Course outline II

- Product differentiation
- Advertising competition
- Compatibility competition

Heterogeneous goods

Competition on variants, locations, and qualities

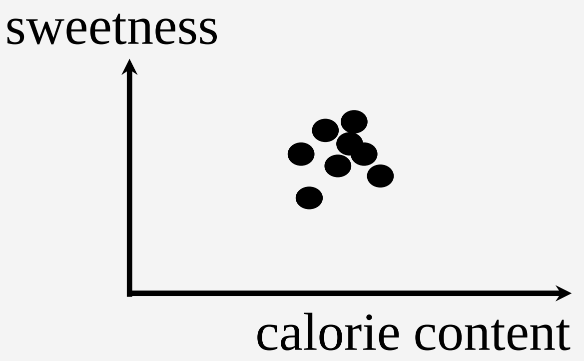
- Basic idea of product differentiation
- The Hotelling Model
- The Schmalensee-Salop model
- Competition on qualities and variations
- Executive summary

Differentiating products in order to overcome the Bertrand paradox

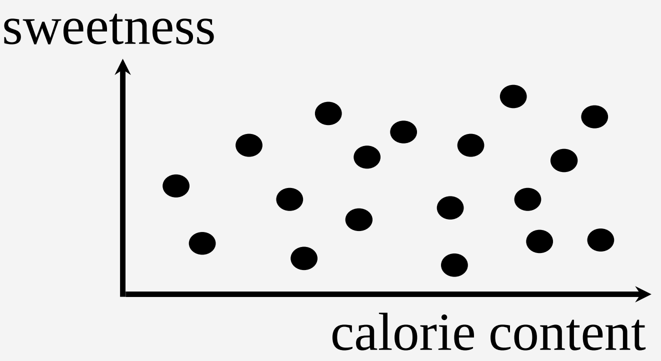
- With homogeneous goods, competition can be quite intense: Even in a market with only two competitors, firms may face a zero-profit situation in a Bertrand-Nash equilibrium.
- Differentiating products may help to achieve positive profits.

Preferences (Example: drinks)

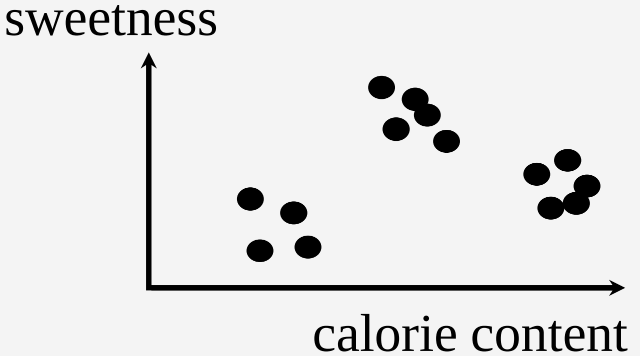
Homogeneous preferences



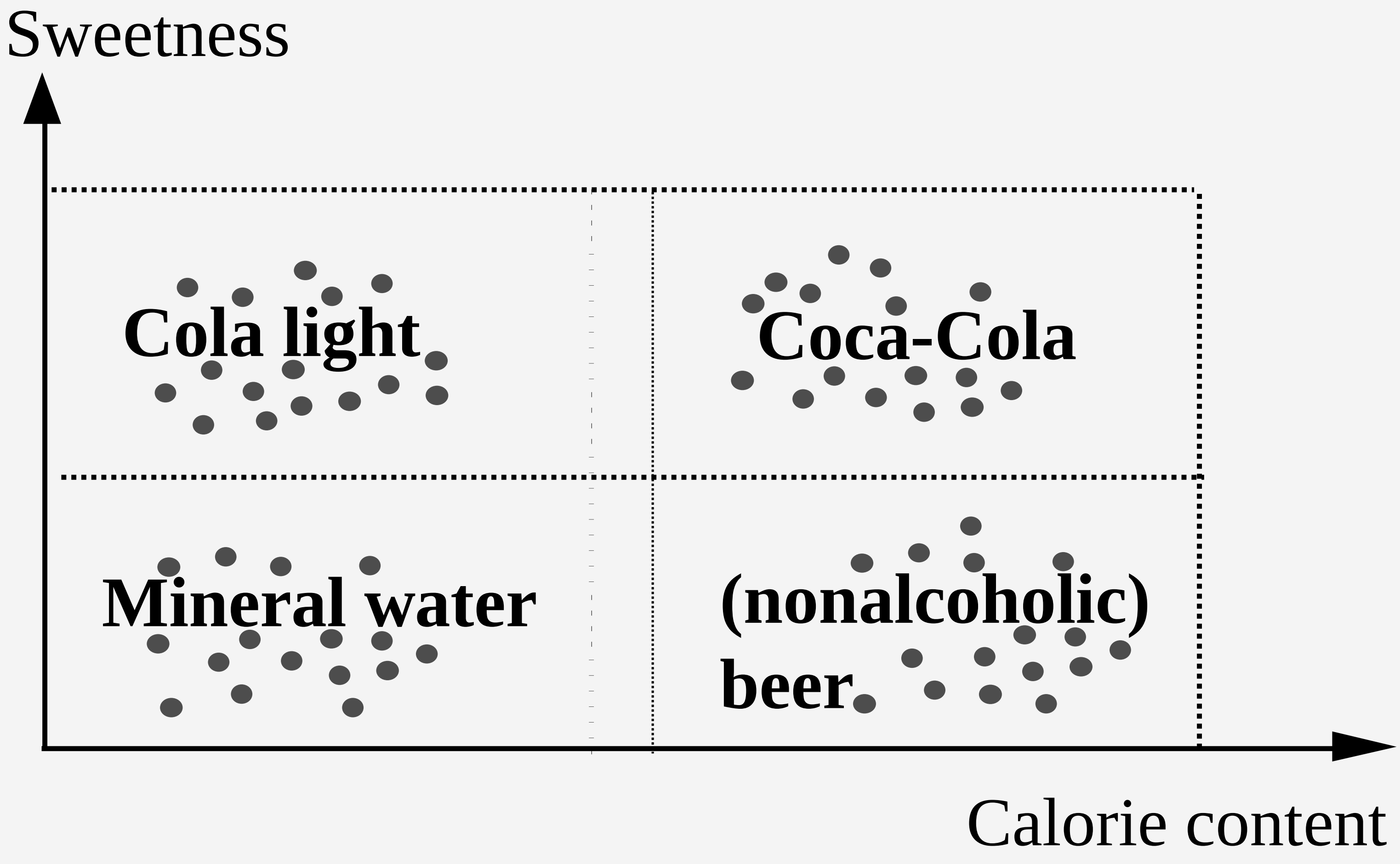
Diffuse preferences



Clustered preferences



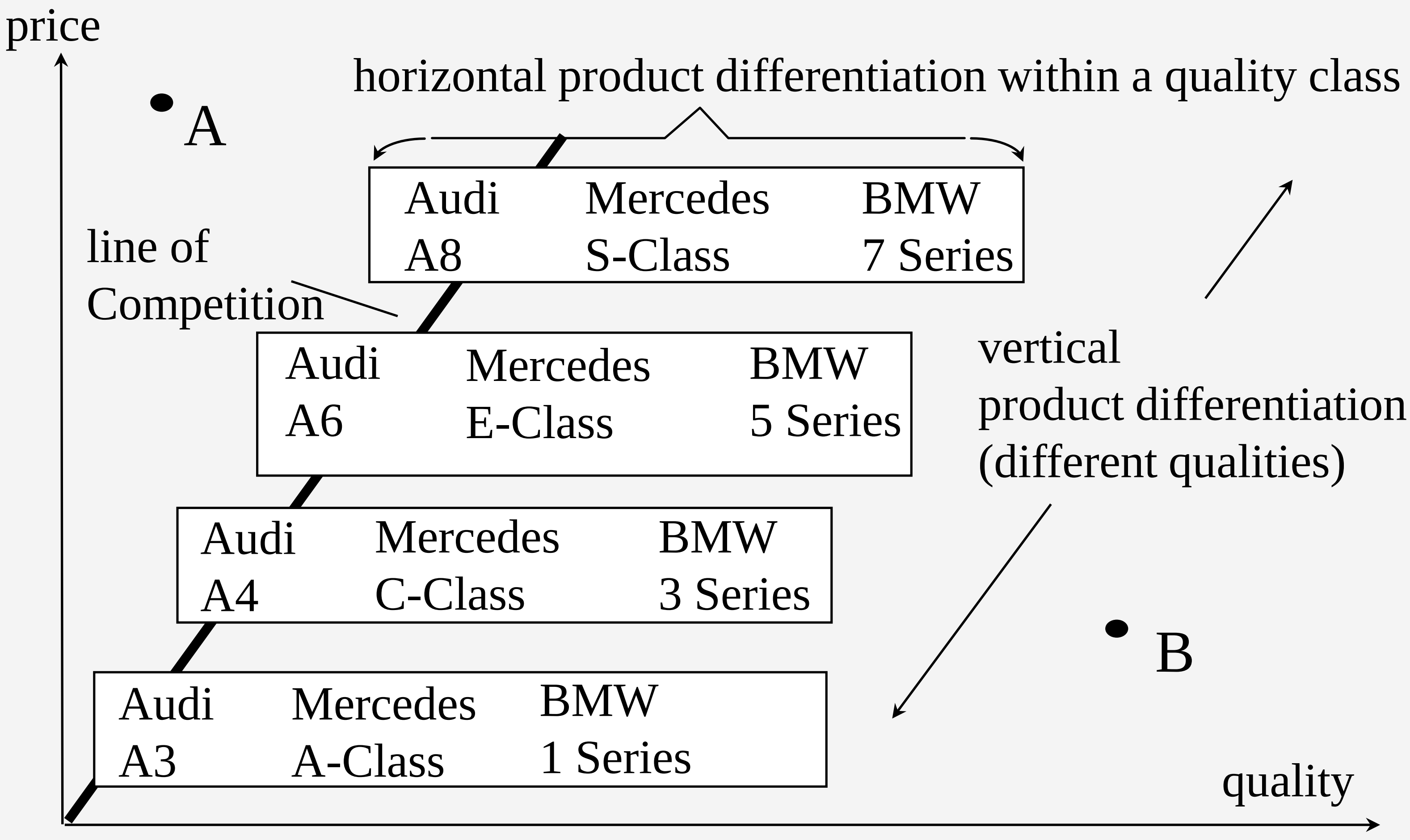
Example: product differentiation of drinks



