Course outline I

- Introduction
- Game theory
- Price setting
  - monopoly
  - oligopoly
- Quantity setting
  - monopoly
  - oligopoly
- Process innovation

Homogeneous goods

Innovation competition

- Product versus process innovation
- Drastic versus non-drastic innovation
- Patent race
- Incentives to innovate
- Executive summary

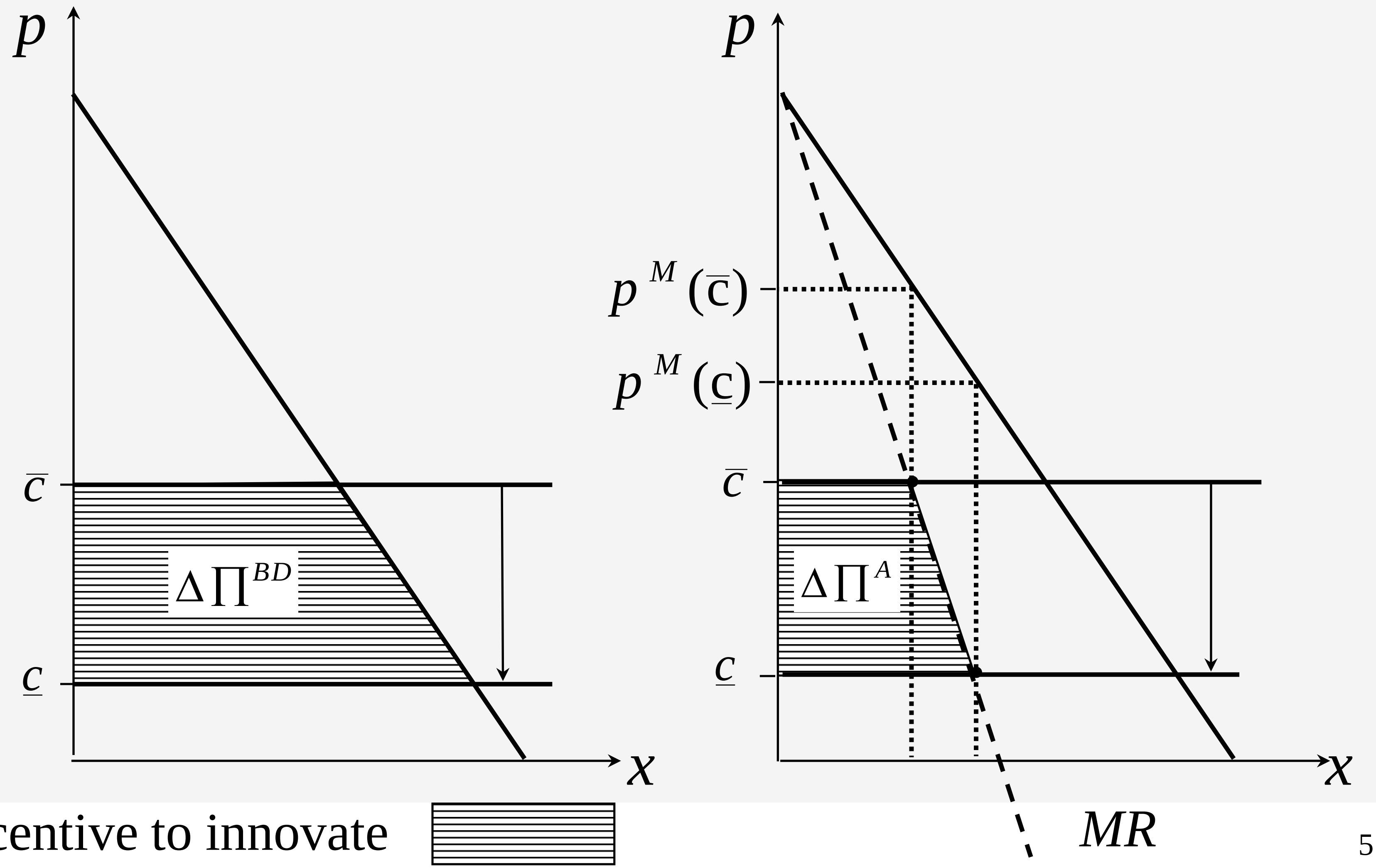
Research and development

- Three levels of research:
  - fundamental research
  - applied research with project planning
  - development of new products and their commercialization
- Innovation of product and process
- Three stages
  - invention
  - adoption
  - diffusion

