Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

# BACHELOR – PRÜFUNG

**DATUM:** **13. Februar 2020**

**FACH:** **Unternehmensstrategien im Wettbewerb**
**Competitive Strategy**
KLAUSURDAUER: 60 Min

**PRÜFER:** **Prof. Dr. Harald Wiese**

**MATRIKEL-NR.:**

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

**ERLÄUTERUNGEN:**
**Maximal erreichbare Punkte / Maximal number of points: 50**
**Lesen Sie die Aufgabenstellung vor dem Bearbeiten gründlich!**
**/ Please read careful before writing!**
**Schreiben Sie, bitte, leserlich! / Write legibly, please!**
**Begründen Sie Ihre Antworten! / Jusity your answers!**
**Machen Sie jeweils Ihren Rechenweg deutlich!**
**/ Make your calulations clear!**
**Sollte der Platz unter den Fragen nicht ausreichen,**
**verwenden Sie bitte jeweils die Rückseite!**
**In case your need more space, please use the reverse side!**
**Hilfsmittel: keine / no devices**

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|---|---|---|---|---|---|---|---|---|
| **PUNKTE:** | | | | | | | | **NOTE:** |

**Unterschrift des Prüfers/der Prüfer:**

**Exercise 1 (8 points)**
The demand function is given by

$$X(p) = 10 - \frac{1}{2}p.$$

Assume firm 1 is already established in the market. It produces a good at constant marginal and average cost of $\bar{c} = 8$ and thereby realizes a profit of 18. An innovation could result in a cost reduction to $\underline{c} = 4$, allowing firm 1 to achieve a profit of 32 by choosing $p^M(\underline{c}) = 12$. These results do not need do be verified.

**a)** Determine firm 1's incentive to innovate.

**b)** There is a potential entrant, firm 2, that could realize the innovation instead of firm 1. Firms would compete in prices. Determine the incentive to innovate (according to Arrow) of firm 2.

**Aufgabe 1 (8 Punkte)**
Gegeben sei die Nachfragefunktion:

$$X(p) = 10 - \frac{1}{2}p.$$

Nehmen Sie an, dass Unternehmen 1 bereits im Markt etabliert ist. Die durchschnittlichen und marginalen Kosten von Unternehmen 1 betragen $\bar{c} = 8$, so dass es einen Gewinn von 18 erzielt. Eine Innovation würde zur Senkung der Kosten auf $\underline{c} = 4$ führen, so dass Unternehmen 1 zum Preis $p^M(\underline{c}) = 12$ einen Gewinn von 32 erzielen würde. Diese Ergebnisse müssen Sie nicht überprüfen.

**a)** Bestimmen Sie den den Innovationsanreiz für Unternehmen 1.

**b)** Anstelle von Unternehmen 1 könnte einem potentiellen Konkurrenten, Unternehmen 2, die Innovation gelingen. Die Unternehmen befinden sich dann im Preiswettbewerb. Bestimmen Sie den Innovationsanreiz nach Arrow für Unternehmen 2.

**Proposal of solution:**

**a)** The incentive to innovate for firm 1 is $\Delta\Pi_1^A = 32 - 18 = 14$.

**b)** Since the monopolistic price $p^M(\underline{c}) = 12 > 8 = \bar{c}$, firm 1 is not blockaded and, thus, firm 2 will deter firm 1 by choosing $p = 8 - \epsilon$:

$$\begin{aligned}
\Pi_2^L(\bar{c} - \epsilon) &= (\bar{c} - \epsilon - \underline{c}) \left(10 - \frac{1}{2}(\bar{c} - \epsilon)\right) \\
&= (8 - \epsilon - 4) \left(10 - \frac{1}{2}(8 - \epsilon)\right) \\
&= (4 - \epsilon) \left(6 - \frac{1}{2}\epsilon\right) \\
&= 24 - 4\epsilon - \frac{1}{2}\epsilon^2 \\
&\approx 24
\end{aligned}$$

The Arrow replacement effect is given by $\Delta\Pi_2^A = 24 - 0 = 24$.

**Exercise 2 (13 points)**
There are three firms competing in quantities. Constant marginal and average cost of all firms are $c = 0$. The inverse demand function is given by

$$p(X) = 60 - X.$$

First firm 1 and firm 2 choose their quantities $x_1$ and $x_2$, respectively, simultaneously. Firm 3 can observe these quantities and chooses its own quantity afterwards. Apply backward induction.