Product differentiation

- Horizontal product differentiation:
  Some consumers prefer a good (or rather a feature), while others prefer a different good (or its feature).
- Vertical product differentiation (quality):
  A good is regarded as better than the other by all consumers (unanimous ranking).

Horizontal vs. vertical differentiation



Long-term and short-term action parameters

- Variants and locations (horizontal differentiation)
- Qualities (vertical differentiation)
- Recognition, image (image differentiation)
- Compatibility (compatibility differentiation)

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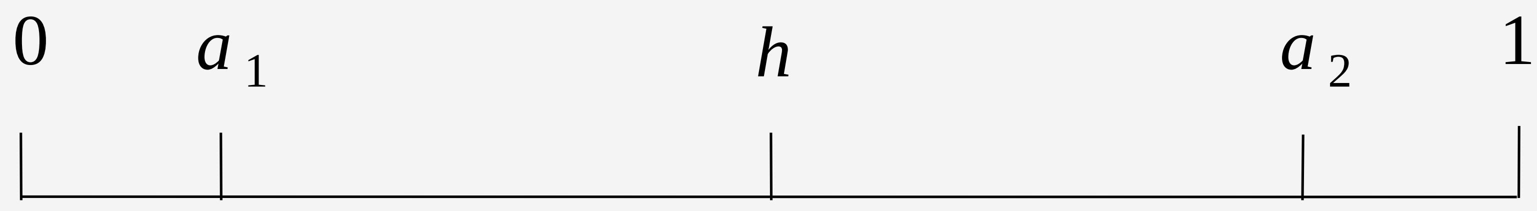
Prices
Quantities

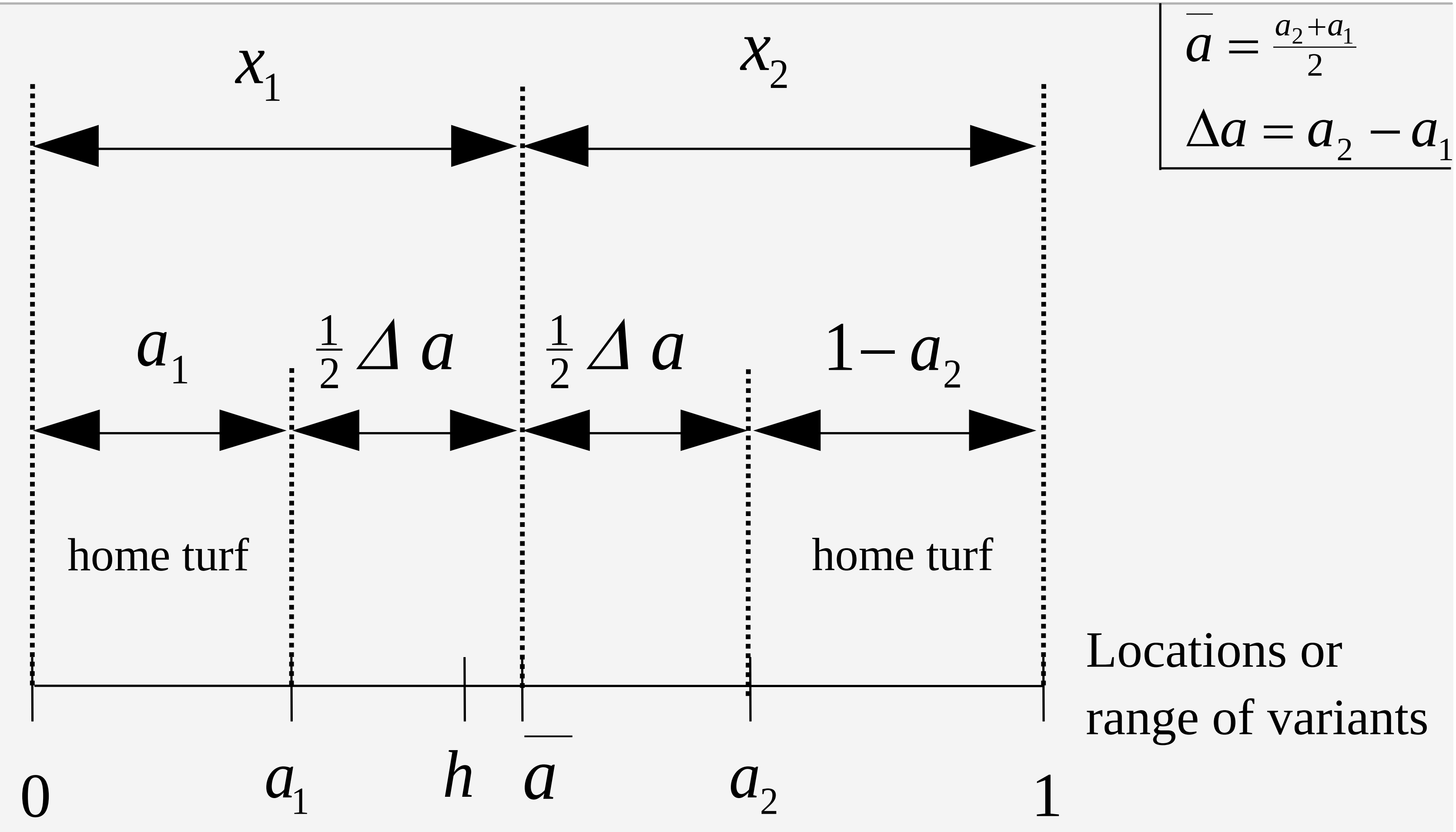
The Hotelling Model

- Linear city of length 1
- Interpretation
  - Competition on location: Two firms offer the same product in different places.
  - Competition on variants: Two firms offer differentiated products in one place.

