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Solidarity and fair taxation in TU games*

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Abstract

We consider an analytic formulation/parametrization of the class of efficient, linear, and symmetric values for TU games that, in contrast to previous approaches, which rely on the standard basis, rests on the linear representation of TU games by unanimity games. Unlike most of the other formulae for this class, our formula allows for an economic interpretation in terms of taxing the Shapley payoffs of unanimity games. We identify those parameters for which the values behave economically sound, i.e., for which the values satisfy desirability and positivity. Put differently, we indicate requirements on fair taxation in TU games by which solidarity among players is expressed.

Key Words: Shapley value, solidarity, taxation, desirability, positivity

JEL code: C71, D60

AMS subject classification: 91A12

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1. Introduction

The Shapley value (Shapley 1953) certainly is the most eminent point-solution concept for TU games. Its standard characterization involves four axioms: efficiency, additivity/linearity, symmetry, and the null player axiom. In a sense, it is mainly the latter property that prevents the Shapley value to allow for solidarity among the players. Irrespective of the productivity of the whole society, an unproductive player obtains a zero payoff. Moreover, together with additivity, the null player property already entails strong marginality (Young 1985), i.e., the players' payoffs depend only on their *own* productivities measured by marginal contributions.

So, if one wishes values to allow for solidarity considerations, one has to drop the null player axiom from the list of required properties. But then we were down to the class of values obeying efficiency, linearity, and symmetry. Obviously, this large class contains a lot of values that do not deviate from the Shapley value just by economically sound solidarity considerations. For example, the equal surplus division value (Driessen & Funaki 1991) and the consensus value (Ju, Borm & Ruys 2007) inhabit this class, but fail positivity (Kalai & Samet 1987). It is possible that these values may assign negative payoffs in monotonic games, i.e., in games where no player ever is destructive in terms his marginal contributions. We feel that this does not fit well our intuitions on solidarity. Moreover, values in this class may not meet desirability (Maschler & Peleg 1966), i.e., a player who is more productive than another one may end up with a lower payoff. Again, this would overstretch our sense of solidarity. At least, one would like to have a value to satisfy weak versions of positivity and desirability as embodied in social acceptability (Joosten, Peters & Thuijsman 1994). Roughly speaking, positivity and desirability should hold for unanimity games.

Formulae/parametrizations for the class of efficient, linear, and symmetric values (ELS values) have been proposed by Ruiz, Valenciano & Zarzuelo (1998), Driessen & Radzik (2003), Chameni-Nembua & Andjiga (2008), and Hernandez-Lamoneda, Juarez & Sanchez-Sanchez (2008). Recently, Chameni-Nembua (2012) and Malawski (2012) come up with a more interpretational one. In essence, the players' marginal contributions within a coalition are taxed at a rate depending on its size, while the tax revenue is distributed evenly among the other players in the coalition under consideration.

We suggest and explore an alternative formula for this class, already indicated by Radzik & Driessen (2009, p. 5), which also is interpretable in terms of taxation. The main idea of our approach is to tax and redistribute the Shapley payoffs of unanimity games. First, the Shapley payoffs are taxed at a certain rate, which depends on the cardinality of the set of productive players in such a game. And second, the overall tax revenue is distributed evenly among *all* players. Linearity extends these payoffs to general TU games.

Radzik & Driessen (2012) provide conditions on the coefficients of the formula due to Driessen & Radzik (2003) such that the resulting value satisfies one or another of the desirable properties above: desirability, positivity combined with desirability, social acceptability, and general acceptability. In this paper, we attempt analogous conditions on the parameters of our formulae.

This paper is organized as follows: In the second section, we introduce basic definitions and notation. The third section surveys formulae for ELS values and introduces a new parametrization for this class. In section four, we provide conditions on the parameters of our formulae such that one or another of the desirable properties mentioned above are satisfied. The appendix contains the lengthier proofs.

2. Basic definitions and notation

A **(TU) game** is a pair (N, v) consisting of a non-empty and finite set of players N and a **coalition function** $v \in \mathbb{V}(N) := \left\{f : 2^N \to \mathbb{R} \mid f(\emptyset) = 0\right\}$. Since we work within a fixed player set, we frequently drop the player set as an argument. In particular, we address $v \in \mathbb{V}$ as a game. Subsets of N are called **coalitions**; $v(K)$ is called the worth of coalition K. For $v, w \in \mathbb{V}$ and $\lambda \in \mathbb{R}$, the coalition functions $v + w \in \mathbb{V}$ and $\lambda \cdot v \in \mathbb{V}$ are given by $(v + w)(K) = v(K) + w(K)$ and $(\lambda \cdot v)(K) = \lambda \cdot v(K)$ for all $K \subseteq N$. For $T \subseteq N$, $T \neq \emptyset$, the game $u_T \in \mathbb{V}$, $u_T(K) = 1$ if $T \subseteq K$ and $u_T(K) = 0$ for $T \nsubseteq K$, is called a **unanimity game**. For $T \subseteq N$, $T \neq \emptyset$, the game $e_T \in \mathbb{V}$, $e_T(K) = 1$ if $T = K$ and $e_T(K) = 0$ for $T \neq K$, is called a **standard game**. A game v is called **monotonic** if $v(K) \geq v(L)$ for all $K, L \subseteq N$ such that $L \subseteq K$. Any $v \in \mathbb{V}$ can be uniquely represented by unanimity games,

$$v = \sum_{T \subseteq N : T \neq \emptyset} \lambda_T(v) \cdot u_T, \tag{1}$$

where the Harsanyi dividends, $\lambda_T(v)$, $T \subseteq N$, $T \neq \emptyset$ (Harsanyi 1959) are given implicitly by

$$v(S) = \sum_{T \subseteq S : T \neq \emptyset} \lambda_T(v), \qquad S \subseteq N,\ S \neq \emptyset. \tag{2}$$

For $v \in \mathbb{V}$, the **dual game** $v^* \in \mathbb{V}$ is defined by

$$v^*(S) = v(N) - v(N \setminus S), \qquad S \subseteq N. \tag{3}$$

It is well-known that

$$u_T^* = \sum_{S \subseteq T : S \neq \emptyset} (-1)^{|S|-1} \cdot u_S, \qquad T \subseteq N,\ T \neq \emptyset. \tag{4}$$

For $v \in \mathbb{V}$, $i \in N$, and $K \subseteq N \setminus \{i\}$, the **marginal contribution** of i to K in v is given by $MC_i^v(K) := v(K \cup \{i\}) - v(K)$. Player $i \in N$ is called a **null player** in $v \in \mathbb{V}$ iff $MC_i^v(K) = 0$ for all $K \subseteq N \setminus \{i\}$; players $i, j \in N$ are called **symmetric** in $v \in \mathbb{V}$ if $MC_i^v(K) = MC_j^v(K)$ for all $K \subseteq N \setminus \{i, j\}$.

A **value** on N is an operator φ that assigns a payoff vector $\varphi(v) \in \mathbb{R}^N$ to any $v \in \mathbb{V}$. For $K \subseteq N$, we denote $\sum_{i \in K} \varphi_i(v)$ by $\varphi_K(v)$ .The **Shapley value** (Shapley 1953), Sh, given by

$$\mathrm{Sh}_i(v) := \sum_{T \subseteq N : i \in T} \frac{\lambda_T(v)}{|T|}, \qquad i \in N,\ v \in \mathbb{V} \tag{5}$$

is the unique value on N that satisfies the axioms **E**, **A** (or **L**), **ET** (or **S**), and **N** below.