**Aufgabe 2 (13 Punkte)**
Auf einem Markt befinden sich drei Unternehmen im Mengenwettbewerb. Die durchschnittlichen und marginalen Kosten für alle Unternehmen sind $c = 0$. Die inverse Nachfrage ist gegeben durch

$$p(X) = 60 - X.$$

Zuerst wählen Unternehmen 1 und Unternehmen 2 ihre Mengen $x_1$, bzw. $x_2$ simultan. Unternehmen 3 kann diese Mengen beobachten und wählt dann seine eigene Menge. Nutzen Sie Rückwärtsinduktion.

**Proposal of solution:**
The game is solved by backward induction. First, we determine the behavior of firm 3 in the second stage:

$$\Pi_3(x_1, x_2, x_3) = (60 - x_1 - x_2 - x_3)x_3$$

$$\begin{aligned}
\frac{\partial \Pi_3}{\partial x_3} = 60 - x_1 - x_2 - 2x_3 &\overset{!}{=} 0 \\
60 - x_1 - x_2 &= 2x_3 \\
x_3^R(x_1, x_2) &= 30 - \frac{1}{2}x_1 - \frac{1}{2}x_2
\end{aligned}$$

In the first stage, firm 1 and 2 anticipate how firm 3 will react in the second stage. The two firms are in a Cournot competition. The profit functions are given by:

$$\begin{aligned}
\Pi_1\left(x_1, x_2, x_3^R(x_1, x_2)\right) &= \left(60 - x_1 - x_2 - x_3^R(x_1, x_2)\right) x_1 \\
&= \left(60 - x_1 - x_2 - (30 - \frac{1}{2}x_1 - \frac{1}{2}x_2)\right) x_1 \\
&= \left(30 - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) x_1,
\end{aligned}$$

$$\Pi_2\left(x_1, x_2, x_3^R(x_1, x_2)\right) = \left(30 - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) x_2$$

By taking the derivative with respect to $x_1$ ($x_2$ respectively), the reaction functions are found to by given by:

$$\begin{aligned}
\frac{\partial \Pi_1}{\partial x_1} = 30 - x_1 - \frac{1}{2}x_2 &\overset{!}{=} 0 \\
x_1^R(x_2) &= 30 - \frac{1}{2}x_2 \\
x_2^R(x_1) &= 30 - \frac{1}{2}x_1
\end{aligned}$$

Plugging $x_2^R(x_1)$ in $x_1^R(x_2)$ gives:

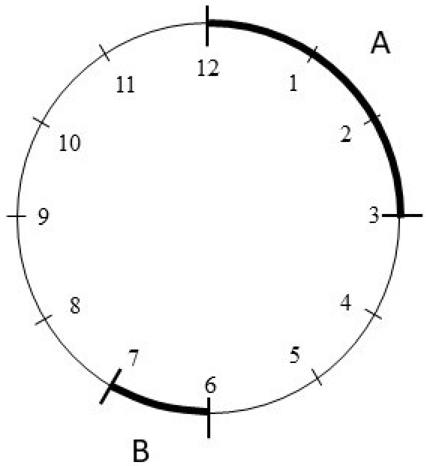
$$x_1^R(x_2) = 30 - \frac{1}{2}\left(30 - \frac{1}{2}x_1\right)$$
$$x_1 = 15 + \frac{1}{4}x_1$$
$$x_1 = 20$$
$$x_2 = 30 - \frac{20}{2} = 20$$

Now, the quantity of firm 3 can be determined:

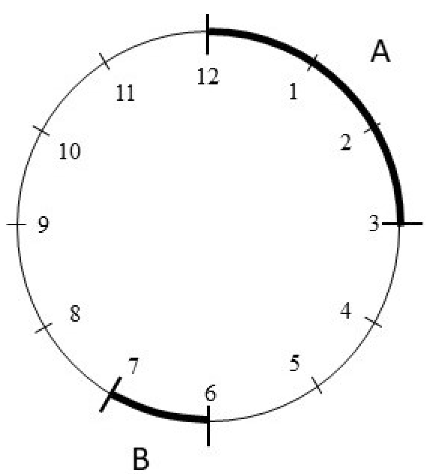
$$x_3^R(x_1 = x_2 = 20) = 30 - \frac{1}{2}(x_1 + x_2) = 10$$

The associated price is $p^S = 60 - (20 + 20 + 10) = 10$. Firm 1 and firm 2 realize a profit of $\Pi_1^S = \Pi_2^S = 10 * 20 = 200$. Firm 3 realizes a profit of $\Pi_3^S = 10 * 10 = 100$.