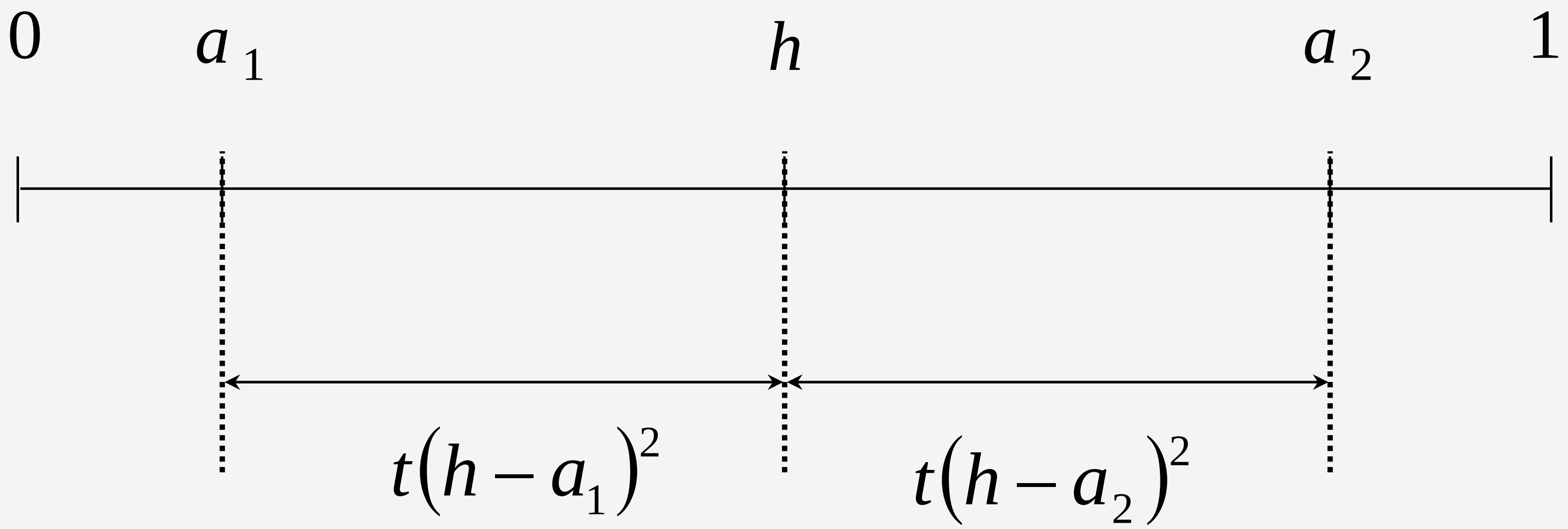


$$0 \le a_1 < a_2 \le 1$$

Demand in the case of identical prices



Costs of transport

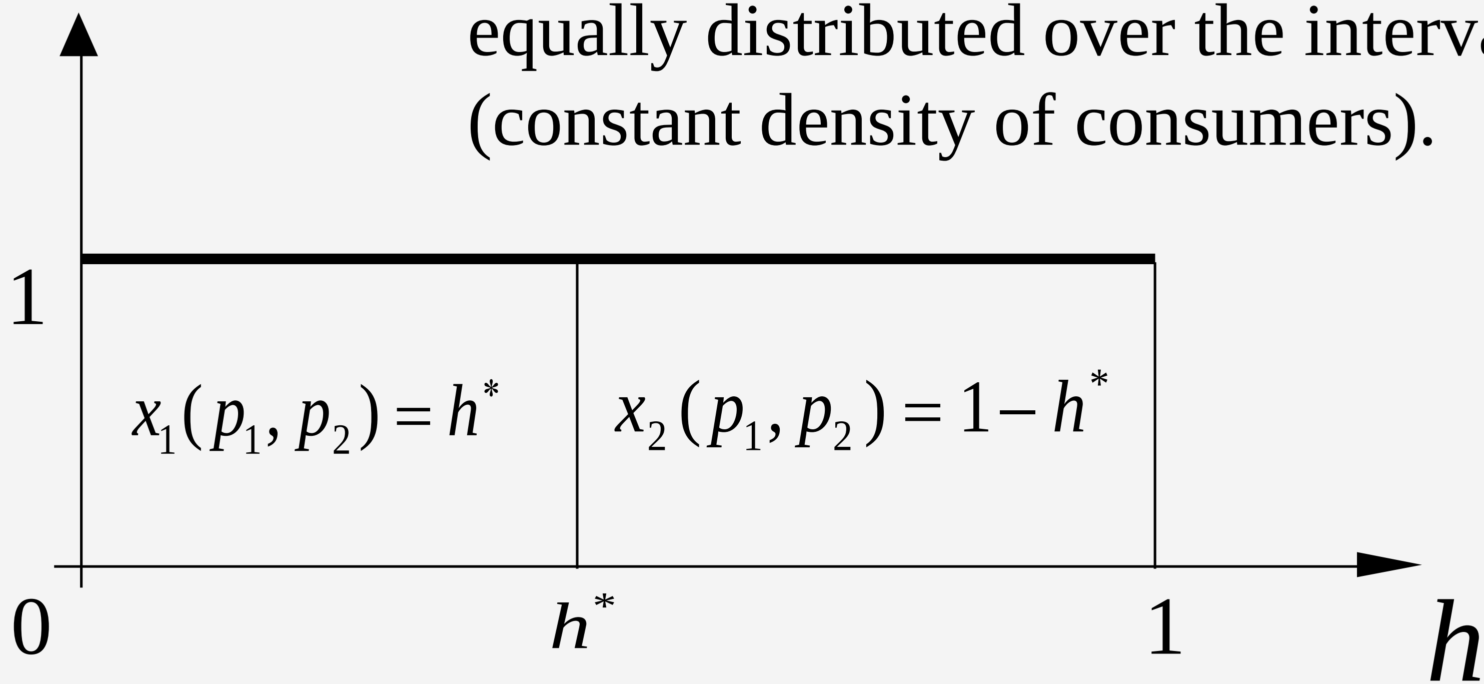


The consumer at *h* prefers producer 1's good if:

$$p_1^{eff} = p_1 + t(h-a_1)^2 \le p_2 + t(h-a_2)^2 = p_2^{eff}$$

Proportionate demand with uniform distribution

The consumers are supposed to be equally distributed over the interval (constant density of consumers).



The consumer in h^* is indifferent between good 1 and good 2.

The demand function

$\bar{a} = \frac{a_2 + a_1}{2}$

$\Delta a = a_2 - a_1$

- Firm 1's demand function:

$$p_1 + t(h - a_1)^2 \leq p_2 + t(h - a_2)^2$$

$$\Leftrightarrow h \leq \frac{a_2 + a_1}{2} + \frac{p_2 - p_1}{2t(a_2 - a_1)} =: h^*$$

$$\Rightarrow x_1(p_1, p_2, a_1, a_2) = h^* = \bar{a} + \frac{1}{2t\Delta a}(p_2 - p_1)$$

consumers in case of equal prices

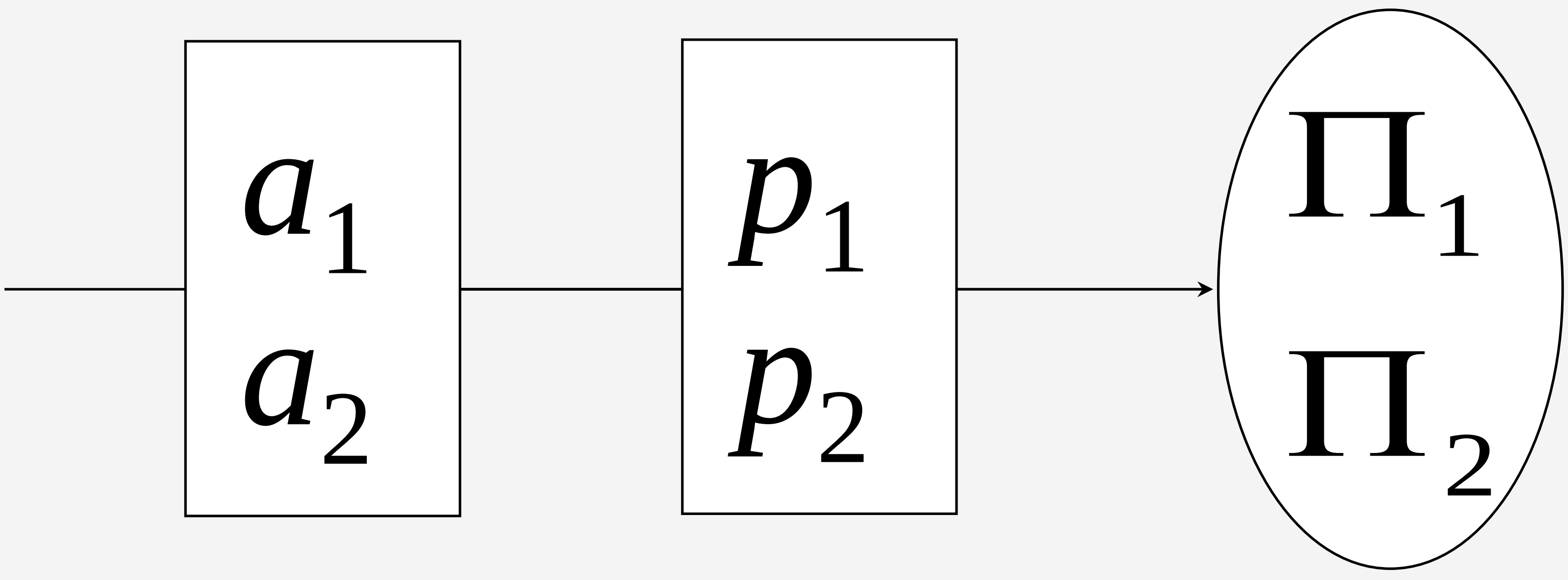
intensity of competition

firm 1's price advantage

Exercise (Hotelling)

- Deduce $x_2(p_1, p_2, a_1, a_2)$!
- Calculate all equilibria in the simultaneous location competition, if prices are given by $p_1 = p_2 > c$

The two-stage differentiation game



Solving the pricing game I

- Profit functions

$$\Pi_1(p_1, p_2) = (p_1 - c)\left(\bar{a} + \frac{p_2 - p_1}{2t\Delta a}\right)$$

$$\Pi_2(p_1, p_2) = (p_2 - c)\left(1 - \bar{a} + \frac{p_1 - p_2}{2t\Delta a}\right)$$

- Reaction functions

$$p_1^R(p_2) = \frac{p_2 + c + 2t\bar{a}\Delta a}{2}$$

$$p_2^R(p_1) = \frac{p_1 + c + 2t(1 - \bar{a})\Delta a}{2}$$

Solving the pricing game II

- Bertrand-Nash equilibrium

$$\left(p_1^B = c + \frac{2}{3} t\left(1 + \bar{a}\right)\Delta a,\; p_2^B = c + \frac{2}{3} t\left(2 - \bar{a}\right)\Delta a \right)$$