Efficiency, E. For all $v \in \mathbb{V}$, $\varphi_N(v) = v(N)$.

Additivity, A. For all $v, w \in \mathbb{V}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$.

Equal treatment, ET. For all $v \in \mathbb{V}$ and $i, j \in N$, who are symmetric in $v \in \mathbb{V}$, $\varphi_i(v) = \varphi_j(v)$.

Null player, N. For all $v \in \mathbb{V}$ and all $i \in N$, who are null players in v, $\varphi_i(v) = 0$.

We further refer to the following standard axioms.

Linearity, L. For all $v, w \in \mathbb{V}$ and $\lambda \in \mathbb{R}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$ and $\varphi(\lambda \cdot v) = \lambda \cdot \varphi(v)$.

Symmetry, S. For all $v \in \mathbb{V}$, $i \in N$, and all bijections $\pi : N \to N$, $\varphi_{\pi(i)}(v \circ \pi^{-1}) = \varphi_i(v)$.

Continuity, C. The mapping $\varphi : \mathbb{V} \to \mathbb{R}^N$ is continuous.

Moreover, we refer to the following values, which also obey **E**, **L**, and **S**. The **equal division value**, ED, is given by

$$\mathrm{ED}_i(v)(v) := \frac{v(N)}{|N|}, \qquad i \in N,\ v \in \mathbb{V}.$$

The **egalitarian Shapley values** (Joosten 1996), Sh^{α}, $\alpha \in [0,1]$, are given by $\mathrm{Sh}^{\alpha} = \alpha \cdot \mathrm{Sh} + (1-\alpha) \cdot \mathrm{ED}$. The **equal surplus division value** (Driessen & Funaki 1991), ES, is given by

$$\mathrm{ES}_i(v) := v(\{i\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{|N|}, \qquad i \in N,\ v \in \mathbb{V}.$$

The **solidarity value** (Nowak & Radzik 1994), So, is given by

$$\mathrm{So}_i(v) := \sum_{S \subseteq N : i \in S} \frac{1}{\binom{|N|}{|S|} \cdot |S|} \sum_{j \in S} \frac{v(S) - v(S \setminus \{j\})}{|S|}, \qquad i \in N,\ v \in \mathbb{V}.$$

The **consensus value** (Ju et al. 2007), Con, is given by $\mathrm{Con} = \frac{1}{2} \cdot \mathrm{Sh} + \frac{1}{2} \cdot \mathrm{ES}$. The **least-square pre-nucleolus** (Ruiz, Valenciano & Zarzuelo 1996), LSPN, is given by

$$\mathrm{LSPN}_i(v) := \mathrm{Ba}_i(v) + \frac{v(N) - \sum_{j \in N} \mathrm{Ba}_j(v)}{|N|}, \qquad i \in N,\ v \in \mathbb{V},$$

where Ba stands for the Banzhaf value (Banzhaf 1965, Owen 1975),

$$\mathrm{Ba}_i(v) := \sum_{T \subseteq N : i \in T} \frac{\lambda_T(v)}{2^{|T|-1}}, \qquad i \in N,\ v \in \mathbb{V}.$$

3. Efficient, linear, and symmetric values

In this section, we first provide the formulae for the class of efficient, linear, and symmetric values (henceforth, **ELS values**) mentioned in the introduction. The formulae below apply to all $v \in \mathbb{V}$ and $i \in N$.

Ruiz et al. (1998): For $\rho = \left(\rho_1, \ldots, \rho_{|N|-1}\right) \in \mathbb{R}^{|N|-1}$, the value RVZ^{ρ} is given by

$$\mathrm{RVZ}_i^{\rho}(v) := \frac{v(N)}{|N|} + \sum_{S \subsetneq N : i \in S} \frac{\rho_{|S|}}{|S|} \cdot v(S) - \sum_{S \subseteq N \setminus \{i\} : S \neq \emptyset} \frac{\rho_{|S|}}{|N| - |S|} \cdot v(S).$$

Driessen & Radzik (2003)[1]: For $b = \left(b_1, \ldots, b_{|N|-1}\right) \in \mathbb{R}^{|N|}$, the value DR^b is given by

$$\mathrm{DR}_i^b(v) := \frac{v(N)}{|N|} + \sum_{S \subsetneq N \setminus \{i\}} \frac{b_{|S|+1} \cdot v(S \cup \{i\})}{\binom{|N|}{|S|+1} \cdot (|S|+1)} - \sum_{S \subseteq N \setminus \{i\} : S \neq \emptyset} \frac{b_{|S|} \cdot v(S)}{\binom{|N|}{|S|+1} \cdot (|S|+1)}. \tag{6}$$

A major disadvantage of the above formulae is that the parameters can hardly be interpreted in economic terms. To remedy this, Chameni-Nembua (2012)[2] proposes another type of parametrization. For $\alpha = \left(\alpha_2, \ldots, \alpha_{|N|}\right) \in \mathbb{R}^{|N|-1}$, the value CN^α is given by

$$\mathrm{CN}_i^\alpha(v) := \frac{v(\{i\})}{|N|} + \sum_{S \subseteq N : i \in S, |S|>1} \frac{AMC_i^v(S, \alpha)}{\binom{|N|}{|S|} \cdot |S|},$$

where

$$AMC_i^v(S, \alpha) := \alpha(|S|) \cdot [v(S) - v(S \setminus \{i\})] + \frac{1 - \alpha(|S|)}{|S| - 1} \sum_{j \in S \setminus \{i\}} [v(S) - v(S \setminus \{j\})].$$

This way a player's payoff is some average of marginal contributions, both of his own ones and the other player ones. Within a coalition S, the marginal contribution of player $i \in S$ is taxed at a rate of $1 - \alpha(|S|)$, leaving him a share of $\alpha(|S|) \cdot [v(S) - v(S \setminus \{i\})]$, while the tax revenue amounting to $(1 - \alpha(|S|)) \cdot [v(S) - v(S \setminus \{i\})]$ is distributed evenly among the *other* players in S.

Despite of the structural differences of the formulae above, they are closely related. By applying these values to standard games, the parameters can be recovered in a similar fashion. In particular, for $T \subsetneq N$, $T \neq \emptyset$, and $i \in T$, we have

$$\begin{aligned} \rho_{|T|} &= |T| \cdot \mathrm{RVZ}_i^\rho(e_T), \\ b_{|T|} &= \binom{|N|}{|T|} \cdot |T| \cdot \mathrm{DR}_i^b(e_T), \\ \alpha(|T|+1) &= \binom{|N|}{|T|} \cdot |T| \cdot \mathrm{CN}_i^\alpha(e_T). \end{aligned} \tag{7}$$

[1]Chameni-Nembua & Andjiga (2008) and Malawski (2012 and personal communication) consider essentially the same formulae, the latter under the name *inversely procedural values*. Moreover, Hernandez-Lamoneda et al. (2008) consider similar parametrizations, which just rescale the parameters. Actually, they consider coutinuous values and require just additivity. Yet, it is well-known that linearity entails continuity and that additivity combined with continuity implies linearity.