Five models

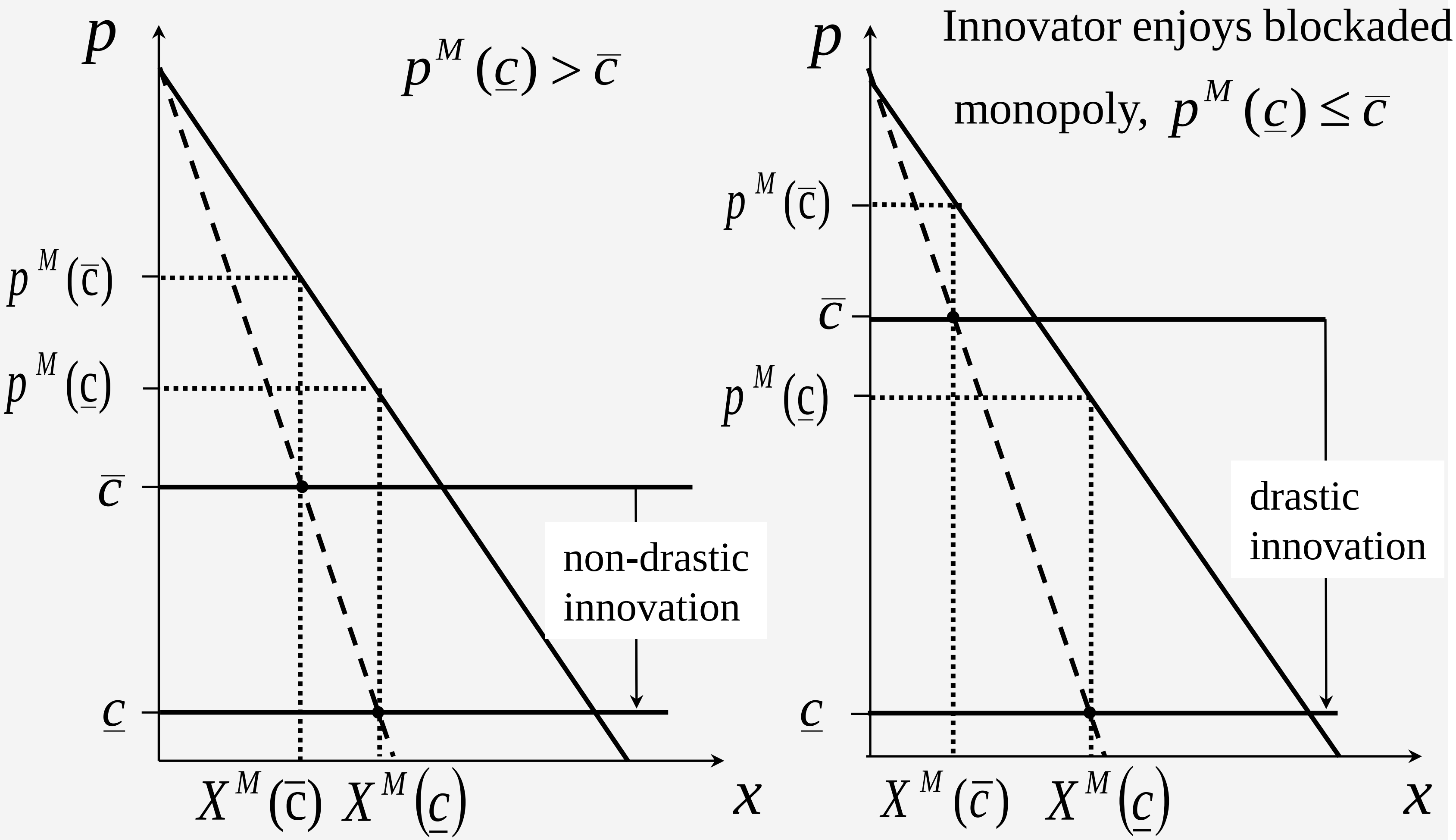
- Patent race with respect to process innovation
- Process innovation leads to reduction of average cost: $\underline{c} < \overline{c}$
- Incentives to innovate for
  - benevolent dictator
  - monopolist
  - perfect competition
  - two symmetric firms
  - two asymmetric firms

Benevolent dictator v. monopolist



incentive to innovate

Drastic v. non-drastic innovation



Exercise (drastic or non-drastic innovation)

- Inverse-demand function $p=a-X$
- All firms have identical unit costs $\bar{c}$, where
  $\bar{c} < a < 2\bar{c}$
- Only one firm reduces its unit cost to
  $\underline{c} = 2\bar{c} - a$

Infer kind of innovation

Perfect competition

- Incentives to innovate
  - for drastic innovation

$$\Delta\Pi^{PC,drastic} = \Pi^{M}(\underline{c}) - 0 > \Pi^{M}(\underline{c}) - \Pi^{M}(\overline{c}) = \Delta\Pi^{A}$$

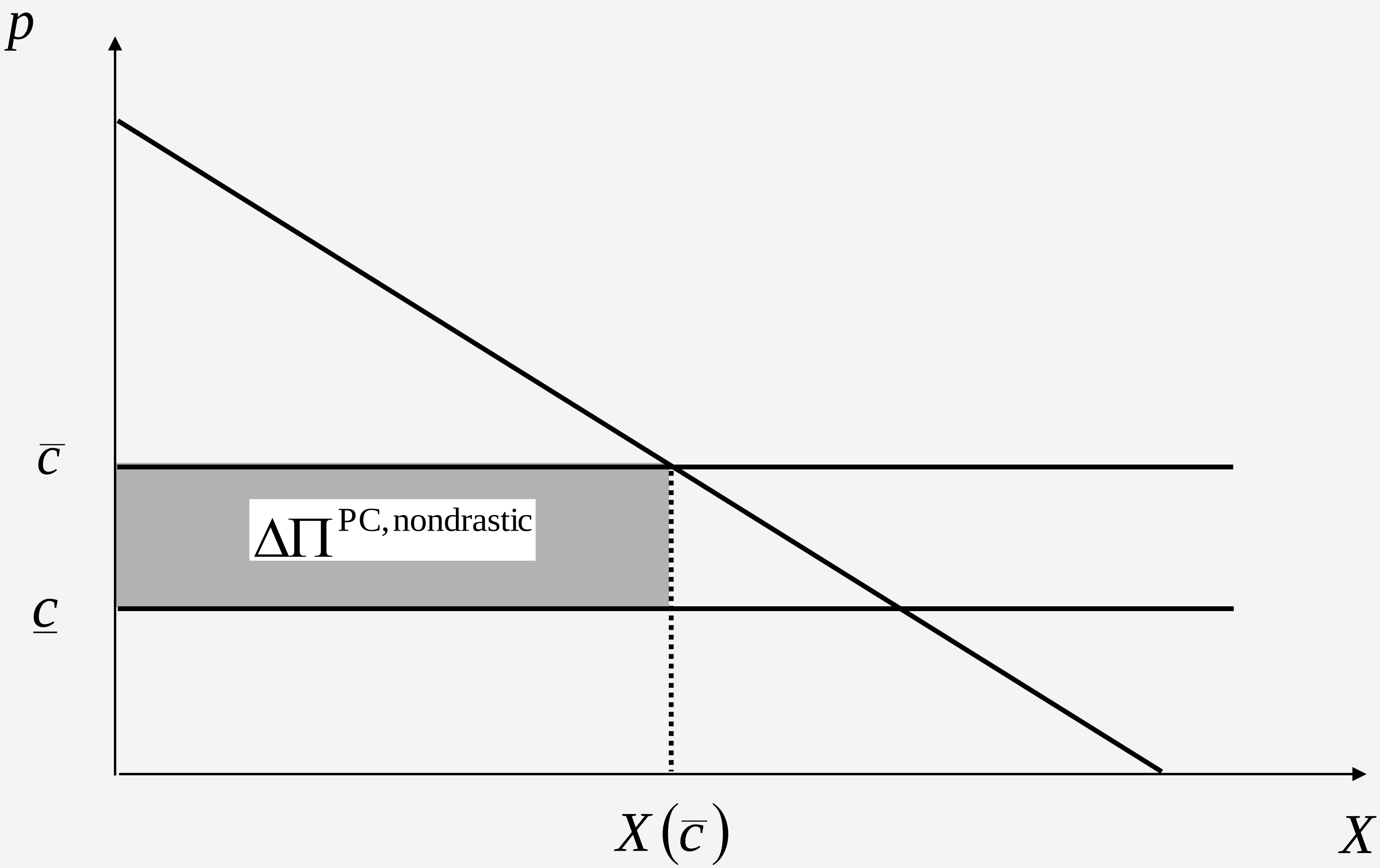
  - for non-drastic innovation

$$\Delta\Pi^{PC,nondrastic} = (\overline{c} - \varepsilon - \underline{c})D(\overline{c} - \varepsilon) = \int_{\underline{c}}^{\overline{c}} D(\overline{c})dc$$

$$\Delta\Pi^{A} < \Delta\Pi^{PC,nondrastic} \leq \Delta\Pi^{BD}$$

(compare next slide and slide „ Benevolent dictator v. monopolist”)

