Course outline I

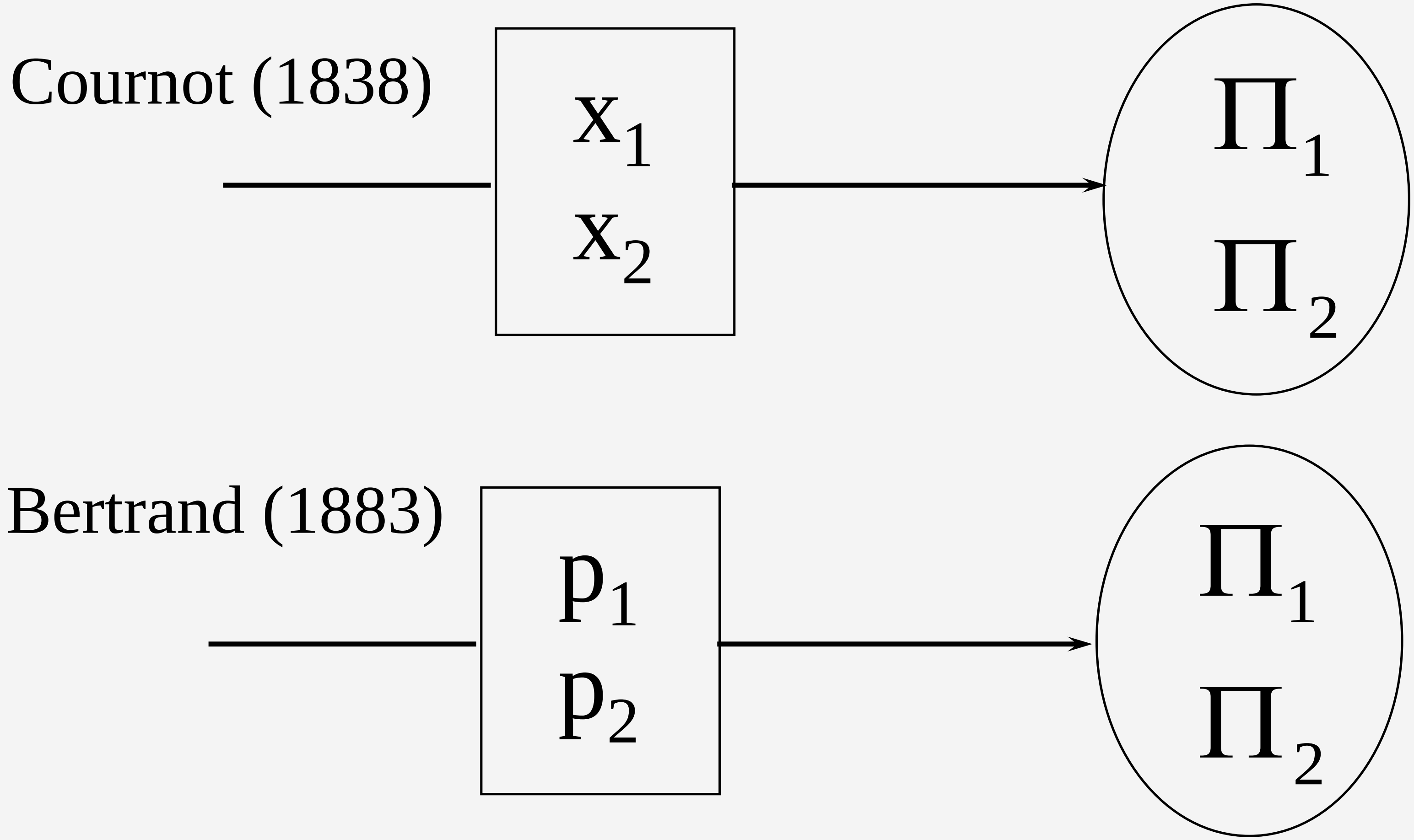
- Introduction
- Game theory
- Price setting
  - monopoly
  - oligopoly
- Quantity setting
  - monopoly
  - oligopoly
- Process innovation

} Homogeneous goods

Quantity and cost competition

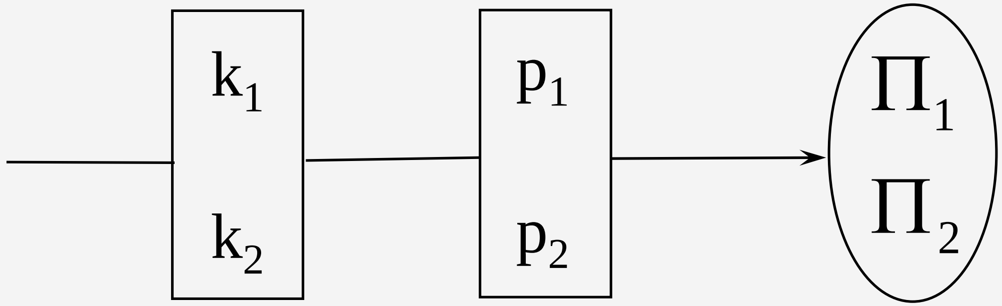
- Bertrand versus Cournot
- Simultaneous quantity competition (Cournot)
- Sequential quantity competition (Stackelberg)
- Quantity Cartel
- Concentration and competition

Price or quantity competition?



Capacity + Bertrand = Cournot

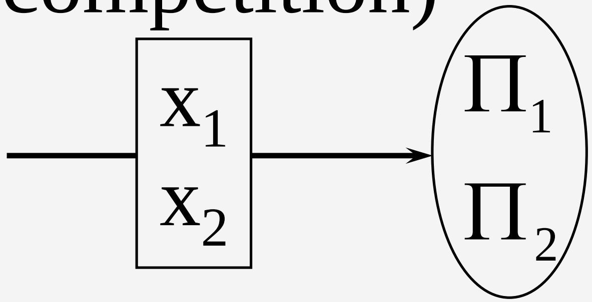
- Bertrand (1883) criticized Cournot's model (1838) on the grounds that firms compete by setting prices and not by setting quantities.
- Kreps and Scheinkman (1983) defended Cournot's model. They developed a two-stage game with capacities



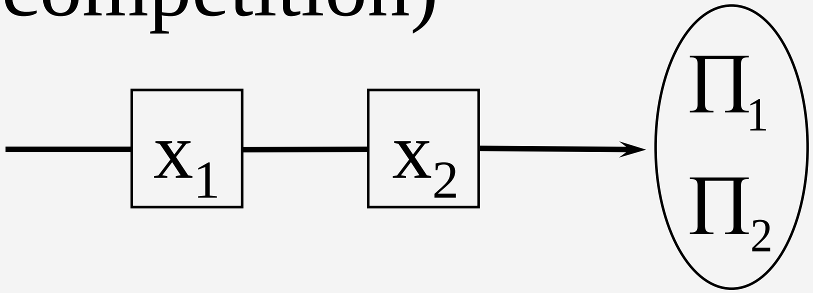
and proved that capacities in a Nash equilibrium are determined by Cournot's model.