- Output levels

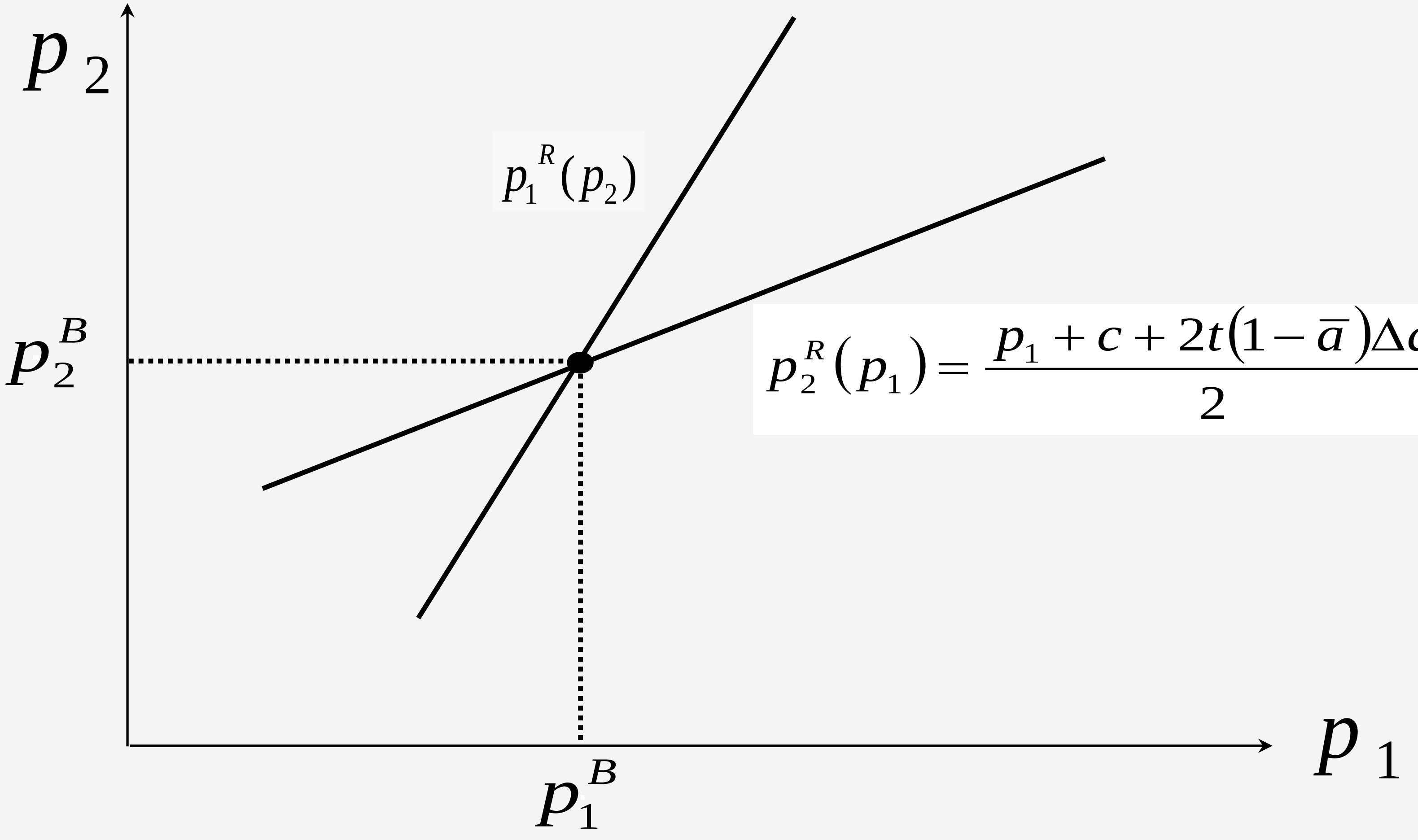
$$x_1^B = \frac{1}{3}(1 + \bar{a}) > 0 \quad and \quad x_2^B = \frac{1}{3}(2 - \bar{a}) > 0$$

- Profits

$$\Pi_1^B(a_1, a_2) = \frac{2}{9} t\left(1 + \bar{a}\right)^2 \Delta a > 0; \quad \Pi_2^B(a_1, a_2) = \frac{2}{9} t(2 - \bar{a})^2 \Delta a > 0$$

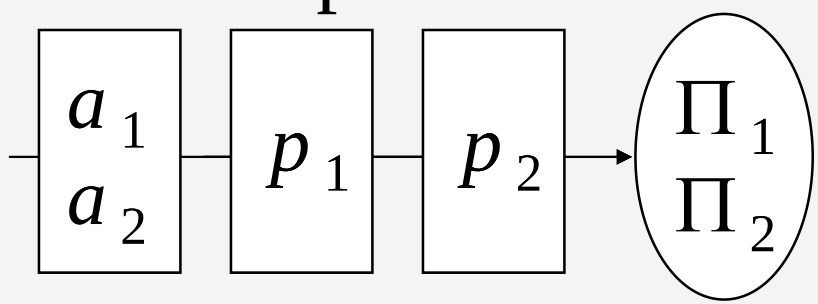
- When do the firms earn the same profits and why?

Equilibrium in the simultaneous competition

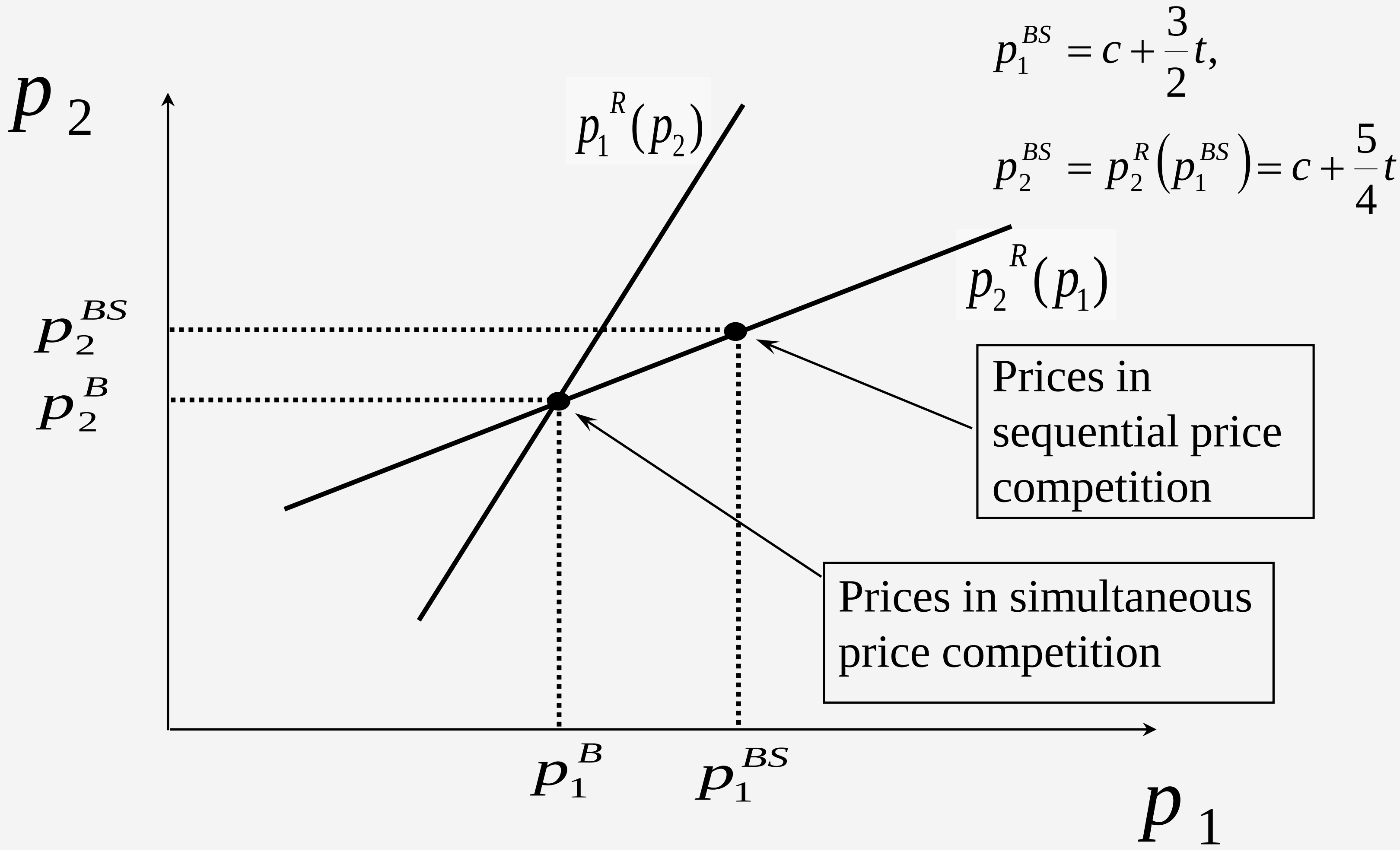


Exercises (elasticity, sequential price competition)

- Find the price elasticity of demand in the case of $p_1 = p_2 \quad and \quad \Delta a = 1$.
- Assume maximal differentiation $(a_1 = 0, a_2 = 1)$. Find the Bertrand equilibrium in the case of sequential price competition. Calculate the profits.