$\Delta\Pi^{PC,nondrastic}$ graphically



Assumptions and notation for dyopolistic innovation competition

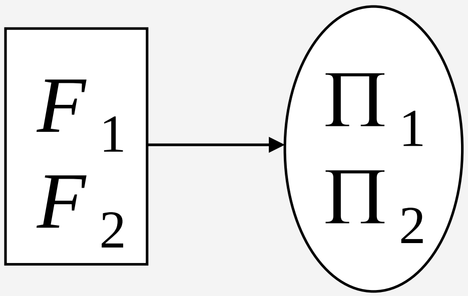
- Patent race with respect to process innovation
- R&D activity of firms 1 and 2: F_1 and F_2 with costs $C(F_i) = F_i$
- A measure of innovation difficulty : F_0
- Innovation probability of firm *i*: $w_i = \dfrac{F_i}{F_0 + F_1 + F_2}$
- Probability of no innovation: $\overline{w} = \dfrac{F_0}{F_0 + F_1 + F_2}$

Exercise (innovation probability)

How does the innovation probability w_1 depend on F_0 , F_1 and F_2.

F_1	F_2	Innovation probability of firm 1
½ F_0	½ F_0	
$6F_0$	F_0	
$34F_0$	F_0	
$100F_0$	F_0	
F_0	0	

Symmetric innovation competition



Initially, none of the firms is in the market. The successful firm enters the market and receives Π^M.

Equilibrium (symmetric case)

- Profit function of firm 1

$$\Pi_1(F_1, F_2) = \frac{F_1}{F_1 + F_2 + F_0}\Pi^M - F_1$$

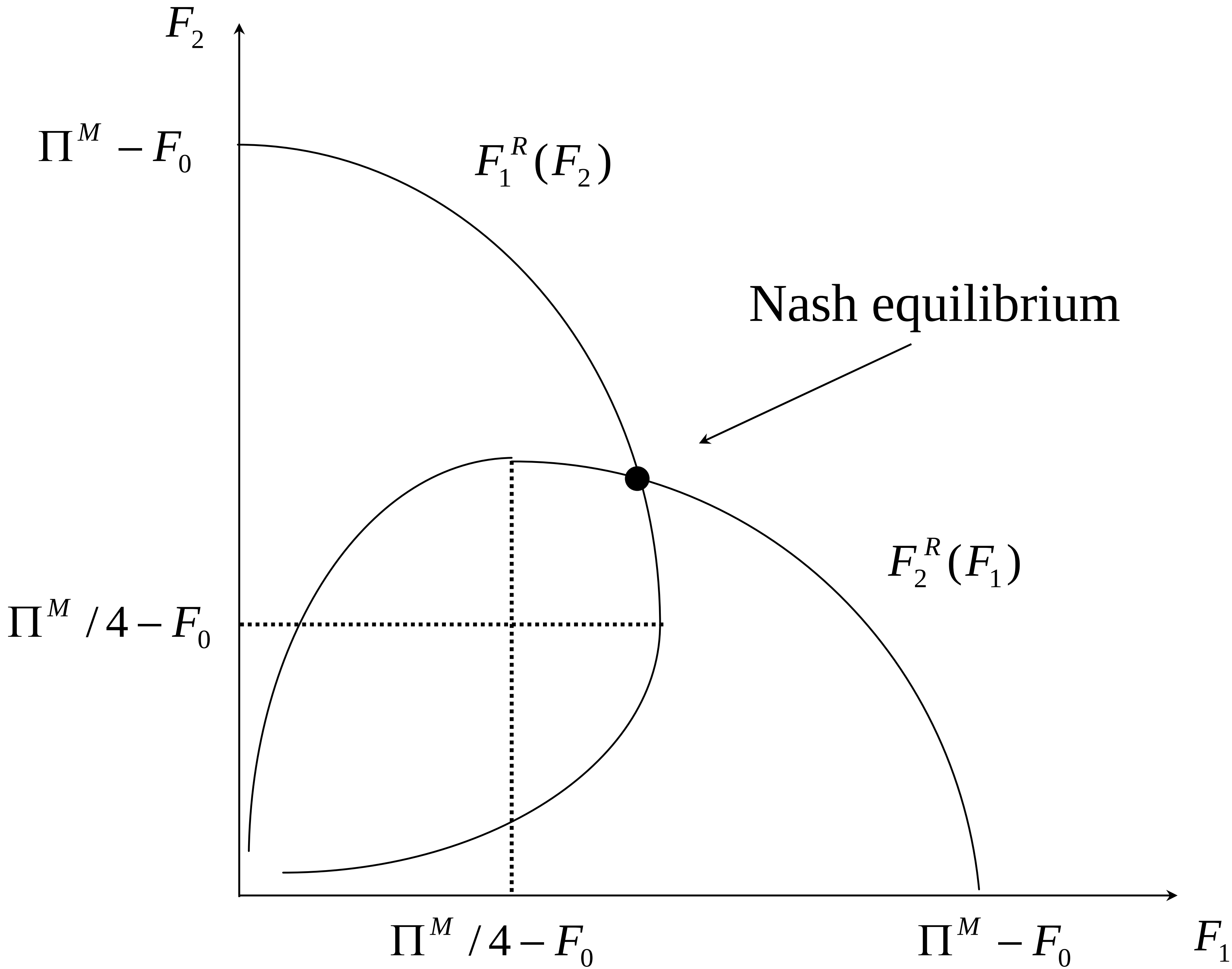
- Reaction function of firm 1

$$F_1^R(F_2) = \sqrt{\Pi^M(F_2 + F_0)} - (F_2 + F_0)$$

- Nash equilibrium

$$(F_1^N, F_2^N), \quad F_1^N = F_2^N = -\frac{1}{2}F_0 + \frac{1}{2}\left(\frac{1}{4}\Pi^M + \frac{1}{4}\sqrt{\Pi^M(\Pi^M + 8F_0)}\right)$$