Cournot versus Stackelberg

- Cournot duopoly (simultaneous quantity competition)



- Stackelberg duopoly (sequential quantity competition)



Homogeneous duopoly (linear case)

- Two firms (i=1,2) produce a homogenous good.
- Outputs: x_1 and x_2, $X= x_1+x_2$
- Marginal costs: c_1 and c_2
- Inverse demand function:

$$p(X) = a - bX = a - b(x_1 + x_2)$$

- Profit function of firm 1:

$$\Pi_1(x_1, x_2) = p(X)x_1 - c_1 x_1 = (a - b(x_1 + x_2) - c_1)x_1$$

Cournot-Nash equilibrium

- Profit functions: $\Pi_1(x_1, x_2), \Pi_2(x_1, x_2)$
- Reaction functions:

$$x_1^R(x_2) = \arg\max_{x_1} \Pi_1(x_1, x_2)$$

$$x_2^R(x_1) = \arg\max_{x_2} \Pi_2(x_1, x_2)$$

- Nash equilibrium: (x_1^C, x_2^C)

$$x_1^R(x_2^C) = x_1^C$$

$$x_2^R(x_1^C) = x_2^C$$

Computing the Cournot equilibrium (accommodation)

- Profit function of firm 1

$$\Pi_1(x_1, x_2) = p(X)x_1 - c_1x_1 = (a - b(x_1 + x_2) - c_1)x_1$$

- Reaction function of firm 1

$$x_1^R(x_2) = \frac{a - c_1}{2b} - \frac{x_2}{2} \qquad \text{analogous} : \ x_2^R(x_1) = \frac{a - c_2}{2b} - \frac{x_1}{2}$$

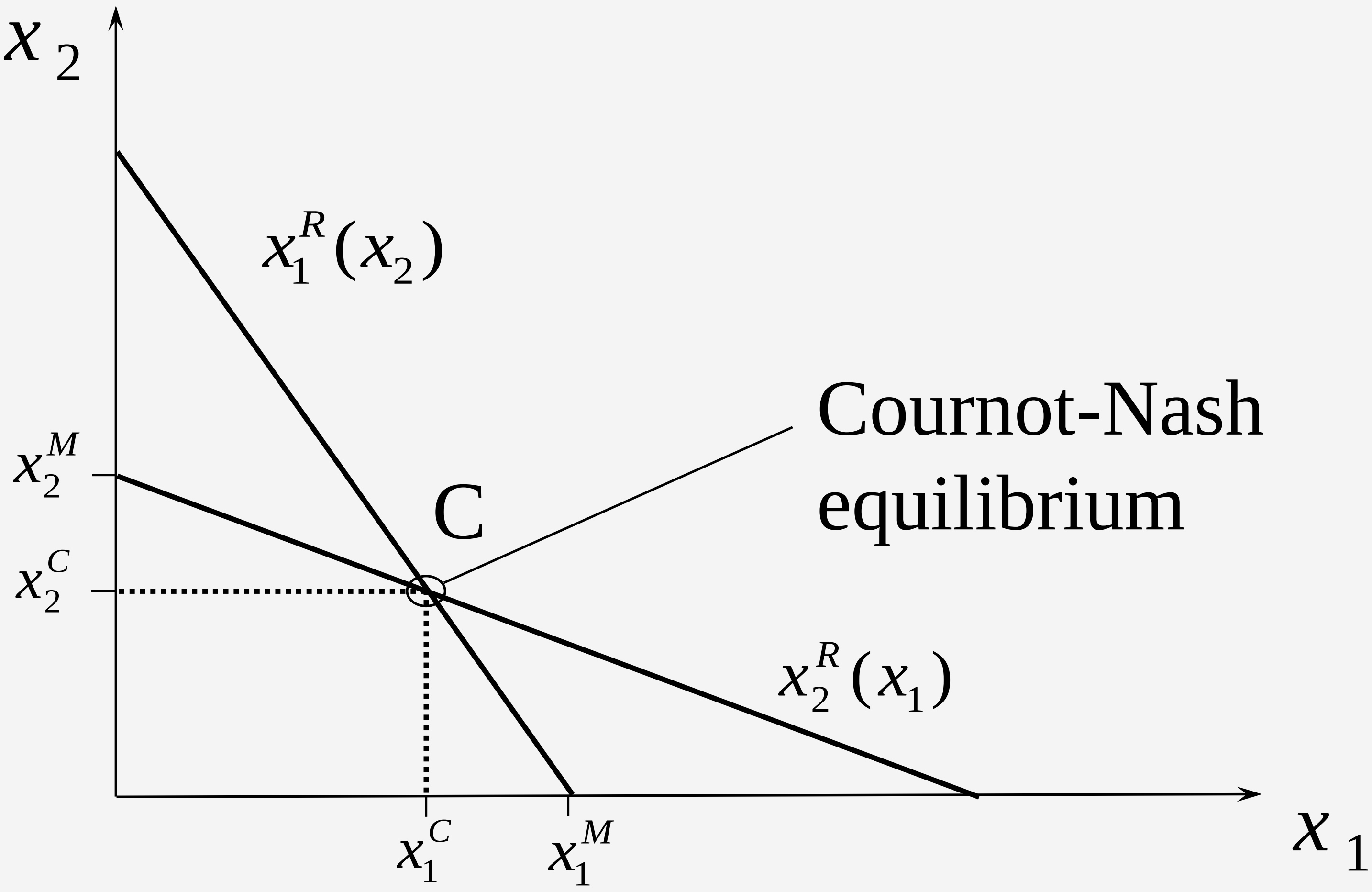
- Nash equilibrium

$$\left(x_1^C = \frac{a - 2c_1 + c_2}{3b}, x_2^C = \frac{a - 2c_2 + c_1}{3b} \right)$$

$$p^C = \frac{(a + c_1 + c_2)}{3}$$

$$\Pi_1^C = \frac{(a - 2c_1 + c_2)^2}{9b} \qquad \text{analogous} : \ \Pi_2^C = \frac{(a - 2c_2 + c_1)^2}{9b}$$