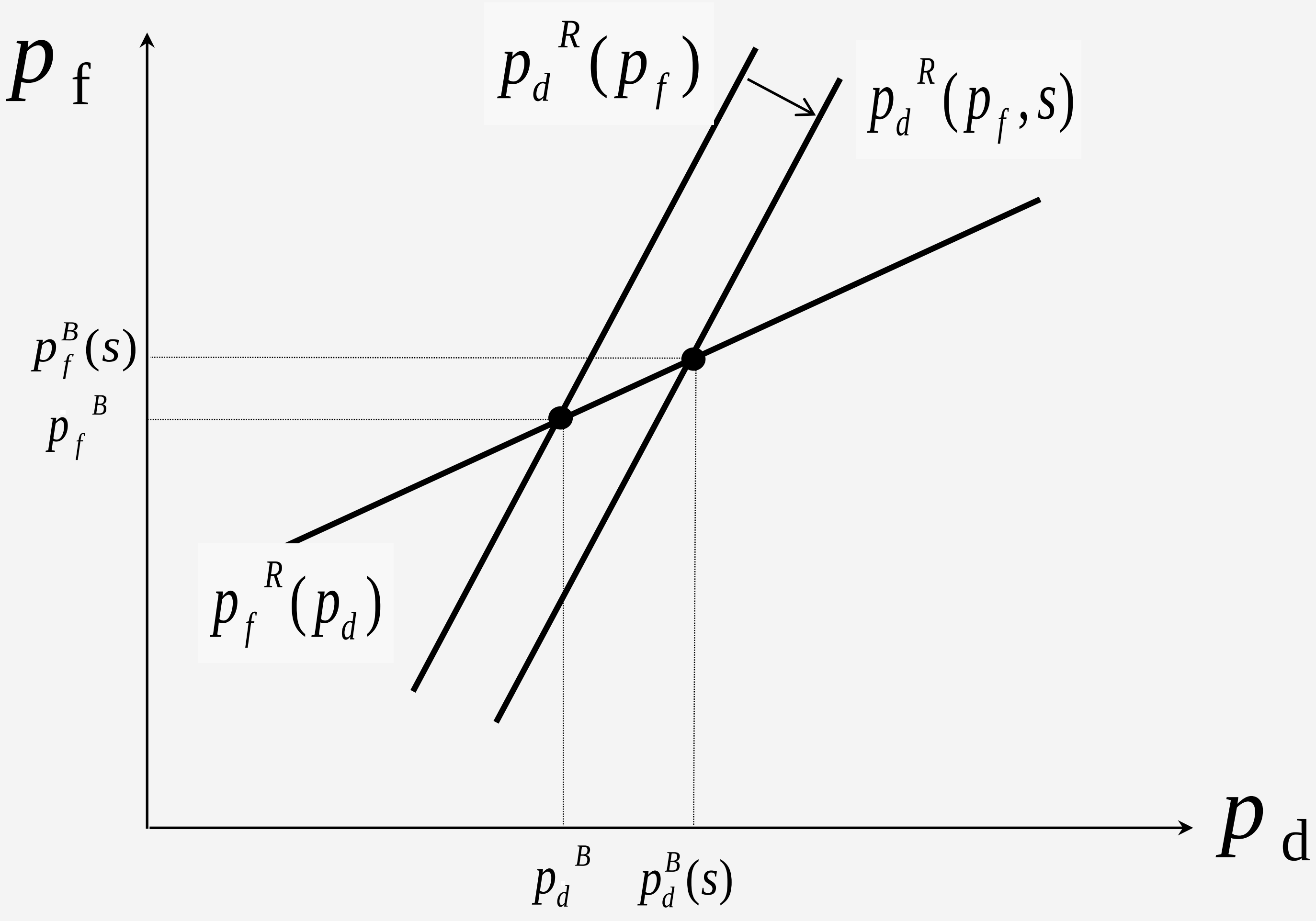
Depicting the equilibria



Exercise (Strategic trade policy)

- Two firms, one domestic (d), the other foreign (f), engage in simultaneous price competition on a market in a third country. Assume $\Delta a = 1$.
- The domestic government subsidizes its firm's exports using a unit subsidy s.
- Which subsidy s maximizes domestic welfare $W(s) = \Pi_d^B(c-s) - s x_d^B(c-s)$?

Depicting the solution



Exercise (linear costs of transport)

Find the demand functions and the Bertrand equilibrium in the case of $\Delta a = 1$ and linear cost of transport, i.e. $t(h-0)$ for buying x_1 and $t(1-h)$ for buying x_2.

Equilibrium locations (1st stage)

- Reduced profit functions:

$$\Pi_1^B(a_1,a_2)=\frac{2}{9}t\left(1+\overline{a}\right)^2\Delta a>0$$

$$\Pi_2^B(a_1,a_2)=\frac{2}{9}t\left(2-\overline{a}\right)^2\Delta a>0$$

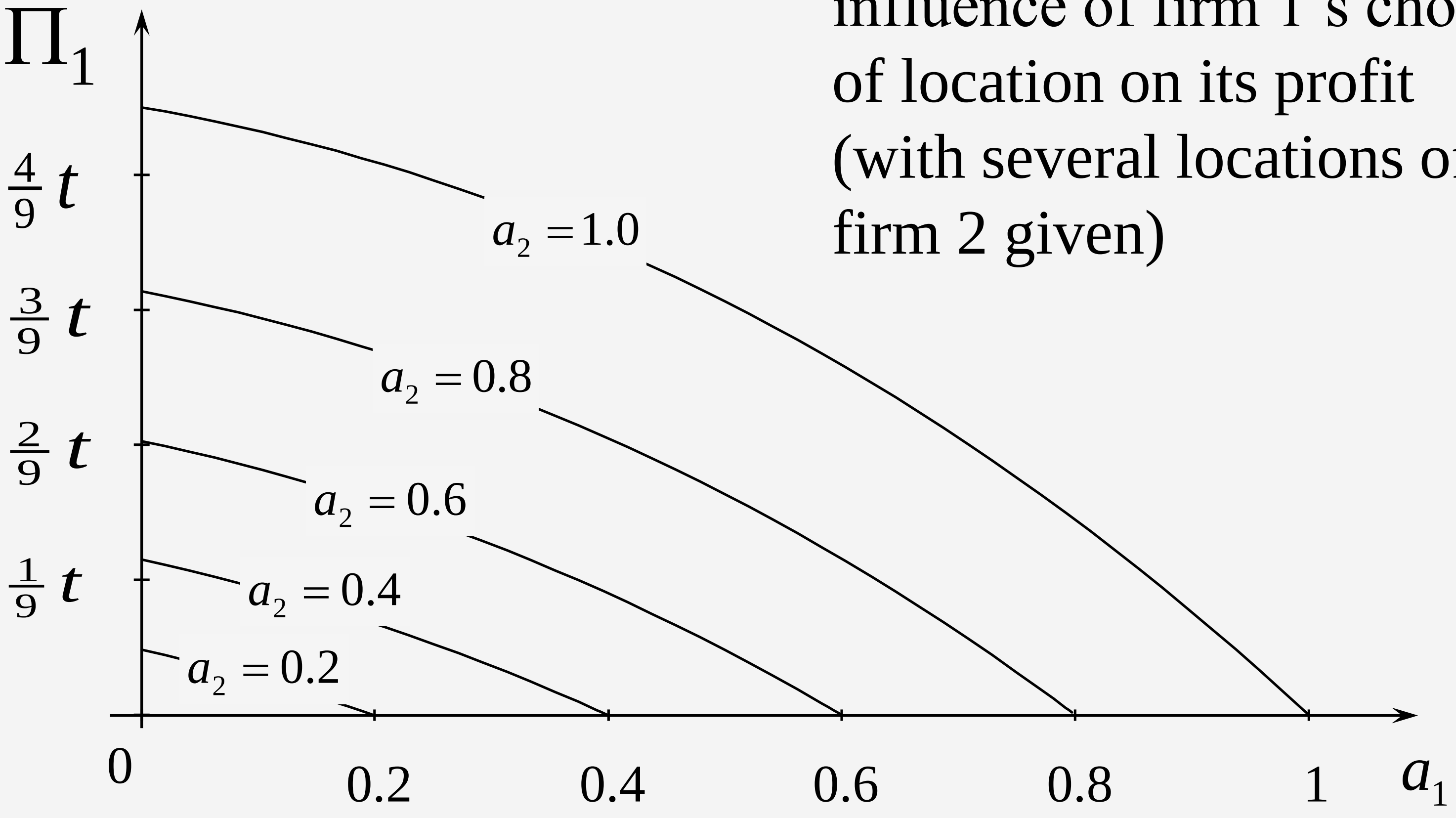
- Influence of location on profit functions:

$$a_1^R\left(a_2\right)=0\quad\left(\text{since}\quad\frac{\partial\Pi_1^B}{\partial a_1}<0\quad for\quad 0\le a_1<a_2\le 1\right)$$

$$a_2^R\left(a_1\right)=1\quad\left(\text{since}\quad\frac{\partial\Pi_2^B}{\partial a_2}>0\quad for\quad 0\le a_1<a_2\le 1\right)$$