[2]Malawski (2012) suggests essentially the fornulae as the *procedural values*. Instead of marginal contributions to coalitions, he considers marginal contributions for orders of the player set.

Hence, conditions on the parameters as for example imposed by (Radzik & Driessen 2012) for the formula suggested by Driessen & Radzik (2003) can easily be translated into conditions for the parameters of the other formulae above.

We advocate another formula for the class of ELS values, already indicated by Radzik & Driessen (2009, p. 5). In contrast to the approaches above, our parametrization applies to unanimity games, i.e., to the Harsanyi dividends $\lambda_T(v)$ in (1). We consider the following class of values on N. For $\tau = \left(\tau_1, \ldots, \tau_{|N|-1}\right) \in \mathbb{R}^{|N|-1}$, the value ζ^τ on N is given by

$$\zeta_i^\tau(v) = \frac{\lambda_N(v)}{|N|} + \sum_{T \subsetneq N: T \neq \emptyset} \frac{\tau_{|T|}}{|N|} \cdot \lambda_T(v) + \sum_{T \subsetneq N: i \in T} \left(\frac{1 - \tau_{|T|}}{|T|}\right) \cdot \lambda_T(v), \quad i \in N,\ v \in \mathbb{V}. \tag{8}$$

While the parametrizations in the previous section are closely related via (7), our parametrization is distinct. Instead of standard games, the unanimity games are employed to recover the coefficients. For $T \subsetneq N$, $T \neq \emptyset$, and $i \in N \setminus T$, we have

$$\tau_{|T|} = |N| \cdot \zeta_i^\tau(u_T). \tag{9}$$

The parameters $\left(\tau_1, \ldots, \tau_{|N|-1}\right)$ can be interpreted as tax rates that are applied to (scaled) unanimity games. For $\lambda \cdot u_T$, $\lambda \in \mathbb{R}$, $T \subseteq N$, $T \neq \emptyset$, we obtain

$$\zeta_i^\tau(\lambda \cdot u_T) = \frac{\tau_{|T|} \cdot \mathrm{Sh}_N(\lambda \cdot u_T)}{|N|} + \left(1 - \tau_{|T|}\right) \cdot \mathrm{Sh}_i(\lambda \cdot u_T), \qquad i \in N.$$

That is, player i's Shapley payoff is taxed at a rate of $\tau_{|T|}$, leaving him a net income of $\left(1 - \tau_{|T|}\right) \cdot \mathrm{Sh}_i(\lambda \cdot u_T)$, while the resulting overall tax revenue amounting to $\tau_{|T|} \cdot \mathrm{Sh}_N(\lambda \cdot u_T)$ is distributed evenly among *all* players. Note that this kind of taxation and redistribution would not affect the payoffs for $\lambda \cdot u_N$. Hence, there is no tax rate $\tau_{|N|}$. The following proposition is immediate from (8) and Malawski (2008, Theorem 2).

Proposition 1. *A value φ on N satisfies **L**, **E**, and **S** iff there is some $\tau \in \mathbb{R}^{|N|-1}$ such that $\varphi = \zeta^\tau$, where ζ^τ is as in (8).*

A number of values in the literature belong to the class of ELS values. In Table 1 below, we provide the tax rates $\tau \in \mathbb{R}^{|N|-1}$ for some of them. Unfortunately, there seems to be no “nice” expressions for the tax rates that produce the solidarity value.

	τ_1	τ_2	$\cdots$	τ_t	$\cdots$	$\tau_{\lvert N\rvert-1}$
Sh	0	0	$\cdots$	0	$\cdots$	0
Sh^α	$1-\alpha$	$1-\alpha$		$1-\alpha$		$1-\alpha$
CON	0	$\frac{1}{2}$	$\cdots$	$\frac{1}{2}$	$\cdots$	$\frac{1}{2}$
ES	0	1	$\cdots$	1	$\cdots$	1
LSPN	$1-\frac{1}{\lvert N\rvert}$	$1-\frac{1}{\lvert N\rvert}$	$\cdots$	$1-\frac{t}{2^{t-1}}\frac{1}{\lvert N\rvert}$	$\cdots$	$1-\frac{\lvert N\rvert-1}{2^{\lvert N\rvert-2}}\frac{1}{\lvert N\rvert}$
ED	1	1	$\cdots$	1	$\cdots$	1

Table 1. Tax rates for some ELS values

4. Solidarity and fair taxation

Within the class of ELS values dwells a huge number of values that do not show certain economically sound properties. In this section, we provide conditions on the parameters of our formula (8) such that one or another of the desirable properties mentioned in the introduction is satisfied. These properties can be viewed as requirements of fair taxation.

4.1. **Technical preliminaries.** Later on, we will make heavy use of the following definitions. For $m \in \mathbb{N}$ and $x \in \mathbb{R}^m$, the backward differences $\Delta_t^k x$, $t \in \{1, \ldots, m\}$, $k \in \{0, \ldots, m-t\}$ are given recursively by

$$\Delta_t^0 x := x_t \quad \text{and} \quad \Delta_t^{k+1} x := \Delta_t^k x - \Delta_{t+1}^k x, \quad t \in \{1, \ldots, m\}, \ k \in \{0, \ldots, m-t\}. \tag{10}$$

It is well-known/easy to show that $\Delta_t^k x$ is given by

$$\Delta_t^k x = \sum_{\ell=0}^{k} (-1)^\ell \cdot \binom{k}{\ell} \cdot x_{t+\ell}, \qquad t \in \{1, \ldots, m\}, \ k \in \{0, \ldots, m-t\}. \tag{11}$$

Moreover, we employ two transformations of $x \in \mathbb{R}^m$, $m \in \mathbb{N}$. We consider $\eta(x), \pi(x) \in \mathbb{R}^m$ defined by

$$\eta_t(x) := \frac{x_t}{t} \qquad \text{and} \qquad \pi_t(x) := \frac{1-x_t}{t}, \qquad t \in \{1, \ldots, m\}. \tag{12}$$

Let $\mathbf{0}, \mathbf{1} \in \mathbb{R}^m$ be given by $\mathbf{0}_t = 0$ and $\mathbf{1}_t = 1$ for all $t \in \{1, \ldots, m\}$. By induction on k, one easily shows

$$\Delta_t^k \eta (\sigma \cdot \mathbf{1}) = \frac{\sigma}{(t+k) \cdot \binom{t+k-1}{k}}, \qquad \sigma \in \mathbb{R}. \tag{13}$$

Hence, the forward differences of both transforms are related by

$$\Delta_t^k \pi (x) = \Delta_t^k \eta (\mathbf{1}) - \Delta_t^k \eta (x) = \frac{1}{(t+k) \cdot \binom{t+k-1}{k}} - \Delta_t^k \eta (x) \tag{14}$$

for all $t \in \{1, \ldots, m\}$, $k \in \{0, \ldots, m-t\}$.

4.2. **Desirability.** Even if players express solidarity among themselves, the payoffs should reflect their individual productivity. At least, payoff differentials should not be opposite to their productivities. This idea is expressed by the desirability axiom.