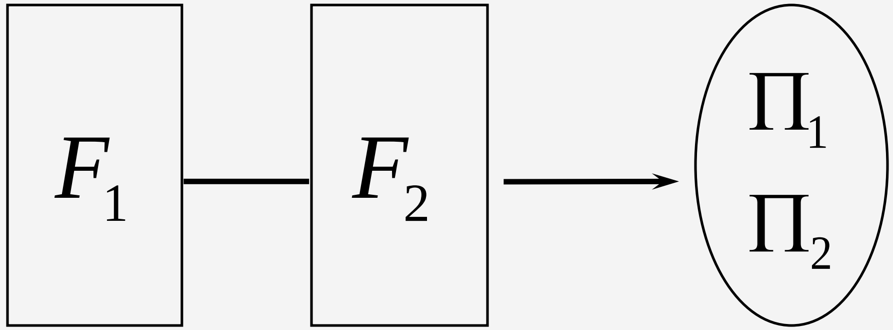
$$\left(\frac{\partial F_1^N}{\partial F_0} = \frac{\partial F_2^N}{\partial F_0} < 0 \qquad with \qquad \frac{\partial F_1^N}{\partial \Pi^M} = \frac{\partial F_2^N}{\partial \Pi^M} > 0\right)$$



F_2
$\Pi^M - F_0$
$F_1^R(F_2)$
Nash equilibrium
$F_2^R(F_1)$
$\Pi^M / 4 - F_0$
$\Pi^M / 4 - F_0$
$\Pi^M - F_0$
F_1

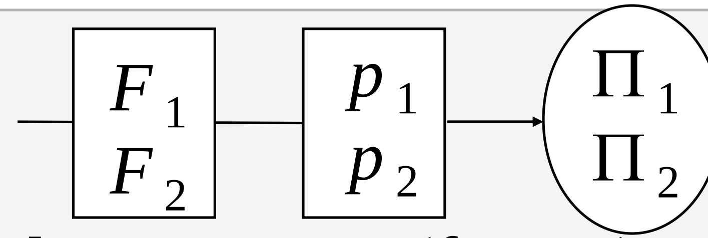
Exercise (sequential symmetric case)

Consider the symmetric case with $F_0=0$. Calculate an equilibrium in the sequential game:



S.: $\left(\frac{1}{4}\Pi^M, F_2^R\right)$

Asymmetric case: one incumbent, one potential competitor



- Monopolist (firm 1) with average cost $\bar{c}$, and potential competitor (firm 2)
- Process innovations leads to reduction of average cost: $\underline{c} < \bar{c}$
- Profits, net of R&D expenditure
  - No firm innovates $\Pi_1 = \Pi^M(\bar{c}), \quad \Pi_2 = 0$
  - Established firm innovates $\Pi_1 = \Pi^M(\underline{c}), \quad \Pi_2 = 0$
  - Entrant innovates (price competition)

$$\Pi_1 = \Pi_1^d = 0 \qquad \textit{and} \qquad \Pi_2 = \Pi_2^d = \begin{cases} \Pi_2^a, & \text{non-drastic innovation} \\ \Pi_2^b, & \text{drastic innovation} \end{cases}$$

Profit functions (asymmetric case)

- Profit function of firm 1 (monopolist)

$$\Pi_1(F_1, F_2) = \frac{F_1}{F_1 + F_2 + F_0}\Pi^M(\underline{c}) + \frac{F_2}{F_1 + F_2 + F_0}\Pi_1^d + \\ + \frac{F_0}{F_1 + F_2 + F_0}\Pi^M(\overline{c}) - F_1$$