- Nash equilibrium: $\left(a_1^N=0,a_2^N=1\right)$

Firm 1's reduced profit function (1st stage)



influence of firm 1's choice of location on its profit (with several locations of firm 2 given)

Equilibrium outcomes

- Maximal differentiation: $\Delta a = 1$
- Prices

$$p_1^B = p_2^B = c + t$$

- Output levels and profits

$$x_1^B = x_2^B = \frac{1}{2} \quad and \quad \Pi_1^B = \Pi_2^B = \frac{1}{2}t$$

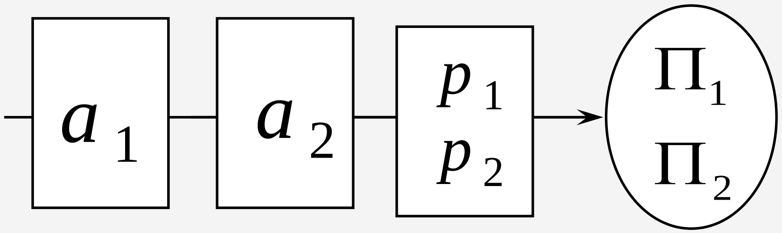
Lerner index (Hotelling)

- Lerner index for one firm
  = Lerner index for the industry (equal costs):

$$\frac{p - MC}{p} = \frac{c + t - c}{c + t} = \frac{t}{c + t} = \frac{1}{\frac{c}{t} + 1}$$

Exercises (sequential choice of location, clusters)

- Which locations would you expect in the case of sequential choice of location?



- Why do firms often form clusters in reality?

Direct and strategic effects for accommodation

- Firm 1's reduced profit function:

$$\Pi_1^B(a_1, a_2) = \Pi_1\left(a_1, a_2, p_1^B(a_1, a_2), p_2^B(a_1, a_2)\right)$$

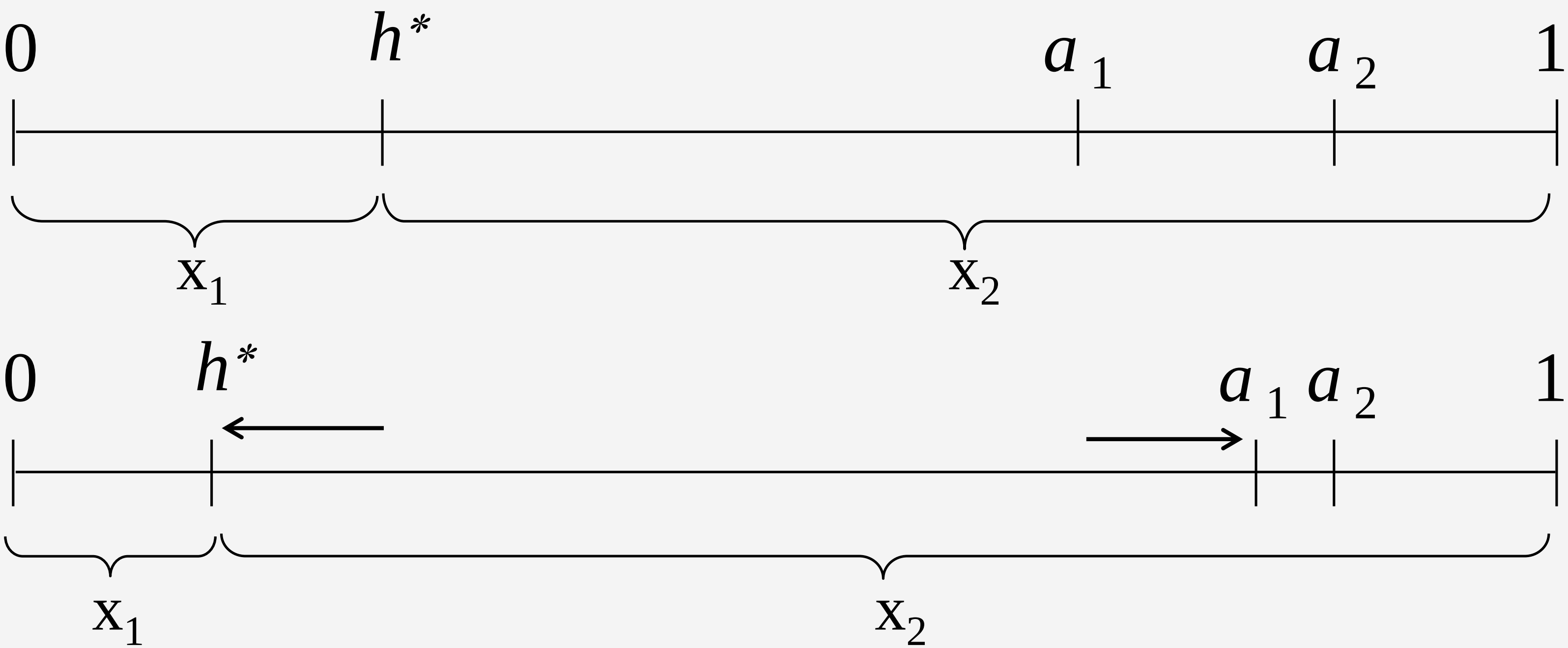
$$\frac{\partial \Pi_1^B}{\partial a_1} = \frac{\partial \Pi_1}{\partial a_1} + \frac{\partial \Pi_1}{\partial p_1} \cdot \frac{\partial p_1^B}{\partial a_1} + \frac{\partial \Pi_1}{\partial p_2} \cdot \frac{\partial p_2^B}{\partial a_1}$$

$\frac{\partial \Pi_1}{\partial a_1}$	$\frac{\partial \Pi_1}{\partial p_1}$	$\frac{\partial \Pi_1}{\partial p_2}$	$\frac{\partial p_2^B}{\partial a_1}$
?*	=0	>0	<0
direct or demand effect	profit maximizing prices in equilibrium of 2nd stage (Envelope theorem)	strategic effect of positioning	

*in most cases >0

Exception: negative direct effect

$p_2 << p_1$:



$$\Rightarrow \frac{\partial \Pi_1}{\partial a_1} < 0$$

Direct and strategic effects for deterrence

$$\Pi_2^B(a_1, a_2) = \Pi_2\left(a_1, a_2, p_2^B(a_1, a_2), p_1^B(a_1, a_2)\right)$$

$$\frac{\partial \Pi_2^B}{\partial a_1} = \frac{\partial \Pi_2}{\partial a_1} + \frac{\partial \Pi_2}{\partial p_2} \cdot \frac{\partial p_2^B}{\partial a_1} + \frac{\partial \Pi_2}{\partial p_1} \cdot \frac{\partial p_1^B}{\partial a_1}$$