Depicting the Cournot equilibrium



Exercise (Cournot)

Find the equilibrium in a Cournot competition. Suppose that the demand function is given by $p(X) = 24 - X$ and the costs per unit by $c_1 = 3$ and $c_2 = 2$.

Common interests

- c_1, c_2 ↓
  Obtaining government subsidies and negotiating with labor unions.

- a ↑, b ↓
  Advertising by the agricultural industry (e.g. CMA).

Exercise (taxes in a duopoly)

Two firms in a duopoly offer petrol. The demand function is given by $p(X)=5-0.5X$. Unit costs are $c_1=0.2$ and $c_2=0.5$.

a) Find the Cournot equilibrium and calculate the price.

b) Now suppose that the government imposes a quantity tax *t* (eco tax). Who ends up paying it?

Two approaches to cost leadership

- Direct approach (reduction of own marginal costs)
  - change of ratio between fixed and variable costs
  - investments in research and development (R&D)
- Indirect approach ("raising rivals' costs")
  - sabotage
  - minimum wages, enviromental legislation

Direct approach, analytically I

- $\Pi_1^C(c_1, c_2) = \Pi_1(c_1, c_2, x_1^C(c_1, c_2), x_2^C(c_1, c_2))$
  $= \left(a - b\left[x_1^C(c_1, c_2) + x_2^C(c_1, c_2)\right] - c_1\right) * x_1^C(c_1, c_2)$

$$x_1^C = \frac{a - 2c_1 + c_2}{3b};\ x_1^C = \frac{a - 2c_2 + c_1}{3b}$$

- Direct approach (reduction of your own marginal costs):

$$\frac{\partial \Pi_1^C}{\partial c_1} = \frac{\partial \Pi_1}{\partial c_1} + \frac{\partial \Pi_1}{\partial x_1}\frac{\partial \mathrm{x}_1^C}{\partial c_1} + \frac{\partial \Pi_1}{\partial x_2}\frac{\partial \mathrm{x}_2^C}{\partial c_1}$$

Direct approach, analytically II

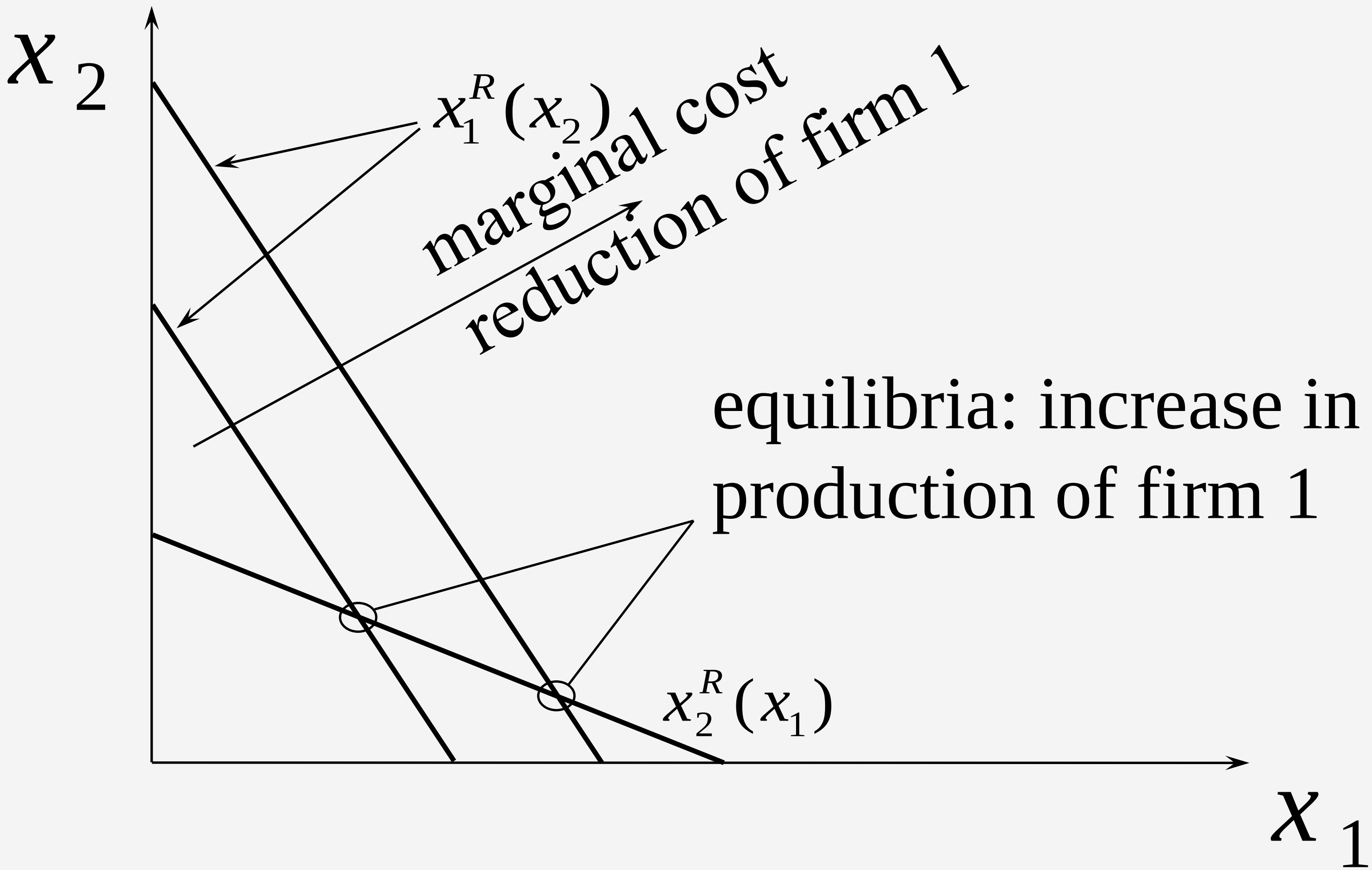
- $$\frac{\partial \Pi_1^C}{\partial c_1} = \underbrace{\frac{\partial \Pi_1}{\partial c_1}}_{<0} + \underbrace{\frac{\partial \Pi_1}{\partial x_1}}_{=0} \frac{\partial x_1^C}{\partial c_1} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_2}}_{<0} \underbrace{\frac{\partial x_2^C}{\partial c_1}}_{>0}}_{<0} < 0$$