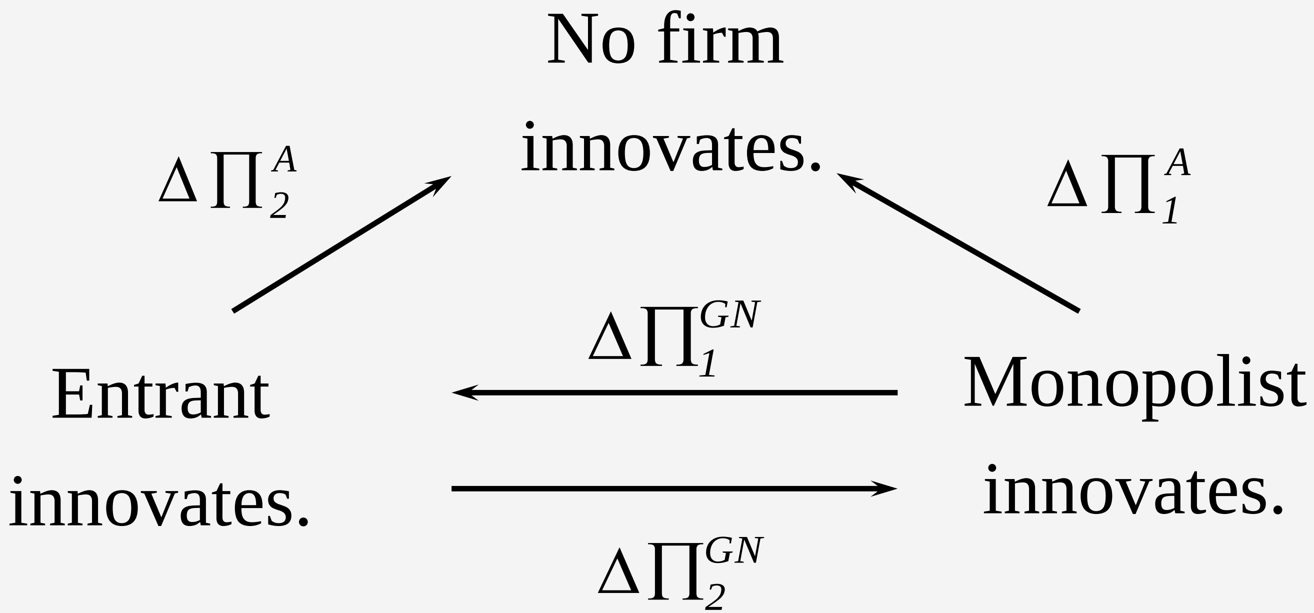
- Profit function of firm 2

$$\Pi_2(F_1, F_2) = \frac{F_2}{F_1 + F_2 + F_0}\Pi_2^d - F_2$$

Incentives to innovate

1) innovation vs. no innovation $\triangleq \Delta\prod^{A}$

2) innovation vs. innovation $\triangleq \Delta\prod^{GN}$

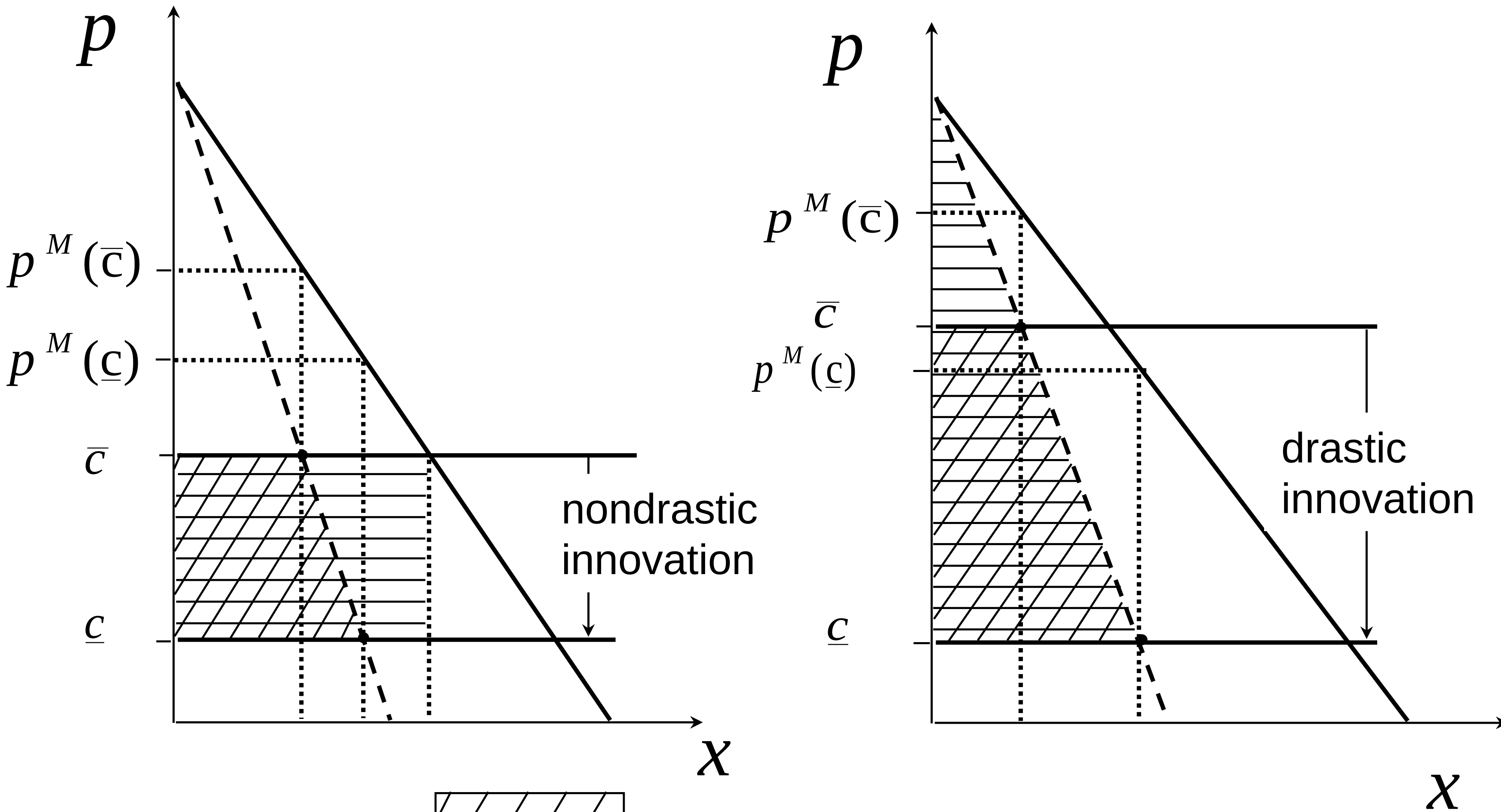


Replacement effect (Arrow)

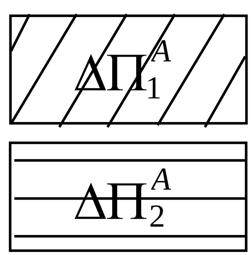
- The Arrow terms are defined as the profit differences a firm enjoys by innovating rather than not innovating.
- If the incumbent innovates, he replaces himself. If the entrant innovates, he achieves positive profits as compared to zero profits:

$$\Delta\Pi_1^A = \Pi^M(\underline{c}) - \Pi^M(\overline{c}) \leq \Pi_2^d - 0 = \Delta\Pi_2^A$$

$$\Delta\Pi_2^d = \begin{cases} \Pi_2^a \approx (\overline{c} - \underline{c}) \cdot X(\overline{c}), & \text{non-drastic innovation} \\ \Pi_2^b = \Pi^M(\underline{c}), & \text{drastic innovation} \end{cases}$$



Incentive to innovate from replacement effect



for established firm

for potential competitor

Efficiency effect (Gilbert, Newbery)

- The Gilbert-Newbery terms are defined as the profit differences a firm enjoys if she herself rather than her competitor innovates.
- The established firm's incentive to remain a monopolist is greater then the entrant's incentive to become a duopolist:

$$\Delta\Pi_1^{GN} = \Pi^M(\underline{c}) - \Pi_1^d \geq \Pi_2^d - 0 = \Delta\Pi_2^{GN}$$

Gilbert-Newbery effect: $\Pi_1^d + \Pi_2^d \leq \Pi^M(\underline{c})$

From previous slide follows: $\Pi_1^d + \Pi_2^d \leq \Pi^M(\underline{c}) \quad (1)$

Drastic innovation firm 2

$\Pi_1^d = 0$ and $\Pi_2^d = \Pi^M(\underline{c})$

blockade and ”=” in equation (1)

Nondrastic innovation firm 2

$\Pi_1^d = 0$ and $\Pi_2^d = \Pi_2^a \approx (\bar{c} - \underline{c}) \cdot X(\bar{c}) < \Pi^M(\underline{c})$

deterrence and ”<” in equation (1)

→ equation on previous slide is true

Replacement versus efficiency effect

- Replacement effect
  - entrant has a greater incentive to innovate

$$\Delta\Pi_1^A \leq \Delta\Pi_2^A$$

- Efficiency effect
  - established firm has a greater incentive to innovate

$$\Delta\Pi_1^{GN} \geq \Delta\Pi_2^{GN}$$

Equilibrium (asymmetric case)

