictions; JB: p-value of the Jarque Bera test of the normality of residuals.

Table 1b: Ex-post estimates of the Bundesbank’s interest rate rule with \( \phi_y = 0 \)

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Table 2a: Real-time estimates of the Bundesbank’s interest rate rule

Estimation equation:

\[
i_t = (1 - \rho^t) \left[ \phi_{dp} \cdot E((\Delta p_{t+n} - \Delta p_{t+n}^*)|\Omega_t) + \phi_y \cdot E((y_t - y_t^*)|\Omega_t) + \phi_{sy} \cdot E((\Delta y_{t+n} - \Delta y_{t+n}^*)|\Omega_t) + \phi_{sm} \cdot E((\Delta m_t - \Delta m_t^*)|\Omega_t) \right] + \rho^t \cdot i_{t-1} + \mu_t
\]

<table>
<thead>
<tr>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=3</th>
<th>n=4</th>
<th>n=5</th>
<th>n=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_{dp})</td>
<td>2.17***</td>
<td>2.19***</td>
<td>2.43***</td>
<td>3.05***</td>
<td>2.64***</td>
<td>2.73***</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(0.36)</td>
<td>(0.33)</td>
<td>(0.45)</td>
<td>(0.71)</td>
<td>(0.81)</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>0.06</td>
<td>0.01</td>
<td>0.11</td>
<td>0.15</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.23)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>(\phi_{sy})</td>
<td>2.41***</td>
<td>1.79***</td>
<td>1.53***</td>
<td>1.72***</td>
<td>2.57***</td>
<td>3.01***</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(0.53)</td>
<td>(0.43)</td>
<td>(0.48)</td>
<td>(0.87)</td>
<td>(1.11)</td>
</tr>
<tr>
<td>(\phi_{sm})</td>
<td>0.98***</td>
<td>0.61***</td>
<td>0.39**</td>
<td>0.17</td>
<td>0.60**</td>
<td>0.80**</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.21)</td>
<td>(0.16)</td>
<td>(0.15)</td>
<td>(0.23)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>(\rho^t)</td>
<td>0.84***</td>
<td>0.82***</td>
<td>0.85***</td>
<td>0.89***</td>
<td>0.91***</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

***(**/*) denotes significance at the 1% (5%/10%) level. Estimation period: 1979Q1 to 1998Q4, estimation method: GMM; HAC-robust standard errors in parentheses. For further notes see Table 1a.

Table 2b: Real-time estimates of the Bundesbank’s interest rate rule with \(\phi_y = 0\)

<table>
<thead>
<tr>
<th>n=0</th>
<th>n=1</th>
<th>n=2</th>
<th>n=3</th>
<th>n=4</th>
<th>n=5</th>
<th>n=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_{dp})</td>
<td>2.30***</td>
<td>2.21***</td>
<td>2.26***</td>
<td>2.57***</td>
<td>2.64***</td>
<td>2.81***</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.25)</td>
<td>(0.25)</td>
<td>(0.39)</td>
<td>(0.58)</td>
<td>(0.71)</td>
</tr>
<tr>
<td>(\phi_{sy})</td>
<td>2.57***</td>
<td>1.82***</td>
<td>1.47***</td>
<td>1.74***</td>
<td>2.57***</td>
<td>3.07***</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(0.49)</td>
<td>(0.39)</td>
<td>(0.49)</td>
<td>(0.86)</td>
<td>(1.09)</td>
</tr>
<tr>
<td>(\phi_{sm})</td>
<td>1.05***</td>
<td>0.61***</td>
<td>0.39**</td>
<td>0.30**</td>
<td>0.60**</td>
<td>0.78**</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.21)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.23)</td>
<td>(0.30)</td>
</tr>
</tbody>
</table>

***(**/*) denotes significance at the 1% (5%/10%) level. Estimation period: 1979Q1 to 1998Q4, estimation method: GMM; HAC-robust standard errors in parentheses. Variables: left-hand-side variable: 3-month money market rate (end-of-quarter); right-hand-side variables: inflation gap according to CPI; level and change in the output gap with Bundesbank’s own estimates of production potential, money growth measured by central bank money stock (until end of 1987) and M3 afterwards. For details on the construction of the real-time data base see Gerberding et al. (2004). To correct for extreme outliers in the residuals, we include a dummy variable in the estimations which is one in the first quarter of 1981 and zero otherwise. The instrument set includes the contemporary values of inflation and the price assumption (which were known to policy makers at the end of each quarter) as well as two lags of each explanatory variable. Pretesting suggests that this instrument structure is sufficient.

R²: adjusted coefficient of determination; SEE: standard error of the regression; J-stat: p-value of the J-statistic on the validity of overidentifying restrictions; JB: p-value of the Jarque Bera test of the normality of residuals.
Table 3: Inferred values of $\delta$ for different values of $\gamma_1$

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_1=1$</th>
<th>$\gamma_1=1.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient estimates based on ex post data</td>
<td>$\delta=0.70$</td>
<td>$\delta=0.64$</td>
</tr>
<tr>
<td>Coefficient estimates based on real-time data</td>
<td>$\delta=0.71$</td>
<td>$\delta=0.65$</td>
</tr>
</tbody>
</table>

***(**/*) denotes significance at the 1% (5%/10%) level. Estimation period: 1979Q1 to 1998Q4.; estimation method: GMM; HAC-robust standard errors in parentheses; for further notes see table 2a.