?* ?* =0 >0 <0

direct effect

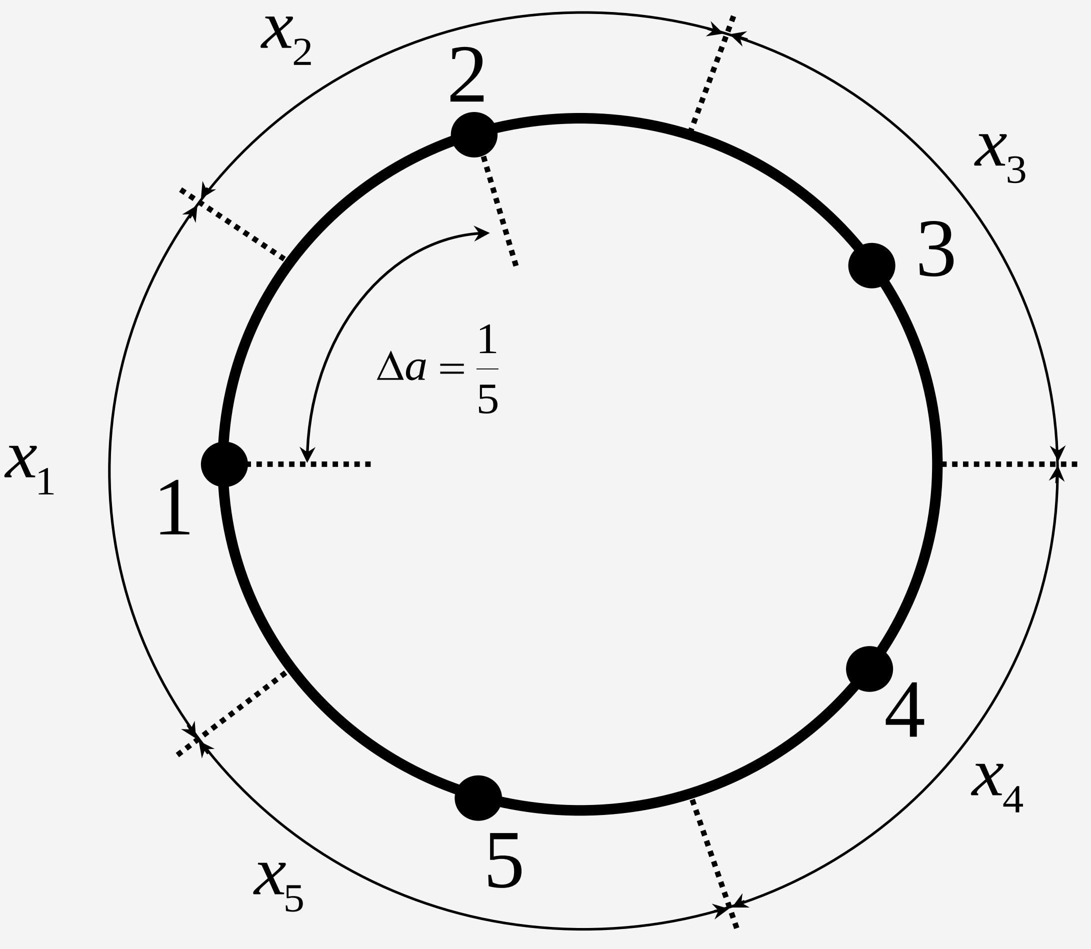
strategic effects

*in most cases <0

Welfare analysis

- Equilibrium locations are a_1=0 and a_2=1
- Total quantity is exogenous, at 1.
- ✹ costs of transport should be minimized.
- Locations a_1=0.25 and a_2=0.75 minimize transportation costs (too much differentiation).

The circular city



The Schmalensee-Salop model

- Model for the analysis of blockade, deterrence (limit number of variants)
- Circular city of length 1
- Firms are uniformly distributed
  - $\Delta a = a_{k+1} - a_k = \frac{1}{n}$
  - $\bar{a} = \frac{1}{2}\frac{1}{n}$
- The circular city can be considered to be made out of n linear cities.

The demand function I

- Indifferent consumer between firm 1 and 2

$$p_1 + t(h - a_1)^2 \leq p_2 + t(h - a_2)^2$$

$$\Leftrightarrow h \leq \frac{a_2 + a_1}{2} + \frac{p_2 - p_1}{2t(a_2 - a_1)} =: h^*_{1,2}$$

- $x_{2_{1,2}} = a_2 - h^*_{1,2} = \frac{p_1 - p_2}{2t(a_2 - a_1)} + \frac{a_2 - a_1}{2}$

$$= \frac{p_1 - p_2}{2t\frac{1}{n}} + \frac{1}{2}\frac{1}{n}$$

The demand function II

- Indifferent consumer between firm 2 and firm 3:

$$\frac{a_3 + a_2}{2} + \frac{p_3 - p_2}{2t(a_3 - a_2)} = h^*_{2,3}$$

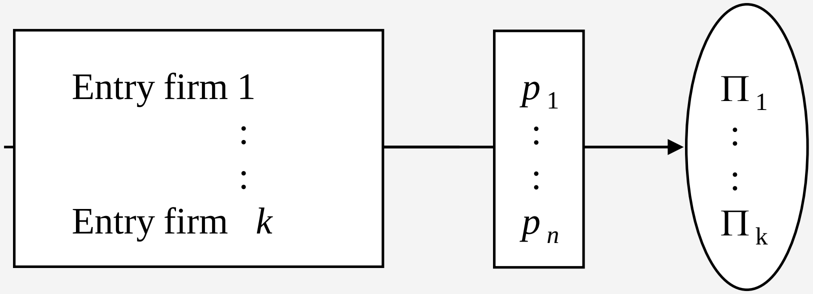
- $$x_{2_{2,3}} = h^*_{2,3} - a_2 = \frac{p_3 - p_2}{2t(a_3 - a_2)} + \frac{a_3 - a_2}{2} = \frac{p_3 - p_2}{2t\frac{1}{n}} + \frac{1}{2}\frac{1}{n}$$

- Firm 2's demand function:

$$x_2 = x_{2_{1,2}} + x_{2_{2,3}} = \frac{1}{n} + \frac{n}{2t}(p_1 + p_3 - 2p_2)$$

Entry and pricing decisions

- The firms decide whether to enter the market:
  - all firms simultaneously and equidistantly
  - potential competitors midway between two established firms.
- Firms incur location costs of C_F.
- Game structure (note $n \leq k$):



Solving the pricing game

- Firm 2's profit function:

$$\Pi_2 = (p_2 - c)\left(\frac{1}{n} + n\frac{1}{2t}(p_1 + p_3 - 2p_2)\right) - C_F$$

- Firm 2's reaction function:

$$p_2^R(p_1, p_3) = \frac{1}{4}(p_1 + p_3) + \frac{c}{2} + \frac{t}{2n^2}$$

- Symmetric Nash equilibrium:

$$\left(p_1^B = c + \frac{t}{n^2}, p_2^B = c + \frac{t}{n^2}, \ldots, p_n^B = c + \frac{t}{n^2}\right)$$

$$\Pi_i^B(n) = \frac{t}{n^3} - C_F$$

Equilibrium number of firms with free entry

- Profit function depending on number of firms:

$$\Pi_i^B(n) = \frac{t}{n^3} - C_F$$

- Entry:

$$\Pi_i^B(n) = \frac{t}{n^3} - C_F \geq 0 \Leftrightarrow n \leq \sqrt[3]{\frac{t}{C_F}} =: n_{\max}$$

$$p^B(n_{\max}) = c + \sqrt[3]{tC_F^2}$$

Lerner index (Schmalensee)

- Lerner index for one firm = Lerner index for the industry (equal costs):

$$\frac{p - MC}{p} = \frac{c + \frac{t}{n^2} - c}{c + \frac{t}{n^2}} = \frac{\frac{t}{n^2}}{c + \frac{t}{n^2}} = \frac{1}{\frac{n^2}{t}c + 1}$$

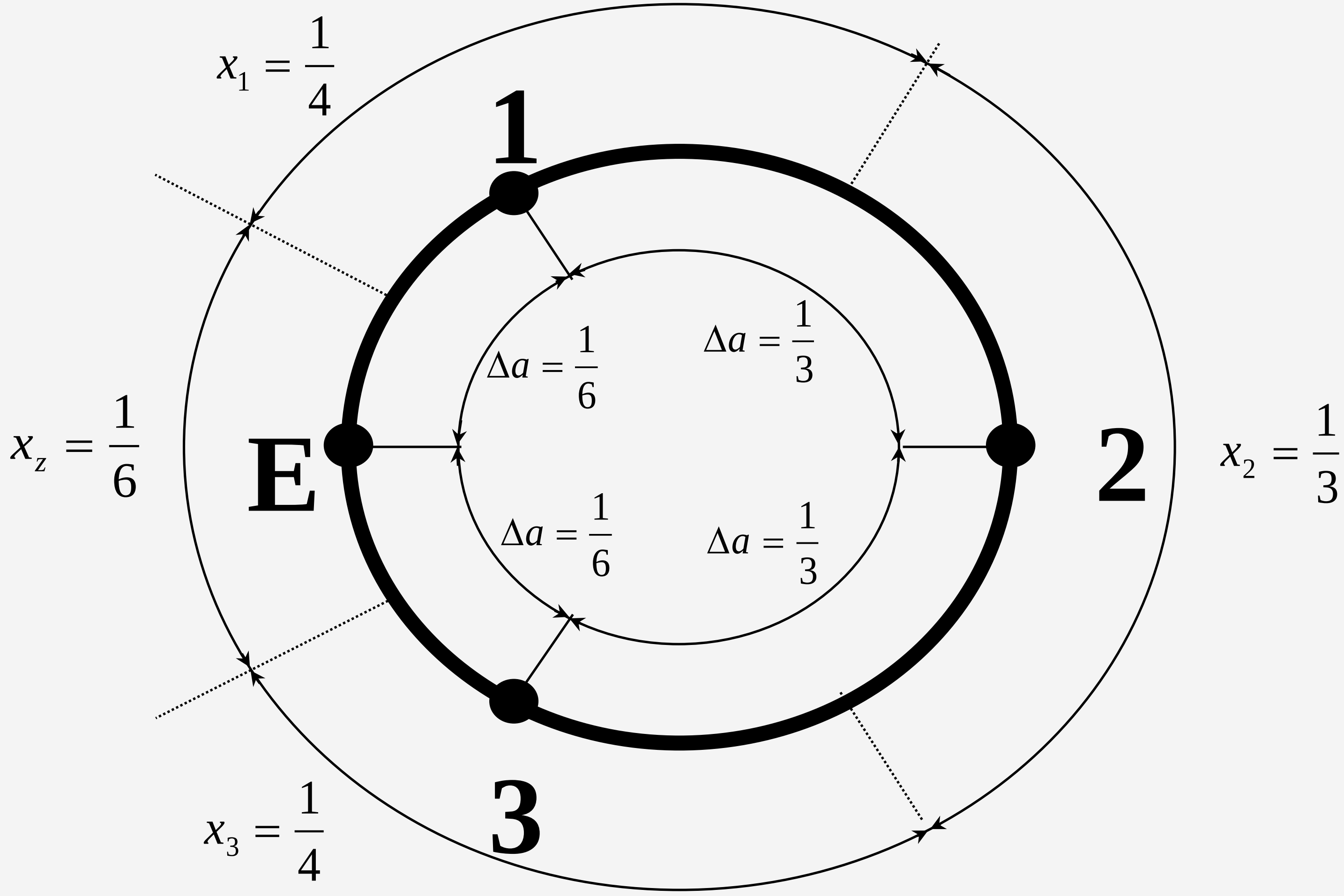
Market equilibrium

- While the price is above marginal costs, entry costs prevent firms from realizing profits.
- An equilibrium with zero profits prevails.
- The lower the costs of entry the higher the number of entering firms.
- The higher the costs of transport the higher the price and the number of entering firms.