Desirability, D (Maschler & Peleg 1966). For all $v \in \mathbb{V}$ and $i, j \in N$ such that $MC_i^v(K) \geq MC_j^v(K)$ for all $K \subseteq N \setminus \{i, j\}$, we have $\varphi_i(N, v) \geq \varphi_j(N, v)$.[3]

The following theorem identifies those tax systems $\tau \in \mathbb{R}^{|N|-1}$ for which the resulting ELS value ζ^τ meets desirability. The lengthy proof the theorem is referred to the appendix.

Theorem 1. *The value ζ^τ, $\tau \in \mathbb{R}^{|N|-1}$ obeys desirability iff $\Delta_t^k \pi(\tau) \geq 0$ for all $t \in \{1, \ldots, |N|-1\}$ and $k \in \{0, \ldots, |N|-t-1\}$, where $\pi(\tau) \in \mathbb{R}^{|N|-1}$ is given by (12).*

Remark 1. Theorem 1 implies the following necessary requirements on $\tau \in \mathbb{R}^{|N|-1}$ for ζ^τ to satisfy desirability. (i) $\Delta_t^0 \pi \geq 0$, i.e., $\tau_t \leq 1$ for all $t \in \{1, \ldots, |N|-1\}$, i.e., the players should not be overtaxed. (ii) $\Delta_t^1 \pi \geq 0$, i.e., $\tau_{t+1} \geq \tau_t + \frac{\tau_t - 1}{t}$ for all $t \in \{1, \ldots, |N|-2\}$. Given $\tau_t \leq 1$, this requires that tax rates do not decrease too much when t increases. In particular, if $\tau_t = 1$ for some t, then $\tau_s = 1$ for all $s \geq t$.

Remark 2. Let $\tau \in \mathbb{R}^{|N|-1}$ be such that $\tau_t = \sigma \leq 1$ for some $\sigma \in \mathbb{R}$ and all $t \in \{1, \ldots, |N|-1\}$. By (13), (14), and Theorem 1, the ELS value ζ^τ meets desirability. Hence by Table 1, the Shapley value, the egalitarian Shapley values, and the equal division value obey desirability. Moreover, one easily checks that the tax systems of the equal surplus division value as well as of the consensus value meet the condition

[3] Desirability is also known as *local monotonicity* (e.g. Levinský & Silársky 2004) or *fair treatment* (e.g. Radzik & Driessen 2012).

in Theorem 1. By induction on k, we obtain $\Delta_t^k \pi(\tau) = \frac{1}{2^{t-1-k}} \frac{1}{|N|} > 0$ for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$ for the least-square pre-nucleolus. Hence, it also satisfies desirability.

Remark 3. The ELS value DR^b in (6) satisfies desirability iff $b_t \geq 0$ for all $t \in \{1, \ldots, |N| - 1\}$ Radzik & Driessen (2012, Theorem 1). Compare this with Theorem 1. By (10) and (12), it is tantamount to requiring $\Delta_t^k \pi(\tau) \geq \Delta_t^k \pi(\mathbf{0})$ for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$. In a strong sense, taxes are required to be non-negative.

4.3. **Positivity for null players.** In monotonic games, no player ever is destructive, i.e., all players always have a non-negative productivity. Hence, even if players show solidarity to less productive, ones nobody should end up with a sub-zero payoff. This idea is expressed by the positivity axiom.

Positivity (Kalai & Samet 1987), P. For all $v \in \mathbb{V}$ that are monotonic and all $i \in N$, we have $\varphi_i(N, v) \geq 0$.[4]

There seems to be no "nice" way to characterize those values ζ^τ, $\tau \in \mathbb{R}^{|N|-1}$ that satisfy positivity. Instead, we provide conditions for a weaker requirement, the restriction of positivity to null players. In the next subsection, however, we will see that positivity for null players combined with desirability already entails positivity for ELS values.

Positivity for null players, PN. For all $v \in \mathbb{V}$ that are monotonic and all $i \in N$ who are null players in v, we have $\varphi_i(N, v) \geq 0$.

The lengthy proof the following theorem is referred to the appendix.

Theorem 2. *The value ζ^τ, $\tau \in \mathbb{R}^{|N|-1}$ on N obeys **PN** iff $\Delta_t^k \eta(\tau) \geq 0$ for all for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$, where $\eta(\tau) \in \mathbb{R}^{|N|-1}$ is given by (12).*

Remark 4. Theorem 2 implies the following necessary requirements on $\tau \in \mathbb{R}^{|N|-1}$ for ζ^τ to satisfy positivity for null players. (i) $\Delta_t^0 \eta(\tau) \geq 0$, i.e., $\tau_t \geq 0$ for all $t \in \{1, \ldots, |N| - 1\}$, i.e., players should not be undertaxed in the sense that tax rates are negative. (ii) $\Delta_t^1 \pi(\tau) \geq 0$, i.e., $\frac{t+1}{t} \tau_t \geq \tau_{t+1}$ for all $t \in \{1, \ldots, |N| - 2\}$. Given $\tau_t \geq 0$, this requires that tax rates do not increase too much for increasing t.

[4] Positivity is also known as *monotonicity* (e.g. Radzik & Driessen 2012).

In particular, if $\tau_t = 0$ for some t, then $\tau_s = 0$ for all $s \geq t$. Hence by Table 1, the equal surplus division value and the consensus value fail positivity for null players.

Remark 5. Let $\tau \in \mathbb{R}^{|N|-1}$ be such that $\tau_t = \sigma \geq 0$ for some $\sigma \in \mathbb{R}$ and all $t \in \{1, \ldots, |N| - 1\}$. By (13), (14), and Theorem 2, the ELS value ζ^τ meets positivity for null players. Hence by Table 1, the Shapley value, the egalitarian Shapley values, and the equal division value obey positivity for null players. Also, it is immediate that the equal surplus division value as well as of the consensus value fail positivity for null players. For the least-square pre-nucleolus, using (13) and by induction on k, we obtain

$$\Delta_t^k \eta(\tau) = \frac{1}{(t+k)\binom{t+k-1}{k}} - \frac{1}{2^{t-1+k} \cdot |N|} \geq 0$$

for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$. Hence, it also satisfies positivity for null players.

4.4. **Social acceptability.** In the previous two subsections, we dealt with two properties that seem to be crucial for (ELS) values to be economically sound, desirability and positivity (for null players). Joosten et al. (1994) consider a weaker version of the combination of these axioms, the social acceptability axiom.

Social acceptability, SA. For all $T \subseteq N$, $T \neq \emptyset$, $i \in T$, and $j \in N \setminus T$, we have $\varphi_i(u_T) \geq \varphi_j(u_T) \geq 0$.

Social acceptability imposes rather weak fairness requirements. Since unanimity games are monotonic, the requirement $\varphi_i(u_T) \geq 0$ and $\varphi_j(u_T) \geq 0$ above is equivalent to positivity restricted to unanimity games. In u_T, the players in T are more productive than those in $N \setminus T$. Hence for ELS values, demanding $\varphi_i(u_T) \geq \varphi_j(u_T)$ for $i \in T$ and $j \in N \setminus T$ is equivalent to desirability restricted to unanimity games.