  direct effect

  strategic effect

- $\frac{\partial \Pi_1}{\partial c_1} = -x_1^C(c_1, c_2) < 0$, $\frac{\partial \Pi_1^C}{\partial x_2} = -bx_1^C(c_1, c_2) < 0$, $\frac{\partial x_2^C}{\partial c_1} = \frac{1}{3b} > 0$

Direct approach, graphically



Exercise (direct approach)

- Who has a higher incentive to reduce own costs, a monopolist or a firm in Cournot-Duopoly?

Indirect approach, analytically

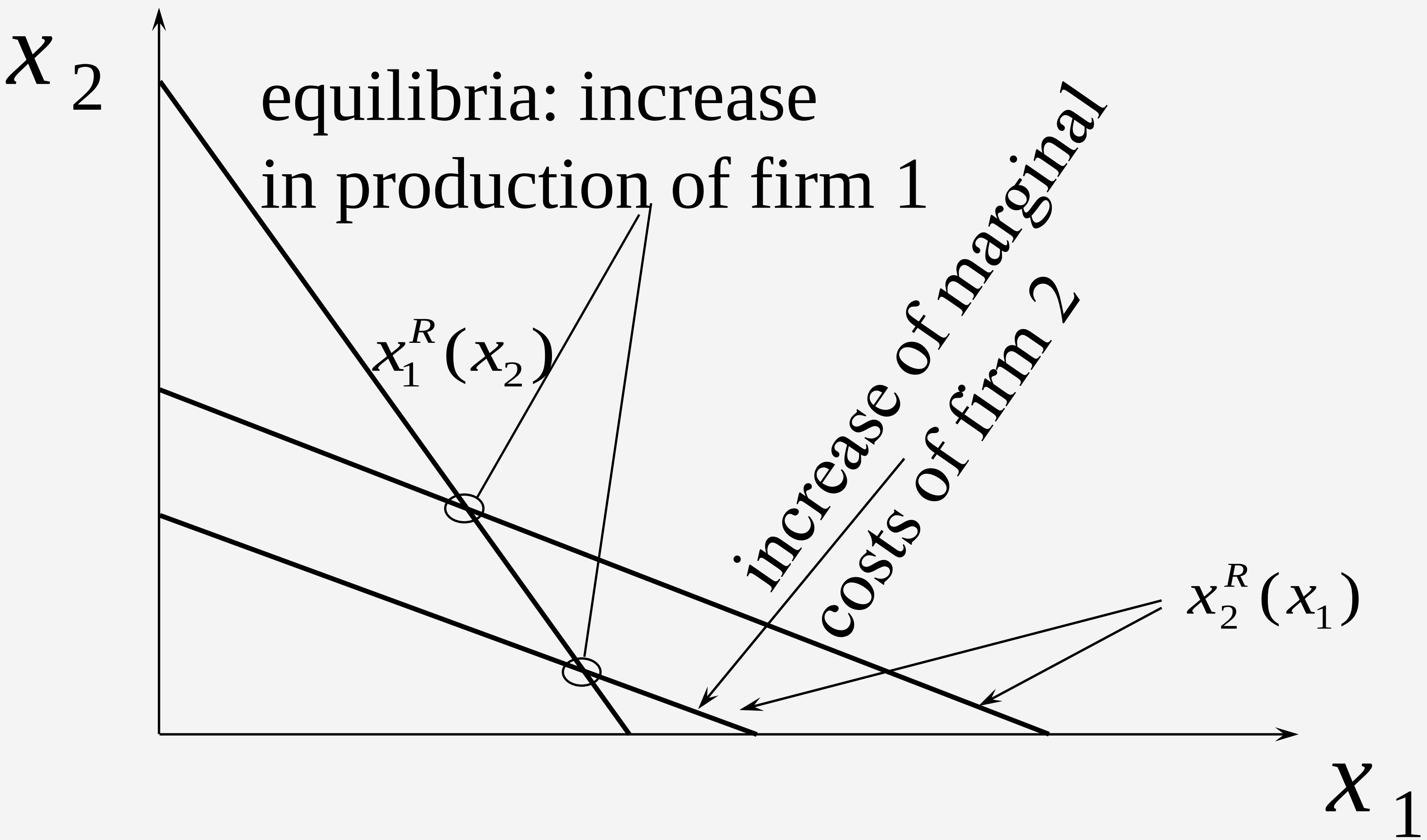
- $\Pi_1^C(c_1, c_2) = \Pi_1\left(c_1, c_2, x_1^C(c_1, c_2), x_2^C(c_1, c_2)\right)$
- Indirect approach (raising rival's cost):

$$\frac{d\Pi_1^C}{dc_2} = \underbrace{\frac{\partial \Pi_1}{\partial c_2}}_{=0} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_2}}_{<0} \underbrace{\frac{dx_2^C}{dc_2}}_{<0}}_{>0} + \underbrace{\frac{\partial \Pi_1}{\partial x_1}}_{=0} \frac{dx_1^C}{dc_2} > 0$$