- Reaction function of firm 1

$$F_1^R(F_2) = -\left(F_2 + F_0\right) + \sqrt{F_0\left(\Pi^M(\underline{c}) - \Pi^M(\overline{c})\right) + F_2\left(\Pi^M(\underline{c}) - \Pi_1^d\right)}$$

$$= -\left(F_2 + F_0\right) + \sqrt{F_0 \Delta\Pi_1^A + F_2 \Delta\Pi_1^{GN}}$$

- Reaction function of firm 2

$$F_2^R(F_1) = -\left(F_1 + F_0\right) + \sqrt{\left(F_0 + F_1\right)\Pi_2^d}$$

$$= -\left(F_1 + F_0\right) + \sqrt{F_0 \Delta\Pi_2^A + F_1 \Delta\Pi_2^{GN}}$$

- Nash equilibrium: “forget it“

Identifying the replacement effect

- The greater the monopoly's profit without innovation, the less are the monopolist's incentives to innovate:

$$F_1^R(F_2) = -(F_2 + F_0) + \sqrt{F_0 \Delta\Pi_1^A + F_2 \Delta\Pi_1^{GN}}$$

$$\frac{\partial F_1^R}{\partial \Pi^M(\overline{c})} < 0$$

Special case: efficiency effect only

- Hypothesis: $F_0 = 0$
  i.e., it is certain that one of the two firms innovates
- Reaction function of firm 1

$$F_1^R(F_2) = -F_2 + \sqrt{F_2(\Pi^M(\underline{c}) - \Pi_1^d)}$$

- Reaction function of firm 2

$$F_2^R(F_1) = -F_1 + \sqrt{F_1 \Pi_2^d(\underline{c})}$$

- Nash equilibrium:

$$\left((\Pi^M(\underline{c}) - \Pi_1^d) \frac{(\Pi^M(\underline{c}) - \Pi_1^d)\Pi_2^d}{\left(\Pi^M(\underline{c}) - \Pi_1^d + \Pi_2^d\right)^2} ; \Pi_2^d \frac{(\Pi^M(\underline{c}) - \Pi_1^d)\Pi_2^d}{\left(\Pi^M(\underline{c}) - \Pi_1^d + \Pi_2^d\right)^2} \right)$$

Executive summary I

- The higher the attainable monopoly profit, the higher the expenditures for R&D in the patent-race Nash equilibrium.
- The less likely successful innovation, the less all firms' expenditures for R&D.
- R&D expenditures might be strategic complements or strategic substitutes.
- Sometimes, it may pay for the monopolist to file a patent but not to actually use it himself (sleeping patent).

Executive summary II

Incentives to innovate for the asymmetric duopoly:

- If the incumbent innovates, he replaces himself. If the entrant innovates, he achieves positive profits as compared to zero profits.
  ✹ replacement effect
- The established firm's incentive to remain a monopolist rather than becoming a duopolist is greater then the entrant's incentive to become a duopolist.
  ✹ efficiency effect

Innovation competition with spill-over effect

- Basic idea
- Simultaneous quantity competition (2nd stage)
- Simultaneous R&D competition (1st stage)
- Simultaneous R&D cooperation (1st stage)
- Comparison of R&D competition and R&D cooperation
- Executive summary

Basic idea

- It is often not possible to internalise the benefits of R&D activities perfectly:
  - employee turnover
  - analysis of patents
- R&D cooperation can be observed in some industries (e.g. PSA has different cooperation projects). On the product market, the firms may still compete.

VW Sharan, Ford Galaxy

https://de.wikipedia.org/wiki/VW_Sharan

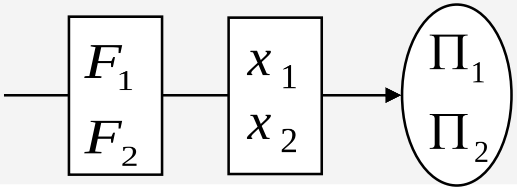
https://de.wikipedia.org/wiki/Ford_Galaxy

The model

- Before the innovation $c = c_1 = c_2$
- The innovation reduces costs by

$$\begin{aligned} \Delta c_1 &= F_1 + \beta F_2 \\ \Delta c_2 &= F_2 + \beta F_1 \end{aligned} \quad \text{to} \quad \begin{aligned} c - \Delta c_1 \\ c - \Delta c_2 \end{aligned}$$

- β measures the spill-over effect.
- F_i...R&D activity; $C(F_i)$...costs of R&D activity
- Structure



Profit function

- Profit functions

$$\Pi_1(F_1, F_2, x_1, x_2) = (a - bX - (c - \Delta c_1))x_1 - C(F_1)$$

$$\Pi_2(F_1, F_2, x_1, x_2) = (a - bX - (c - \Delta c_2))x_2 - C(F_2)$$

assume: $a - c = 1$ and $b = 1$

$$\Pi_1(F_1, F_2, x_1, x_2) = (1 + \Delta c_1 - x_1 - x_2)x_1 - C(F_1)$$

$$= (1 + [F_1 + \beta F_2] - x_1 - x_2)x_1 - C(F_1)$$

$$\Pi_2(F_1, F_2, x_1, x_2) = (1 + \Delta c_2 - x_1 - x_2)x_2 - C(F_2)$$

$$= (1 + [F_2 + \beta F_1] - x_1 - x_2)x_2 - C(F_2)$$

Cournot competition (2^{nd} stage)

- Reaction functions

$$x_1^R(x_2) = \frac{(1+\Delta c_1 - x_2)}{2} \quad and \quad x_2^R(x_1) = \frac{(1+\Delta c_2 - x_1)}{2}$$

- Cournot equilibrium

$$x_1^C(F_1, F_2) = \frac{(1+2\Delta c_1 - \Delta c_2)}{3} = \frac{1+(2-\beta)F_1 + (2\beta-1)F_2}{3}$$

$x_2^C(F_1, F_2)$ analogous

- Reduced profit functions

$$\Pi_1^C(F_1, F_2) = \left[\tfrac{1+2\Delta c_1 - \Delta c_2}{3}\right]^2 - C(F_1)$$

$$\Pi_2^C(F_1, F_2) = \left[\tfrac{1+2\Delta c_2 - \Delta c_1}{3}\right]^2 - C(F_2)$$