Since the values ζ^τ are closely related to the linear representation of games by unanimity games, we state the following obvious proposition with some diffidence and mainly for completeness' sake. Economically, it just says that there should be no undertaxing and no overtaxing. One easily checks that all ELS values listed in Table 1 meet social acceptability.

Proposition 2. *The value ζ^τ, $\tau \in \mathbb{R}^{|N|-1}$ obeys **SA** iff $\tau_t \in [0,1]$ for all $t \in \{1, \ldots, |N| - 1\}$.*

Proof. Fix $T \subseteq N$, $|T| = t < |N|$. Let $i \in T$, $j \in N \setminus T$. By (8), we have $\zeta_i^\tau(u_T) - \zeta_j^\tau(u_T) = \frac{1-\tau_{|T|}}{|T|} \geq 0$ iff $\tau_t \leq 1$ and $\zeta_j^\tau(u_T) = \frac{\tau_t}{|N|} \geq 0$ iff $\tau_t \geq 0$. Further, $\zeta_i^\tau(u_N) = |N|^{-1} > 0$ for all $i \in N$. □

Remark 6. Compare the results of the proposition with analogous findings for the parametrizations based on standard games. The ELS value DR^b in (6) satisfies social acceptability iff

$$0 \leq \frac{|N| \cdot t}{|N| - t} \cdot \binom{|N|}{t}^{-1} \cdot \sum_{s=t}^{|N|-1} \binom{s}{t} \cdot \frac{b_s}{s} \leq 1$$

for all $t \in \{1, \ldots, |N| - 1\}$ (Radzik & Driessen 2012, Theorem 3).

4.5. **Strong social acceptability.** While the restriction of desirability and positivity to unanimity games is of some interest because unanimity games have a simple structure and can easily be interpreted in economic terms, one may wonder "Why stop here? Why not go the whole way?" In the following, we explore a strong version of social acceptability, the combination of desirability and positivity.

Strong social acceptability, $\mathbf{SA}^+$. The value φ obeys **D** and **P**.

The following lemma entails that an ELS value obeying **D** and **PN** also satisfies $\mathbf{SA}^+$.

Lemma 1. ***E**, **A**, **D**, and **PN** imply **P**.*

Proof. Let φ on N obey **E**, **A**, **D**, and **PN** and let $v \in \mathbb{V}$ be monotonic. For $i \in N$, let $v^i \in \mathbb{V}$ be given by

$$v^i(S) = v(S \setminus \{i\}), \ S \subseteq N. \tag{15}$$

Then, v^i is monotonic too. Moreover, i is a null player in (N, v^i). Since v is monotonic and by (15), we have

$$MC_j^{v-v^i}(K) = 0 \leq MC_i^{v-v^i}(K), \qquad j \in N \setminus \{i\}, \ K \subseteq N \setminus \{i, j\}.$$

By **D** and **A**, this implies

$$\varphi_j(v) - \varphi_i(v) \leq \varphi_j(v^i) - \varphi_i(v^i).$$

Summing up over $j \in N$ and applying **E** and (15), we obtain

$$v(N) - |N| \cdot \varphi_i(v) \leq v(N \setminus \{i\}) - |N| \cdot \varphi_i(v^i).$$

Since v is monotonic, this entails

$$\varphi_i(N, v) \geq \varphi_i(N, v^i) \overset{\mathbf{PN}}{\geq} 0$$

and we are done. □

In view of Theorems 1 and 2, and Lemma 1, the following theorem is immediate.

Theorem 3. *The value* ζ^{τ}, $\tau \in \mathbb{R}^{|N|-1}$ *on* N *obeys* $\boldsymbol{SA}^{+}$ *iff* $\Delta_t^k \pi(\tau) \geq 0$ *and* $\Delta_t^k \eta(\tau) \geq 0$ *for all for all* $t \in \{1, \dots, |N|-1\}$ *and* $k \in \{0, \dots, |N|-t-1\}$, *where* $\pi(\tau), \eta(\tau) \in \mathbb{R}^{|N|-1}$ *are as in (12).*

Remark 7. The ELS value DR^b in (6) satisfies $\mathbf{SA}^+$ iff $1 \geq b_t \geq 0$ for all $t \in \{1, \dots, |N|-1\}$ Radzik & Driessen (2012, Theorem 2). Compare this with Theorem 3. By (10), (12), and (14), it tantamount to $\Delta_t^k \eta(\mathbf{1}) \geq \Delta_t^k \pi(\tau) \geq \Delta_t^k \pi(\mathbf{0})$ for all $t \in \{1, \dots, |N|-1\}$ and $k \in \{0, \dots, |N|-t-1\}$. In a strong sense, taxes are required to fall between 0 and 1.

Remark 8. From the remarks in the previous two subsections it is clear that all ELS values listed in Table 1, except the equal surplus division value and the consensus value, are strongly socially acceptable.

We now demonstrate the power of Theorem 3 with some examples. The following technical lemma facilitates the application of the theorem.

Lemma 2. *Let* $m \in \mathbb{N}$ *and* $f : [1, m] \to \mathbb{R}$ *be differentiable up to order* $m-1$ *and such that* $(-1)^k \cdot f^{(k)}(\xi) \geq 0$ *for all* $\xi \in [1, m]$ *and* $k \in \{0, \dots, m-t\}$. *For* $x \in \mathbb{R}^m$ *given by* $x_t = f(t)$ *for all* $t \in \{1, \dots, m\}$, *we have* $\Delta_t^k x \geq 0$ *for all* $t \in \{1, \dots, m\}$ *and* $k \in \{0, \dots, m-t\}$.

Proof. Let m and f be as in the lemma. For $t \in \{1, \dots, m\}$, we have

$$\Delta_t^0 x = x_t = f(t) = f^{(0)}(t) = (-1)^0 \cdot f^{(0)}(t) \geq 0.$$

By induction on k, one easily shows

$$\Delta_t^k x = (-1)^k \int_t^{t+1} \int_{i_2}^{i_2+1} \int_{i_3}^{i_3+1} \dots \int_{i_k}^{i_k+1} f^{(k)}(\xi)\, d\xi di_k \dots di_3 di_2$$

for all $t \in \{1, \dots, m\}$ and $k \in \{1, \dots, m-t\}$. The claim now follows from $(-1)^k \cdot f^{(k)}(\xi) \geq 0$ for all $\xi \in [1, m]$ □

Example 1. Consider the tax rates $\tau \in \mathbb{R}^{|N|-1}$, $\tau_t = \frac{|N|-t}{|N|}$, $t \in \{1, \dots, |N|-1\}$, i.e., the tax rate amounts to the share of unproductive players in u_T, $|T| = t$. The resulting value ζ^{τ} meets $\mathbf{SA}^+$. To see this, let $f : [1, |N|-1] \to \mathbb{R}$ be given by

$$f(\xi) = \frac{|N| - \xi}{|N| \cdot \xi}, \qquad \xi \in [1, |N|-1].$$

By (12), we have $f(t) = \frac{\tau_t}{t} = \eta(\tau)$ for $t \in \{1, \ldots, |N| - 1\}$. Moreover, one obtains $f^{(0)}(\xi) = f(\xi) \geq 0$ and

$$f^{(k)}(\xi) = \frac{(-1)^k \cdot k!}{\xi^{k+1}},$$

hence, $(-1)^k \cdot f^{(k)}(\xi) \geq 0$ for all $\xi \in [1, |N| - 1]$, $k \in \{0, \ldots, |N| - t - 1\}$. By Lemma 2, we have $\Delta_t^k \eta(\tau) \geq 0$ for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$. By (12), we obtain $\pi_t(\tau) = |N|^{-1}$ for all $t \in \{1, \ldots, |N| - 1\}$. Hence, $\Delta_t^k \pi(\tau) \geq 0$ for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$. Finally, the claim follows from Theorem 3.