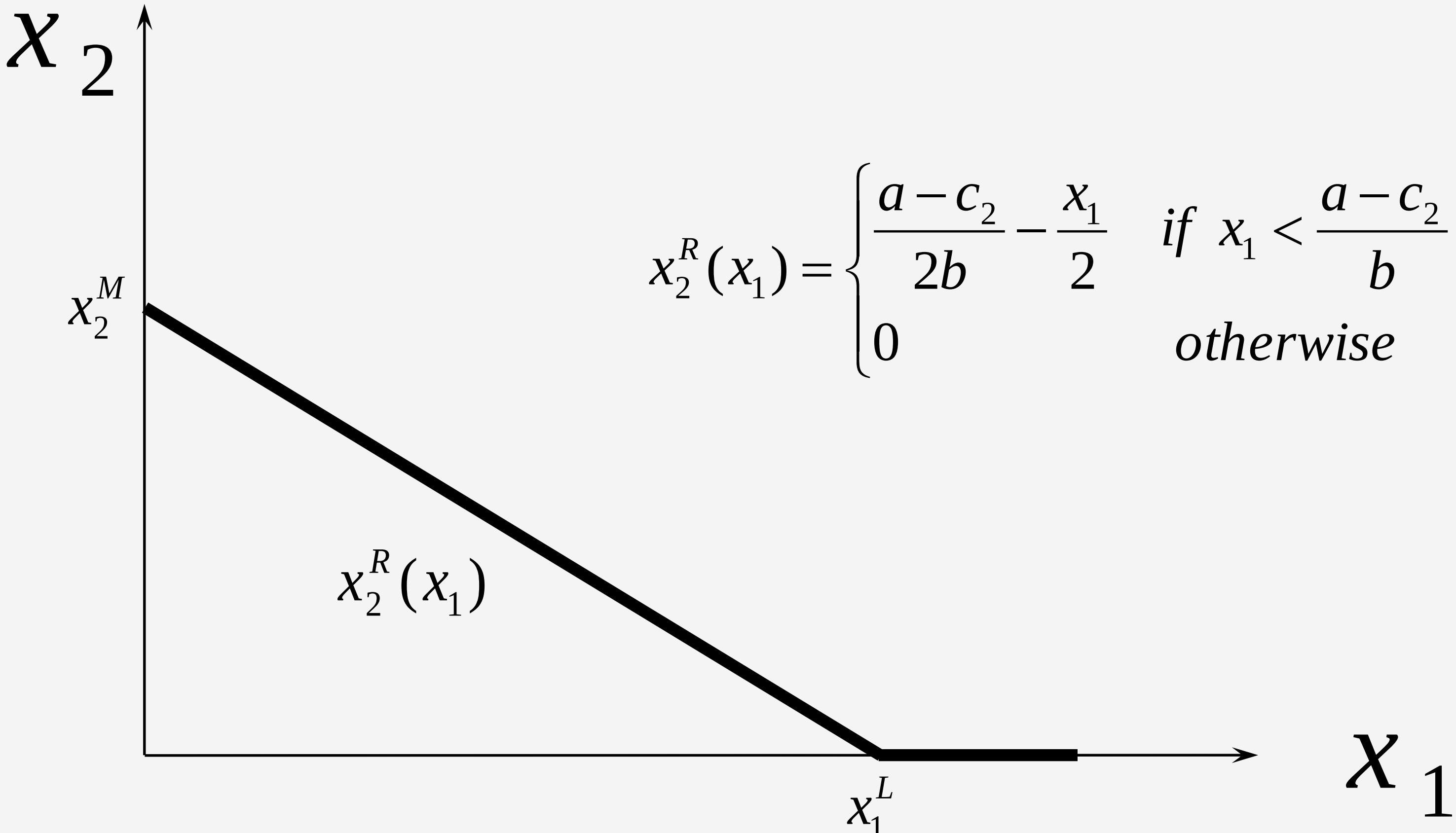
direct effect

strategic effect

Indirect approach, graphically



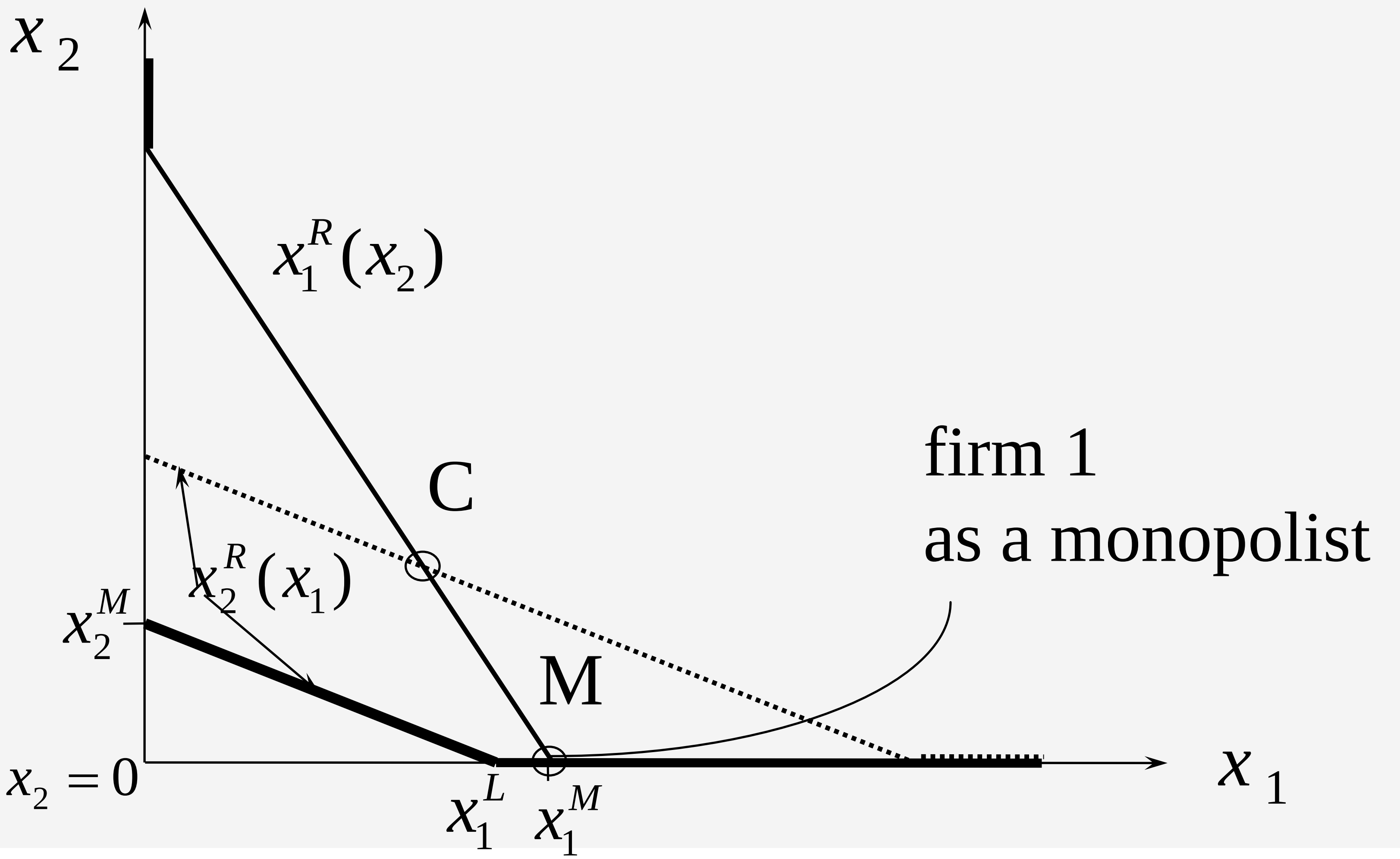
Reaction curve in the linear case



$$x_2^R(x_1) = \begin{cases} \frac{a-c_2}{2b} - \frac{x_1}{2} & \text{if } x_1 < \frac{a-c_2}{b} \\ 0 & \text{otherwise} \end{cases}$$

Note: $x_1^L = \frac{a-c_2}{b}$ alone leads to a price of c_2.