Entry deterrence

- 1st stage:
  The established firms choose the number of variants/locations.
- 2nd stage:
  Potential competitors decide whether to enter the market.
- 3rd stage:
  All firms compete in prices.

Entry by a potential competitor



Product proliferation

- If there are n established firms, the potential entrant‘s profit expectation is determined by 2n.

$$\Pi_E^B(n) \approx \Pi_i^B(2n) = \frac{t}{(2n)^3} - C_F$$

- Limit variants or limit locations: $n^L := \frac{1}{2}\sqrt[3]{\frac{t}{C_F}}$

- The established firms are able to realize positive profits while deterring entry.

$$\frac{n_{\max}}{2} = n^L \leq n < n_{\max} = 2n^L$$

Linear costs of transport

- Consider linear cost of transport $C_i(h) = t\left|h - a_i\right|$ and keep all other assumptions of our models.
- Firm 2‘s demand functions (located between firms 1 and 3):

$$p_2^{eff}\left(h_{2,3}^*\right) = p_3^{eff}\left(h_{2,3}^*\right)$$

$$p_2 + t\left(h_{2,3}^* - a_2\right) = p_3 + t\left(a_3 - h_{2,3}^*\right)$$

$$h_{2,3}^* = \frac{a_2 + a_3}{2} + \frac{p_3 - p_2}{2t}$$

$$x_{2_{2,3}} = h_{2,3}^* - a_2 = \frac{1}{2n} + \frac{p_3 - p_2}{2t} \text{ analogous : } x_{2_{1,2}} = \frac{1}{2n} + \frac{p_1 - p_2}{2t}$$

$$\Rightarrow x_2 = x_{2_{2,3}} + x_{2_{1,2}} = \frac{1}{n} + \frac{p_1 + p_3 - 2p_2}{2t}$$

Exercise (linear costs of transport)

- Calculate the price reaction function for firm 2 and the symmetric Bertrand equilibrium ($p_1=p_2=...=p_n$).
- Find the maximal number of firms and the limit locations.

S.: $$p_2^R = \frac{1}{4}(p_1 + p_3) + \frac{c}{2} + \frac{t}{2n}$$

$$p^B = c + \frac{t}{n}$$

$$n^L = \frac{1}{2}\sqrt{\frac{t}{C_F}}$$

Executive summary I

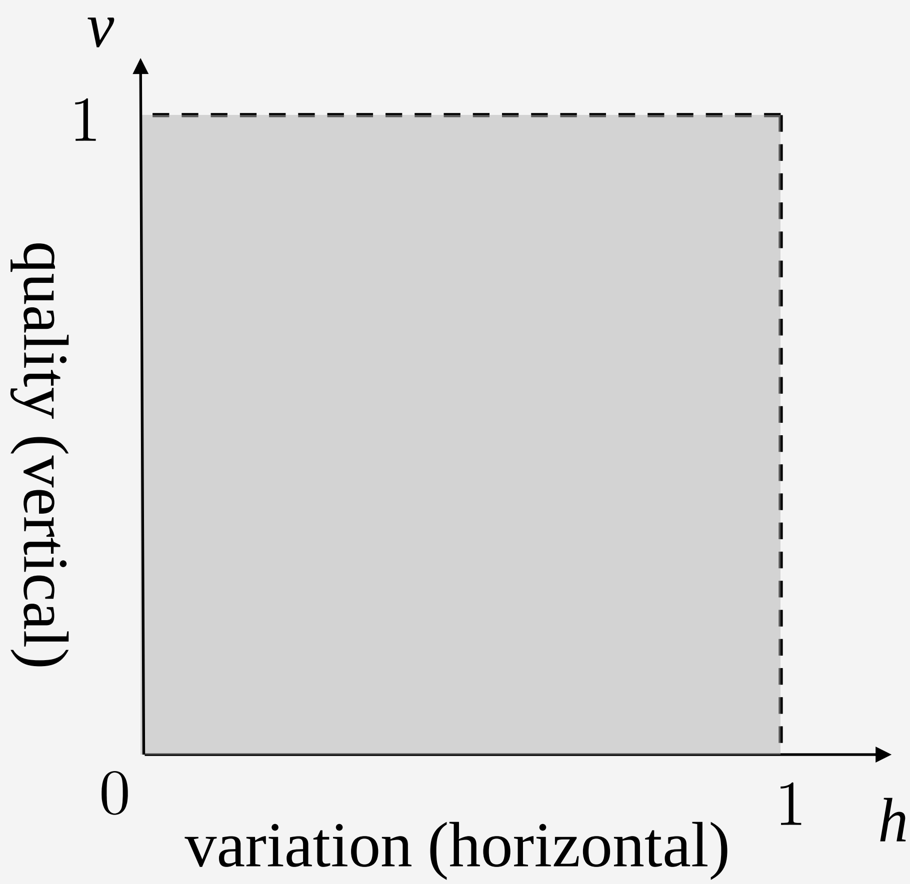
- Differentiation of products gives some monopolistic power to firms.
- Direct effect: If prices are fixed, “moving towards” the other firm pays in terms of sales and profits (direct effect). However: geographical nearness may enhance business (furniture shops clustered together).
- Strategic effect: Prices go down because of diminished differentiation (strategic effect).
- Both effects work towards deterrence.

Executive summary II

- The more firms are in the market, the lower prices, outputs and profits. Therefore, incumbent firms may try to drive competitors out of business and deter entry by product proliferation.
- From the social welfare point of view, competition on locations and variants need not lead to optimal product differentiation.

Competition on qualities and variants

- Maximal horizontal product differentiation: h position of consumer in horizontal product space.
- Quality differentiation, $0 \le q_2 < q_1 \le 1$: v consumers'
- willingness to pay for quality.



Competition on qualities and variations - demand function

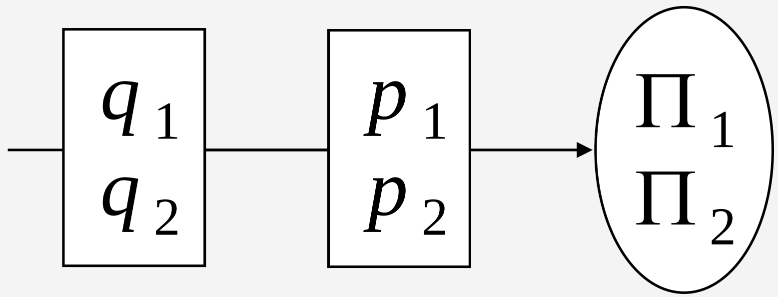
- Linear costs of transport: *t(h-0)* for firm 1 and *t(1-h)* for firm 2
- Consumer buys product 1 if

$$p_1 + th - vq_1 \le p_2 + t(1-h) - vq_2$$

$$h \le \frac{1}{2} + \frac{\Delta p + v\Delta q}{2t} \quad \text{where} \quad \Delta p = p_2 - p_1, \Delta q = q_1 - q_2$$

- Derivation of demand curve:

Competition on qualities and variations - results



- Low costs of transport
- ✹ maximum quality differentiation

$$q_1^N = 1 \qquad q_2^N = 0$$

$$p_1^B = c + \tfrac{2}{3} \qquad p_2^B = c + \tfrac{2}{3}$$

$$\Pi_1^B = \tfrac{4}{9} \qquad \Pi_2^B = \tfrac{1}{9}$$

- High costs of transport
- ✹ maximum (costless!) quality

$$q_1^N = 1 \qquad q_2^N = 1$$

$$p_1^B = c + t \qquad p_2^B = c + t$$

$$\Pi_1^B = \tfrac{1}{2} t \qquad \Pi_2^B = \tfrac{1}{2} t$$

Executive summary

- Horizontal product differentiation pays.
- If horizontal product differentiation is expensive or difficult, vertical product differentiation may also help to avoid the Bertrand paradox.
- If horizontal product differentiation is possible, firms will choose the maximal quality in case of costless quality.