How Cournot outputs depend on

- „real“ R&D activity Δc_i :

$$\frac{\partial x_i^C}{\partial \Delta c_i} > 0 \quad and \quad \frac{\partial x_j^C}{\partial \Delta c_i} < 0$$

- R&D activity F_i :

$$\frac{\partial x_i^C}{\partial F_i} = \frac{2-\beta}{3} > 0$$

$$\frac{\partial x_j^C}{\partial F_i} = \frac{2\beta - 1}{3} \begin{cases} < 0, & \beta < \frac{1}{2} \\ > 0, & \beta > \frac{1}{2} \end{cases}$$

Exercise (R&D competition on 1st stage)

- Assume $C(F_i) = \frac{1}{2}\gamma F_i^2 \quad i = 1,2.$
  Find the symmetric equilibrium in the R&D game.

$$S.:\ F_1^N = F_2^N = \frac{2(2-\beta)}{9\gamma - 2(1+\beta)(2-\beta)}$$

Analyzing direct and indirect effects (R&D competition) I

$$\Pi_1^C(F_1) = \Pi_1\left(F_1, x_1^C(F_1), x_2^C(F_1)\right)$$

$$\Pi_2^C(F_1) = \Pi_2\left(F_1, x_1^C(F_1), x_2^C(F_1)\right)$$

Influence of F_1 on firm 1's profit

$$\frac{d\Pi_1^C}{dF_1} = \underbrace{\frac{\partial \Pi_1}{\partial F_1}}_{\text{direct effect}} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_1}}_{=0}\underbrace{\frac{dx_1^C}{dF_1}}_{>0}}_{=0} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_2}}_{<0}\underbrace{\frac{dx_2^C}{dF_1}}_{\frac{\partial x_2^C}{\partial F_1} = \frac{2\beta - 1}{3}\begin{cases}<0, & \beta < \frac{1}{2}\\ >0, & \beta > \frac{1}{2}\end{cases}}}_{\text{strategic effect}\begin{cases}>0, & \beta < \frac{1}{2}\\ <0, & \beta > \frac{1}{2}\end{cases}}$$

$$\frac{\partial \Pi_1}{\partial F_1} = x_1^C - \frac{\partial C(F_1)}{\partial F_1}$$

Analyzing direct and indirect effects (R&D competition) II

- Influence of F_1 on firm 2's profit

$$\frac{d\Pi_2^C}{dF_1} = \underbrace{\frac{\partial \Pi_2}{\partial F_1}}_{\text{direct effect}} + \underbrace{\underbrace{\frac{\partial \Pi_2}{\partial x_2}}_{=0} \underbrace{\frac{dx_2^C}{dF_1}}_{?}}_{=0} + \underbrace{\frac{\partial \Pi_2}{\partial x_1} \frac{dx_1^C}{dF_1}}_{\text{strategic effect}}$$

$$= \underbrace{\beta x_2^C}_{>0} \underbrace{- x_2^C \frac{2-\beta}{3}}_{<0}$$

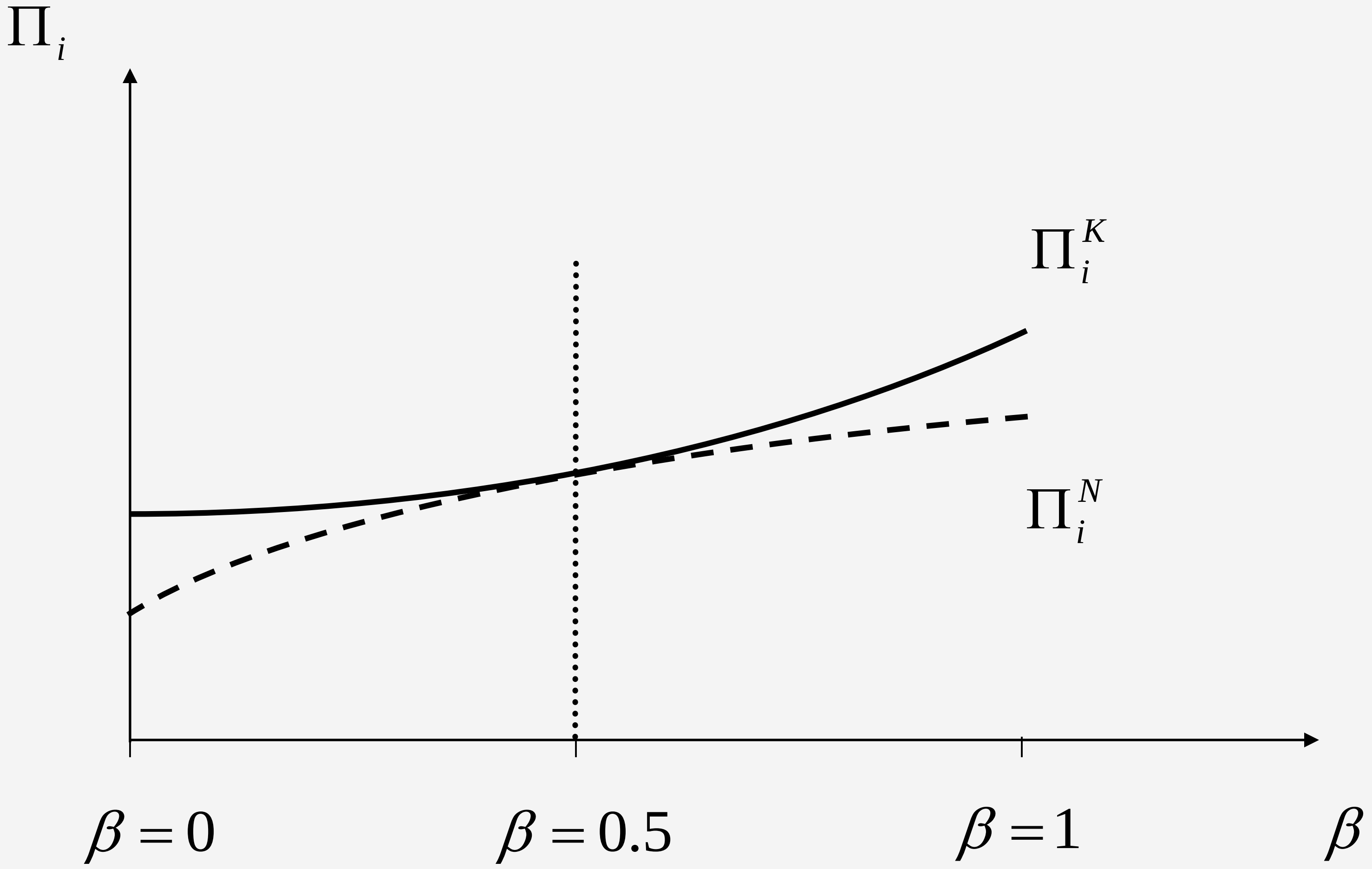
$$= \left(\frac{4}{3}\beta - \frac{2}{3}\right) x_2^C \begin{cases} < 0, & \beta < \frac{1}{2} \\ > 0, & \beta > \frac{1}{2} \end{cases}$$