Blockaded entry, graphically



Blockaded entry

- Entry is blockaded for each firm:

$$c_1 \geq a \ and \ c_2 \geq a$$

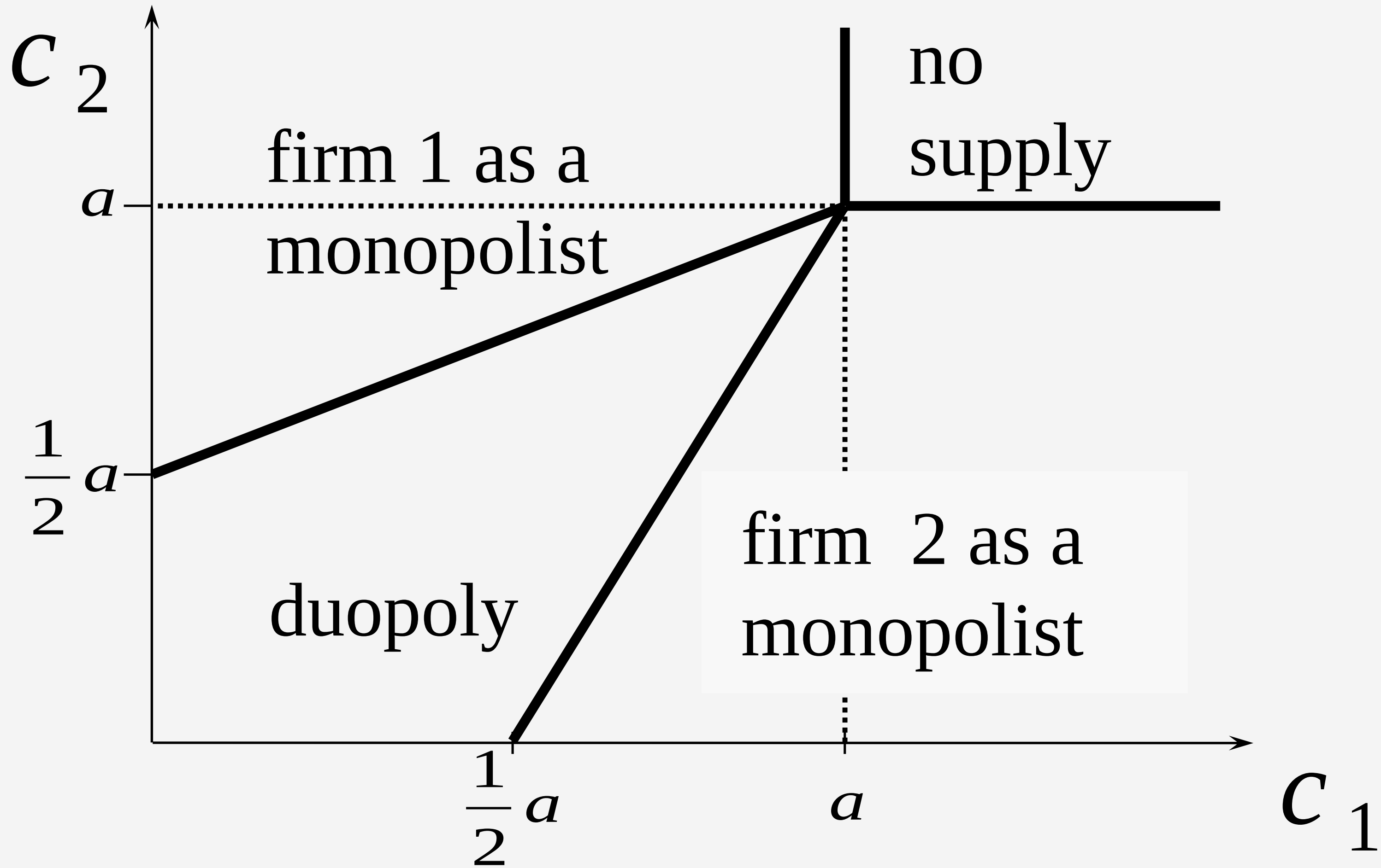
- Entry is blockaded for firm 2:

$$c_1 < a \quad and$$

$$x_1^L \leq x_1^M \qquad \text{i.e.} \qquad \frac{a-c_2}{b} \leq \frac{a-c_1}{2b}$$

$$\Leftrightarrow c_2 \geq p^M(c_1) = \frac{1}{2}(a+c_1)$$

Blockaded entry (overview)