Example 2. We now consider the tax system $\tau \in \mathbb{R}^{|N|-1}$ such that

$$\zeta_i^\tau(u_T) = \frac{1}{2} \cdot \zeta_j^\tau(u_T), \qquad T \subsetneq N,\ T \neq \emptyset,\ i \in N \setminus T,\ j \in T.$$

That is, in unanimity games, unproductive players obtain one half of the payoff of productive players. By (8), we obtain

$$\tau_t = \frac{|N|}{t + |N|}, \qquad t \in \{1, \ldots, |N| - 1\}.$$

The resulting value ζ^τ meets $\mathbf{SA}^+$. To see this, let $f, g : [1, |N| - 1] \to \mathbb{R}$ be given by

$$\begin{aligned} f(\xi) &= \frac{|N|}{\xi + |N|} \cdot \frac{1}{\xi}, \\ g(\xi) &= \left(1 - \frac{|N|}{\xi + |N|}\right) \cdot \frac{1}{\xi}, \qquad \xi \in [1, |N| - 1]. \end{aligned}$$

By (12), we have $f(t) = \frac{\tau_t}{t} = \eta(\tau)$ and $g(t) = \frac{1-\tau_t}{t} = \eta(\tau)$ for $t \in \{1, \ldots, |N| - 1\}$. Moreover, one obtains $f^{(0)}(\xi) \geq 0$, $g^{(0)}(\xi) \geq 0$, and

$$\begin{aligned} f^{(k)}(\xi) &= \frac{(-1)^k \cdot k!}{(|N| + t)^{k+1}} \cdot \frac{|N|}{t^{k+1}} \cdot \sum_{\ell=0}^{k} \binom{k+1}{\ell} \cdot |N|^{k-\ell} \cdot t^\ell, \\ g^{(k)}(\xi) &= \frac{(-1)^k \cdot k!}{(|N| + t)^{k+1}}, \end{aligned}$$

hence, $(-1)^k \cdot f^{(k)}(\xi) \geq 0$ and $(-1)^k \cdot g^{(k)}(\xi) \geq 0$ for all $\xi \in [1, m]$, $k \in \{0, \ldots, |N| - t - 1\}$. By Lemma 2, we have $\Delta_t^k \eta(\tau) \geq 0$ and $\Delta_t^k \pi(\tau) \geq 0$ for all $t \in \{1, \ldots, |N| - 1\}$ and $k \in \{0, \ldots, |N| - t - 1\}$. Finally, the claim follows from Theorem 3.

4.6. **General acceptability.** Radzik & Driessen (2012) consider another rather weak notion of acceptability, general acceptability.

General acceptability, GA. For all $S, T \subseteq N$ and $i \in N$ such that $S \subseteq T$ and $i \in S$, we have $\varphi_i(u_S) \geq \varphi_i(u_T)$.

Within the class of ELS values, general acceptability coincides with strong monotonicity for unanimity games. Note that on the domain of all TU games, there is a unique ELS value that meets strong monotonicity, the Shapley value (Young 1985, Theorem 2).

Strong monotonicity, Mo$^+$ (Young 1985). For all $v, w \in \mathbb{V}$ and $i \in N$ such that $v(K \cup \{i\}) - v(K) \geq w(K \cup \{i\}) - w(K)$ for all $K \subseteq N \setminus \{i\}$, $\varphi_i(v) \geq \varphi_i(w)$.

Proposition 3. *The value ζ^τ, $\tau \in \mathbb{R}^{|N|-1}$ obeys **GA** iff (i) $\tau_t \leq 1$ for all $t \in \{1, \ldots, |N|-1\}$ and (ii)*

$$\tau_{t+1} - \tau_t \geq \frac{\tau_t - 1}{t} \cdot \frac{|N|}{|N| - t - 1}$$

for all $t \in \{1, \ldots, |N|-2\}$.

Proof. Let $t \in \{1, \ldots, |N|-1\}$ and $T \subseteq N$, $|T| = t$. By (8), $\zeta_i^\tau(u_T) \geq \zeta_i^\tau(u_N)$ iff $\tau_t \leq 1$. Let $s \in \{1, \ldots, |N|-2\}$ and $S, T \subseteq N$, $S \subseteq T$, $|S| = s$, $|T| = s+1$. By (8), $\zeta_i^\tau(u_S) \geq \zeta_i^\tau(u_T)$ iff

$$\tau_{s+1} - \tau_s \geq \frac{\tau_s - 1}{s} \cdot \frac{|N|}{|N| - s - 1},$$

which entails the second part of the requirement. □

Remark 9. Proposition 3 first requires that there is no overtaxing, $\tau_t \leq 1$. Given this, the second requirement says that tax rates should not decrease too much when t increases. In particular, if $\tau_t = 1$ for some t, then $\tau_s = 1$ for all $s \geq t$. Recall some necessary requirements for desirability due to Theorem 1, (i) $\tau_t \leq 1$ for all $t \in \{1, \ldots, |N|-1\}$ and (ii) $\tau_{t+1} \geq \tau_t + \frac{\tau_t - 1}{t}$ for all $t \in \{1, \ldots, |N|-2\}$. Since $\tau_t - 1 \leq 0$ and $\frac{|N|}{|N|-t-1} > 1$, desirability implies general acceptability for ELS values.

Remark 10. Compare the results of the proposition with analogous findings for the parametrizations based on standard games. The ELS value DR^b in (6) satisfies general acceptability iff

$$0 \leq \sum_{s=t}^{|N|-1} \frac{|N| - s}{s} \cdot \binom{s}{t} \cdot b_s$$

for all $t \in \{1, \ldots, |N|-1\}$ Radzik & Driessen (2012, Theorem 4).

5. Appendix

In the following, we employ a technical lemma. Note that the lemma is much stronger than we need. Actually, we just make use of $b = m$ or $b = m-1$. Moreover, we do not employ (ii) $\Rightarrow$ (i) for $b = m - 1$.

Lemma 3. *Let M be a non-empty and finite set, $m = |M|$. For $x \in \mathbb{R}^{m+1}$ and $b \in \{0, \ldots, m\}$ the following statements are equivalent:*

(i) For all $t \in \{1, \ldots, b+1\}$ and $k \in \{0, \ldots, b+1-t\}$, $\Delta_t^k x \geq 0$.