Exercise (R&D cooperation on 1st stage)

- We assume that firms cooperate on the first stage and compete on the second. Therefore:
- Firms want to maximize the joint reduced profit function $\Pi^C(F_1, F_2) := \Pi_1^C(F_1, F_2) + \Pi_2^C(F_1, F_2)$.
- While assuming the same quadratic cost function as before, calculate the cartel solution.

$$S.: F_1^K = F_2^K = \frac{2(\beta+1)}{9\gamma - 2(1+\beta)^2}$$

Profit comparison of R&D competition and R&D cooperation



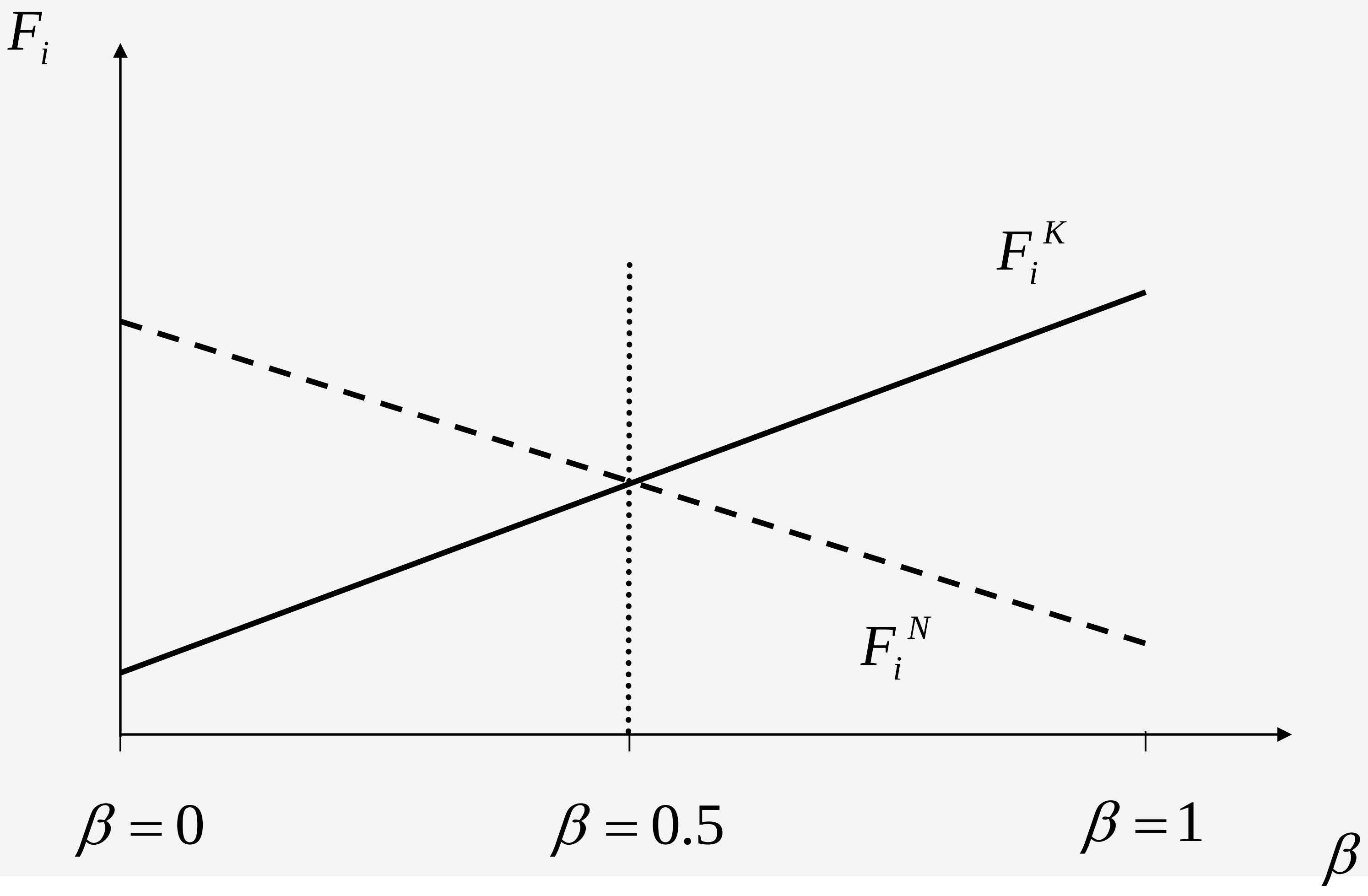
Analytical comparison of competition v. cooperation

- Does R&D competition yield higher R&D activities than cooperation?
- In our concrete model, we find

$$F_1^N + F_2^N - \left(F_1^K + F_2^K\right)$$

$$= \frac{2(2-\beta)}{9\gamma - 2(1+\beta)(2-\beta)} - \frac{2(1+\beta)}{9\gamma - 2(1+\beta)^2} \begin{cases} > 0 & ,\beta < \frac{1}{2} \\ < 0 & ,\beta > \frac{1}{2} \end{cases}$$

Graphical comparison of competition v. cooperation



Interpreting the comparison by way of external effects

- The influence of firm 1 on firm 2's profit is an external effect; see slide „Analyzing direct and indirect effects (R&D competition) II"
- We found

$$\frac{\partial \Pi_2}{\partial F_1} + \frac{\partial \Pi_2}{\partial x_1}\frac{dx_1^C}{dF_1}\begin{cases} < 0, & \beta < \frac{1}{2} \rightarrow \text{neg. ext. effect } F_i^K < F_i^N \\ > 0, & \beta > \frac{1}{2} \rightarrow \text{pos. ext. effect } F_i^K > F_i^N \end{cases}$$

- Bingo!

Executive summary: If spillover effects are sufficiently important,

- Firms want to underinvest in R&D for strategic reasons;
- Firms want to cooperate in order to prevent suboptimal R&D activities;
- Governments may allow R&D cooperation in order to enhance R&D activities.