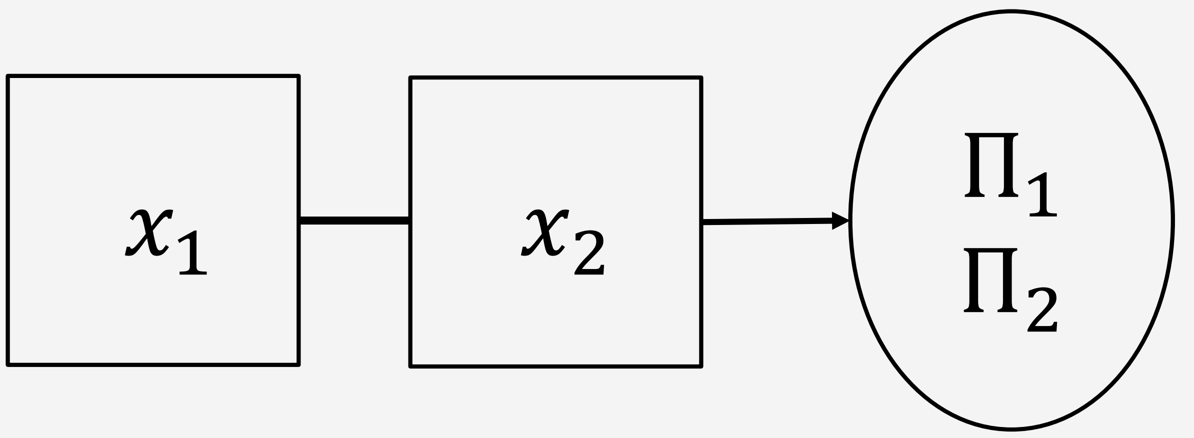


Cournot – Executive summary

- A duopoly will occur only, if entry is blockaded for other firms.
- Firms have common and competing interests with respect to demand and cost functions.
- There are two approaches to cost leadership. The direct approach is to lower your own marginal cost. The indirect approach is known as “raising rivals’ costs“.

Sequential quantity competition (Stackelberg)

- Game structure:



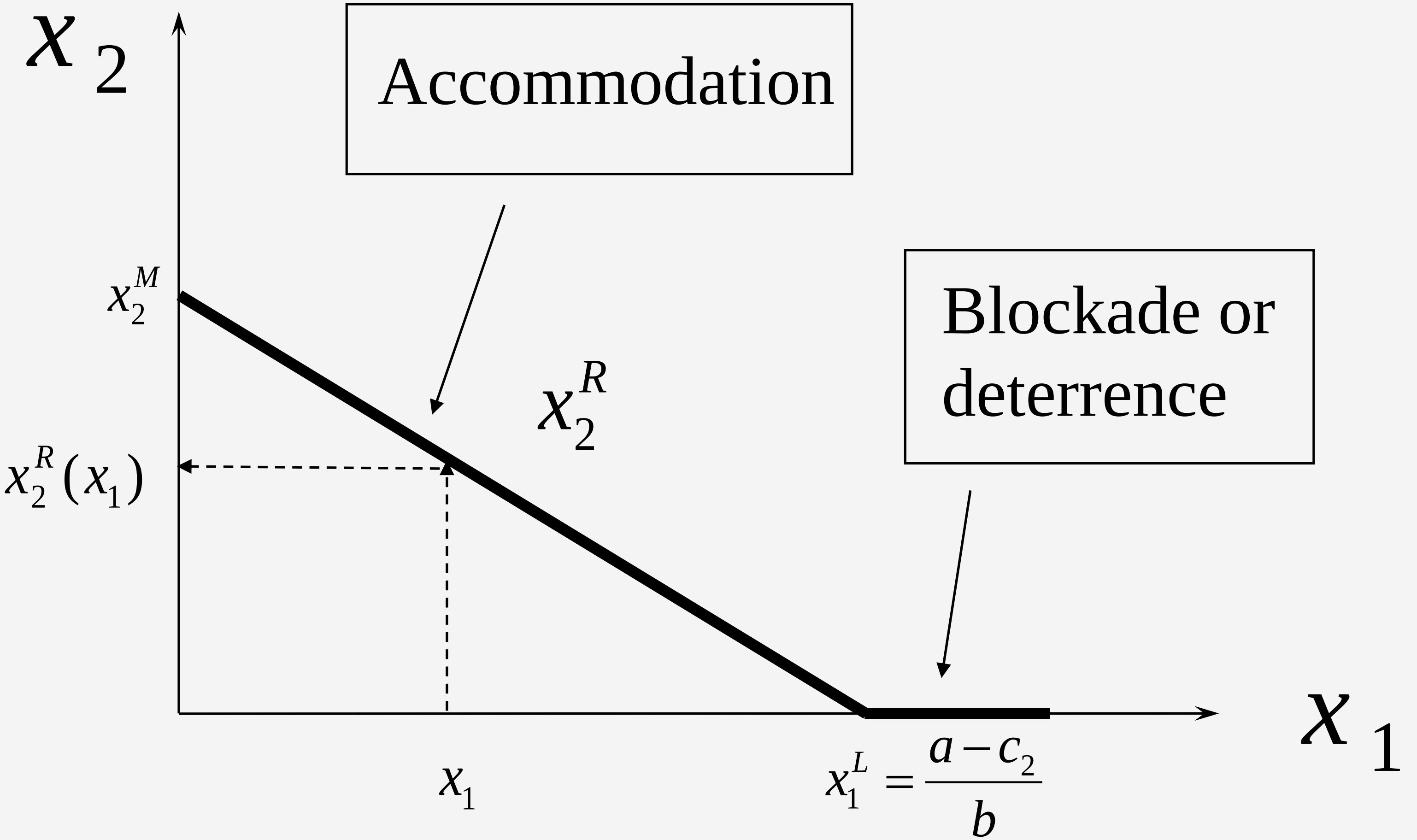
Stackelberg equilibrium

- Profit functions $\Pi_1(x_1, x_2), \Pi_2(x_1, x_2)$
- Follower's reaction function (2nd stage)