(ii) For all $f : 2^M \to \mathbb{R}$ such that

$$\sum_{S \subseteq T} f(S) \geq 0, \qquad T \subseteq M : |T| \leq b, \tag{16}$$

we have

$$\sum_{T \subseteq M : |T| \leq b} x_{|T|+1} \cdot f(T) \geq 0. \tag{17}$$

Proof. Let M, m, and b be as in the lemma. (i)$\Rightarrow$(ii): Consider $x \in \mathbb{R}^{m+1}$ as in (i) and f as in (16). Hence, we have

$$\sum_{T \subseteq M : |T| \leq b} \Delta_{|T|+1}^{b-|T|} x \cdot \sum_{K \subseteq T} f(K) \geq 0$$

and therefore

$$\sum_{K \subseteq M : |K| \leq b} f(K) \cdot \sum_{K \subseteq T \subseteq M : |T| \leq b} \Delta_{|T|+1}^{b-|T|} x \geq 0. \tag{18}$$

For $K \subseteq M$, $k := |K| \leq b$, we have

$$\begin{aligned}
&\sum_{K\subseteq T\subseteq M:|T|\leq b} \Delta_{|T|+1}^{b-|T|}x \\
&= \sum_{t=k}^{b} \binom{m-k}{t-k} \cdot \Delta_{t+1}^{b-t}x \\
&\overset{(11)}{=} \sum_{t=k}^{b}\sum_{\ell=0}^{b-t} (-1)^{\ell} \binom{m-k}{t-k}\binom{b-t}{\ell} \cdot x_{t+1+\ell} \\
&\overset{q=t+\ell+1}{=} \sum_{q=k+1}^{b+1} x_q \cdot \sum_{\ell=0}^{q-(k+1)} (-1)^{\ell} \binom{m-k}{q-1-\ell-k}\binom{b-q+\ell+1}{\ell} \\
&= x_{k+1} + \sum_{q=k+2}^{b+1} x_q \cdot \sum_{\ell=0}^{q-(k+1)} (-1)^{\ell} \binom{m-k}{q-1-\ell-k}\binom{b-q+1+\ell}{\ell} \\
&= x_{k+1} + \sum_{q=k+2}^{b+1} x_q \cdot \binom{m-k}{q-k-1} \sum_{\ell=0}^{q-(k+1)} (-1)^{\ell} \binom{q-(k+1)}{\ell} \\
&= x_{k+1},
\end{aligned}$$

where the last equation drops from the well-known fact that $\sum_{\ell=0}^{a} (-1)^{\ell} \binom{a}{\ell} = 0$ for $a \in \mathbb{N}$, $a > 0$. By (18), we are done.

(ii)⇒(i): Let $x \in \mathbb{R}^{m+1}$ be such that $\Delta_t^k x < 0$ for some $t \in \{1, \ldots, b+1\}$ and $k \in \{0, \ldots, b+1-t\}$. Fix $T, K \subseteq N$, such that $T \subseteq K$, $|T| = t-1$ and $|K| = t-1+k$. Consider $f : 2^M \to \mathbb{R}$ given by

$$f(S) = \begin{cases} (-1)^{|S|-|T|}, & T \subseteq S \subseteq K, \\ 0, & \text{else,} \end{cases} \qquad S \subseteq M. \tag{19}$$

Let $S \subseteq M$, $|S| \leq b$. It is immediate that $\sum_{L\subseteq S} f(L) = 0$ whenever $T \nsubseteq S$. If $T \subseteq S$, then

$$\begin{aligned}
\sum_{L\subseteq S} f(L) &= \sum_{C\subseteq(S\cap K)\setminus T} f(T\cup C) \\
&\overset{(19)}{=} \sum_{C\subseteq(S\cap K)\setminus T} (-1)^{|C|} \\
&= \sum_{c=0}^{|S\cap K|-|T|} \binom{|S\cap K|-|T|}{c} (-1)^{c}.
\end{aligned}$$

This implies $\sum_{L\subseteq S} f(S) = 1$ for $S\cap K = T$ and $\sum_{L\subseteq S} f(S) = 0$ for $|S\cap K| > |T|$. Hence, f is as in (16). Yet, we have

$$\begin{aligned}
\sum_{S\subseteq M:|S|\leq b} x_{|S|+1}\cdot f(S) &\overset{|K|\leq b}{=} \sum_{C\subseteq K\setminus T} (-1)^{|C|}\, x_{|T|+1+|C|} \\
&= \sum_{c=0}^{|K|-|T|} (-1)^c \binom{|K|-|T|}{c} x_{|T|+1+c} \\
&= \sum_{c=0}^{k} (-1)^c \binom{k}{c} x_{|T|+1+c} \\
&= \sum_{c=0}^{k} (-1)^c \binom{k}{c} x_{t+c} \\
&\overset{(11)}{=} \Delta_t^k x < 0.
\end{aligned}$$

Done. □

Proof of Theorem 1. For $|N| = 1$, nothing is to show. Let $|N| > 1$ and $\tau = (\tau_1,\ldots,\tau_{|N|-1}) \in \mathbb{R}^{|N|-1}$ be such that

$$\Delta_t^k \pi \geq 0, \qquad t\in\{1,\ldots,|N|-1\},\ k\in\{0,\ldots,|N|-t-1\}. \tag{20}$$

Further, let $i,j\in N$ and $v\in\mathbb{V}$ be such that $MC_i^v(K)\geq MC_j^v(K)$ for all $K\subseteq N\setminus\{i,j\}$. Hence by (2),

$$\sum_{S\subseteq K} \left(\lambda_{S\cup\{i\}}(v) - \lambda_{S\cup\{j\}}(v)\right) \geq 0, \qquad K\subseteq N\setminus\{i,j\}. \tag{21}$$

Set now $M = N\setminus\{i,j\}$, $b = |N|-2$, $x_t = \pi_t = \frac{1-\tau_t}{t}$, $t\in\{1,\ldots,|N|-1\}$ and $f: 2^M\to\mathbb{R}$, $f(S) = \lambda_{S\cup\{i\}}(v) - \lambda_{S\cup\{j\}}(v)$, $S\subseteq N\setminus\{i,j\}$. By (20) and (21), the former data meet Lemma 3(i) and (16). Thus, the lemma implies

$$\begin{aligned}
\zeta_i^\tau(v) - \zeta_j^\tau(v) &\overset{(8)}{=} \sum_{T\subseteq N\setminus\{i,j\}} \frac{1-\tau_{|T|+1}}{|T|+1}\cdot\left(\lambda_{S\cup\{i\}}(v) - \lambda_{S\cup\{j\}}(v)\right) \\
&\overset{\text{def. } f}{=} \sum_{T\subseteq M:|T|\leq b} x_{|T|+1}\cdot f(T) \geq 0.
\end{aligned}$$

$$\sum_{S\subseteq T} f(S) \geq 0, \qquad T\subseteq 2^{N\setminus\{i,j\}}, \tag{22}$$

but

$$\sum_{T\subseteq N\setminus\{i,j\}} x_{|T|+1}\cdot f(T) < 0. \tag{23}$$

Fix $i, j \in N$. Let $v \in \mathbb{V}$ be such that

$$\lambda_{S\cup\{i\}}(v) = f(S) \qquad \text{and} \qquad \lambda_{S\cup\{j\}}(v) = 0 \qquad \text{for all } S \subseteq N\setminus\{i,j\}. \tag{24}$$