**Exercise 3 (10 points)**
Consider a circular city of length 12 (like a clock) where 60% of the consumers are located in city A (between 12 and 3) and 40% of the consumers in city B (between 6 and 7). Within the cities consumers are equally distributed. Two firms 1 and 2 maximize their sales. Firms can freely choose their locations $a_1$ and $a_2$, respectively, but are not allowed to locate within the cities. They can locate at the border of the cities , e.g. at 3, 6, 7 or 12. Determine all Nash equilibria! Why are these strategy combinations Nash equilibria? Why are there no further Nash equilibria?




**Aufgabe 3 (10 Punkte)**
Gegeben sei ein Ringstraßendorf der Länge 12 (wie eine Uhr), in dem 60% der Konsumenten in Stadt A (zwischen 12 und 3) und 40% der Konsumenten in Stadt B (zwischen 6 und 7) angesiedelt sind. Innerhalb der Städte sind die Konsumenten gleichverteilt. Zwei Unternehmen 1 und 2 maximieren jeweils ihre Absätze. Die Unternehmen können ihren jeweiligen Standort $a_1$ bzw. $a_2$ frei wählen, dürfen sich allerdings nicht innerhalb der Städte niederlassen. Sie können sich aber genau am Stadtrand, z.B. bei 3, 6, 7, oder 12 positionieren. Bestimmen Sie alle Nash-Gleichgewichte! Warum sind diese Strategiekombinationen Nash-Gleichgewichte? Warum gibt es keine weiteren Nash-Gleichgewichte?




**Proposal of solution:**

The only Nash equilibrium is $(a_1, a_2) = (3, 3)$. Unilaterally deviating clockwise would decrease sales from $\frac{1}{2}$ to 0.4. Choosing a location left of the two cities (between 7 and 12) leads to maximal sales between 0.2 and 0.4. So no firm has an incentive to deviate unilaterally from $(3, 3)$.
There are no other Nash equilibria. We can distinguish between i) a situation where both firms are initally located in the same location and ii) a situation where the initial positions differ. For i), both firms are realizing sales of 0.5. If they are located between $]3, 6]$, each firm could improve by unilaterally deviating to $a_i = 3$, realizing sales of 0.6. If they are located between $[7, 12]$, unilaterally deviating to $a_i = 3$ is always an improvment, realizing sales between 0.6 and 0.8 (depending on the initial location). Thus all strategy combinations where $a_1 = a_2 \neq 3$, are no Nash equilibria bacause unilaterally deviating pays. For ii), consider that choosing $a_i = 3$, garantuees sales $\geq 0.5$, thus whenever the firms do not split demand equally, the worse-off firm can decide to move to $a_i = 3$ to improve. Both firms might be located in the interval $[3, 6]$.The firm closer to city A realizes sales of 0.6, the other firm could improve from 0.4 to at least 0.5 by choosing $a_i = 3$. Alternatively, both firms might be located in the interval $[7, 12]$. If demand is not split equally, the worse-off firm will move. If it is split equally (e.g. $(a_1, a_2) = (7, 10)$), each firm could improve to sales between 0.6 and 0.8 by jumping into the interval $[3, 6]$. A third initial position might be that one firm is located in each intervall. Assuming firm 1 is located in $[3, 6]$ and firm 2 in $[7, 12]$. If $a_1 = 3$, firm 2 gets a maximum of 0.4 sales and improves by choosing $a_2 = 3$. If firm 1 is located between $]3, 6]$, the other firm's best response is to chose $a_2 \in [3, a_1[$ to realize sales of 0.6. There is thus no Nash equilibrium for situation ii).
In short, both firms want to approach the larger city A from the side where they are still closer to the consumers in city B. So if, initially, firm 1 is located between $]3, 6]$, the other firm's best response is to chose $a_2 \in [3, a_1[$ to realize sales of 0.6. Firm 1 could improve by deviating. If firm 1 is initially located between $[7, 12]$, firm 2's best response is to choose a location between $[3, 6]$, realizing sales between 0.6 and 0.8. Firm 1 could again profitably deviate from its initial position. The situation is analogous for firm 2.
There is thus no other Nash equilibrium.

**Exercise 4 (4 points)**
Consider the advertising model, we have seen in class. There are two firms producing good 1 and good 2, respectively. Firms can only sell to informed consumers. The share of consumers informed about product 1 is given by $A_1$, the share of consumers informed about product 2 is given by $A_2$. Interpret the brand demand of firm 1 given by

$$x_1 = A_1(1 - A_2) + A_1 A_2 \left( \frac{1}{2} + \frac{p_2 - p_1}{2t} \right)$$

by commenting on:
- $A_1(1 - A_2)$:
- $A_1 A_2$:
- $\frac{A_1 A_2}{2t}$:
- $p_2 - p_1$:

**Aufgabe 4 (5 Punkte)**
Betrachten Sie das Modell des Werbewettbewerbs, das Sie in der Vorlesung kennengelernt haben. Zwei Firmen, 1 und 2, produzieren jeweils Gut 1 und Gut 2. Die Unternehmen können ihre Produkte nur an informierte Konsumenten verkaufen. Der Anteil der Konsumenten, der über Gut 1 informiert ist, ist $A_1$, der Anteil der Konsumenten, der über Gut 2 informiert ist, ist $A_2$. Interpretieren Sie die folgende Markennachfrage von Unternehmen 1

$$x_1 = A_1(1 - A_2) + A_1 A_2 \left( \frac{1}{2} + \frac{p_2 - p_1}{2t} \right),$$

indem Sie Folgendes kommentieren:
- $A_1(1 - A_2)$:
- $A_1 A_2$:
- $\frac{A_1 A_2}{2t}$:
- $p_2 - p_1$:

**Proposal of solution:**

$$\begin{aligned} x_1 &= A_1(1 - A_2) + A_1 A_2 \left( \frac{1}{2} + \frac{p_2 - p_1}{2t} \right) \\ &= A_1(1 - A_2) + \frac{1}{2} A_1 A_2 + \frac{A_1 A_2}{2t} (p_2 - p_1) \end{aligned}$$

- $A_1(1 - A_2)$: monopolistic part of demand, share of consumers only informed about product 1, but not knowing product 2.
- $A_1 A_2$: share of consumers informed about both products.
- $\frac{A_1 A_2}{2t}$: intensity of competition.
- $p_2 - p_1$: price advantage of firm 1.

**Exercise 5 (8 points)**
Consider the following game:

| | | Player 2 c | Player 2 d |
|---|---|---|---|
| Player 1 | a | $5,6$ | $1,7$ |
| | b | $8,2$ | $3,4$ |

**a)** What is the name of this game? Show the specific characteristics of the game!

**b)** State the reaction function of player 2!

**Aufgabe 5 (8 Punkte)**
Betrachten Sie das folgende Spiel:

| | | Spieler 2 c | Spieler 2 d |
|---|---|---|---|
| Spieler 1 | a | $5,6$ | $1,7$ |
| | b | $8,2$ | $3,4$ |

**a)** Wie wird dieses Spiel bezeichnet? Weisen Sie dessen charakteristische Eigenschaften nach!

**b)** Geben Sie die Reaktionsfunktion von Spieler 2 an!

**Proposal of solution:**

**a)** It is the prisoner's dilemma. The specific characteristics are the existance of strictly dominant strategies of both players ($8 > 5; 3 > 1$ and $7 > 6; 4 > 2$), the existance of a unique Nash-equilibrium ($4 > 2; 3 > 1$), and the Pareto-inefficiency of it ($5 > 3; 6 > 4$).

**b)** The reaction function of player 2 is given by:

$$x_2^R(x_1) = d \text{ for } x_1 \in \{a, b\}$$

**Exercise 6 (7 points)**
Consider the following inverse demand function: $p(X) = 17 - 2X$. A monopolist can produce $X$ at costs:

$$C(X) = \begin{cases} F + 9X, & X > 0 \\ 0, & X = 0 \end{cases}$$

with quasi-fix costs $F \geq 0$.

**a)** Determine the monopolistic quantity depending on $F$.

**b)** Assuming $F = 0$, determine the Herfindahl-index.

**Aufgabe 6 (7 Punkte)**
Betrachten Sie die folgende inverse Nachfragefunktion: $p(X) = 17 - 2X$. Ein Monopolist produziert Gut $X$ zu Kosten von:

$$C(X) = \begin{cases} F + 9X, & X > 0 \\ 0, & X = 0 \end{cases}$$

mit quasifixen Kosten $F \geq 0$.

**a)** Bestimmen Sie die Monopolmenge in Abhängigkeit von $F$.

**b)** Nehmen Sie $F = 0$ an. Bestimmen Sie den Herfindahl-Index.

**Proposal of solution:**

**a)** The monopolist maximizes its profits such that $MR = 17 - 4X$ and $MC = 9$. Setting $MR \stackrel{!}{=} MC$, gives the monopolist quantity of $X^M = 2$ und $p^M = 13$. We check if the profit is positive:

$$\Pi^M(X = 2) = (13 - 9) * 2 - F = 8 - F$$

So we get:

$$X^M = \begin{cases} 2, & F \leq 8 \\ 0, & F \geq 8. \end{cases}$$

**b)** $H = \sum_{i=1}^{N} s_i^2 = \sum_{i=1}^{N} \left(\frac{x_i}{X}\right)^2 = 1.$