Hence,

$$\begin{aligned} MC_i^v(K) - MC_j^v(K) &\overset{(2)}{=} \sum_{S\subseteq K} \left(\lambda_{S\cup\{i\}}(v) - \lambda_{S\cup\{j\}}(v)\right) \\ &\overset{(24)}{=} \sum_{S\subseteq K} f(S) \overset{(22)}{\geq} 0, \qquad K \subseteq 2^{N\setminus\{i,j\}}, \end{aligned}$$

i.e., i, j, and v meet the hypothesis of **D**, but

$$\begin{aligned} \zeta_i^\tau(v) - \zeta_j^\tau(v) &\overset{(8)}{=} \sum_{T\subseteq N\setminus\{i,j\}} \frac{1-\tau_{|T|+1}}{|T|+1} \cdot \left(\lambda_{T\cup\{i\}}(v) - \lambda_{T\cup\{j\}}(v)\right) \\ &\overset{(24)}{=} \sum_{T\subseteq N\setminus\{i,j\}} x_{|T|+1} \cdot f(T) \overset{(23)}{<} 0. \end{aligned}$$

Hence, ζ^τ fails **D**. □

Proof of Theorem 2. For $|N| = 1$, the claim is empty. Let now $|N| > 1$. Let $\tau = \left(\tau_1, \ldots, \tau_{|N|-1}\right) \in \mathbb{R}^{|N|-1}$ be such that

$$\Delta_t^k \eta \geq 0, \qquad t \in \{1, \ldots, |N|-1\},\ k \in \{0, \ldots, |N|-t-1\}. \tag{25}$$

Further, let $v \in \mathbb{V}$ be monotonic and let $i^* \in N$ be a null player in v. The former entails

$$MC_i^v(K) \overset{(2)}{=} \sum_{S\subseteq K} \lambda_{S\cup\{i\}}(v) \geq 0, \qquad i \in N,\ K \subseteq N\setminus\{i\}. \tag{26}$$

For $i \in N$, set now $M = N\setminus\{i\}$, $b = |N|-2$, $x_t = \eta_t = \frac{\tau_t}{t}$, $t \in \{1, \ldots, |N|-1\}$ and $f : 2^M \to \mathbb{R}$, $f(S) = \lambda_{S\cup\{i\}}(v)$, $S \subseteq N\setminus\{i\}$. By (25) and (26), the former data meet Lemma 3(i) and (16). Thus, the lemma implies

$$\begin{aligned} \sum_{T\subsetneq N: i\in T} \frac{\tau_{|T|}}{|T|} \cdot \lambda_T(v) &= \sum_{T\subsetneq N\setminus\{i\}} \frac{\tau_{|T|+1}}{|T|+1} \cdot \lambda_{T\cup\{i\}}(v) \\ &= \sum_{T\subseteq M: |T|\leq b} x_{|T|+1} \cdot f(T) \geq 0, \qquad i \in N. \end{aligned} \tag{27}$$

Since i^* is a null player in v, we have $\lambda_N(v) = 0$. This entails

$$\zeta_{i^*}^\tau(v) \overset{(8)}{=} \sum_{T\subsetneq N: T\neq\emptyset} \frac{\tau_{|T|}}{|N|} \cdot \lambda_T(v) = \frac{1}{|N|} \sum_{i\in N} \sum_{T\subsetneq N: i\in T} \frac{\tau_{|T|}}{|T|} \cdot \lambda_T(v) \overset{(27)}{\geq} 0.$$

Hence, ζ^τ meets **PN**.

Let now $\tau = \left(\tau_1, \ldots, \tau_{|N|-1}\right) \in \mathbb{R}^{|N|-1}$ be such that $\Delta_t^k \eta < 0$ for some $t \in \{1, \ldots, |N|-1\}$, $k \in \{0, \ldots, |N|-t-1\}$. Fix $T \subseteq N$, $|T| = t+k+1$ and set

$$v = \sum_{S \subseteq T : |S| \geq t+1} \binom{|S|-1}{|S|-1-t} (-1)^{|S|-1-t} u_S.$$

Then, all $i \in N \setminus T$ are Null players in (N, v), i.e.,

$$v(C) = v(C \cap T), \qquad C \subseteq N.$$

Let now $C \subseteq T$. If $|C| < t$ then $MC_i^v(C) = 0$ for all $i \in T \setminus C$. For $|C| \geq t$ and $i \in T \setminus C$, we have

$$\begin{aligned}
MC_i^v(C) &= \sum_{S \subseteq C : |S| \geq t} \binom{|S|}{|S|-t} (-1)^{|S|-t} \\
&= \sum_{k=t}^{|C|} \binom{|C|}{k} \binom{k}{k-t} (-1)^{k-t} \\
&= \sum_{k=t}^{|C|} \frac{|C|!}{k!\,(|C|-k)!} \frac{k!}{t!\,(k-t)!} (-1)^{k-t} \\
&= \frac{1}{t!} \sum_{k=t}^{|C|} \frac{(|C|-t)!}{(|C|-k)!\,(k-t)!} \frac{|C|!}{(|C|-t)!} (-1)^{k-t} \\
&= \frac{1}{t!} \frac{|C|!}{(|C|-t)!} \sum_{k=t}^{|C|} \frac{(|C|-t)!}{(|C|-k)!\,(k-t)!} (-1)^{k-t} \\
&= \binom{|C|}{t} \sum_{k=t}^{|C|} \binom{|C|-t}{k-t} (-1)^{k-t} \\
&= \binom{|C|}{t} \sum_{j=0}^{|C|-t} \binom{|C|-t}{j} (-1)^{j},
\end{aligned}$$

i.e., $MC_i^v(C) = 1$ if $|C| = t$ and $MC_i^v(C) = 0$ if $|C| > t$. Hence, v is monotonic.

Let $i \in N \setminus T$, i.e., i is a Null player. We then have

$$\zeta_i^\tau(N, v) = \frac{1}{|N|} \sum_{j \in T} \sum_{j \in S \subseteq T} \frac{\tau_{|S|}}{|S|} \cdot \lambda_S(v) \tag{28}$$

and

$$\begin{aligned}
\sum_{j\in S\subseteq T}\frac{\tau_{|S|}}{|S|}\cdot\lambda_S(v) &= \sum_{j\in S\subseteq T:|S|\geq t+1}\frac{\tau_{|S|}}{|S|}\cdot\lambda_S(v)\\
&= \sum_{j\in S\subseteq T:|S|\geq t+1}\frac{\tau_{|S|}}{|S|}\cdot\binom{|S|-1}{|S|-1-t}(-1)^{|S|-t-1}\\
&= \sum_{k=t+1}^{|T|}\frac{\tau_k}{k}\cdot\binom{|T|-1}{k-1}\binom{k-1}{k-1-t}(-1)^{k-t-1}\\
&= \sum_{k=t+1}^{|T|}\frac{\tau_k}{k}\cdot\binom{|T|-1}{t}\binom{|T|-1-t}{k-1-t}(-1)^{k-t-1}\\
&= \binom{|T|-1}{t}\sum_{k=t+1}^{|T|}\frac{\tau_k}{k}\cdot\binom{|T|-1-t}{k-1-t}(-1)^{k-t-1}\\
&= \binom{|T|-1}{t}\cdot\Delta_{t+1}^{|T|-t-1}\eta\\
&< 0
\end{aligned}$$

for all $j\in T$. Hence, $\zeta_i^\alpha(N,v)<0$, contradicting **PN**. □

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