  $x_2^R(x_1) = \arg\max_{x_2} \Pi_2(x_1, x_2)$
- Leader's optimal quantity (1st stage)

  $x_1^S = \arg\max_{x_1} \Pi_1\left(x_1, x_2^R(x_1)\right)$
- Nash equilibrium: (x_1^S, x_2^R)

Finding the profit-maximizing point on the follower's reaction curve



Computing the Stackelberg equilibrium (accommodation)

- Reaction function of firm 2:

$$x_2^R(x_1) = \frac{a - c_2}{2b} - \frac{x_1}{2}$$

- Profit function of firm 1:

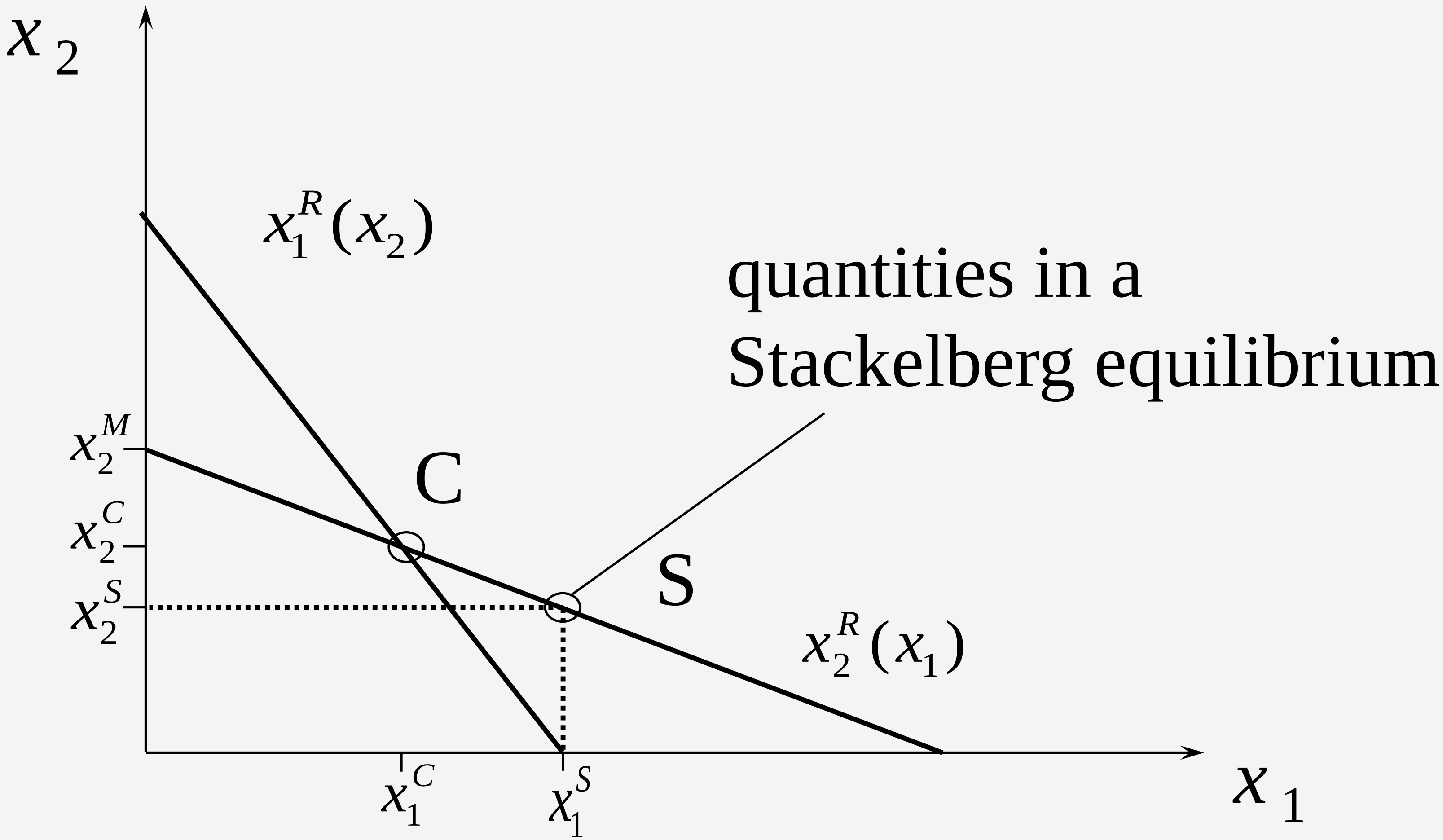
$$\Pi_1(x_1, x_2^R(x_1)) = \left(a - b\left(x_1 + \frac{a - c_2}{2b} - \frac{x_1}{2} \right) - c_1 \right) x_1$$

- Nash equilibrium

$$\left(x_1^S = \frac{a - 2c_1 + c_2}{2b}, x_2^R \right)$$

with $x_2^S = \dfrac{a + 2c_1 - 3c_2}{4b}$ and $p^S = \dfrac{a + 2c_1 + c_2}{4}$

Depicting the Stackelberg outcome (both firms produce)



Exercise (equilibria)

- Which is an equilibrium in the Stackelberg model?

$$\left(x_1^S, x_2^R\left(x_1^S\right)\right),$$
$$\left(x_1^S, x_2^R\right),$$
$$\left(x_1^C, x_2^C\right)?$$

- Are there any additional Nash equilibria ?

Cournot versus Stackelberg

- Profit function of firm 1

$$\Pi_1(x_1, x_2) = p(X)x_1 - C_1(x_1)$$

- First order condition for firm 1

$$\frac{dR_1}{dx_1} = p(X) + x_1 \frac{dp}{dX}\frac{dX}{dx_1} = p(X) + x_1 \frac{dp}{dX}\left(\frac{dx_1}{dx_1} + \frac{dx_2^R}{dx_1}\right)$$

$$= \underbrace{p(X) + x_1 \frac{dp}{dX}}_{\text{direct effect}} + \underbrace{x_1 \frac{dp}{dX}\frac{dx_2^R}{dx_1}}_{\text{follower or strategic effect}} \overset{!}{=} MC_1(x_1)$$

direct effect

follower or strategic effect,
Cournot: 0, Stackelberg: >0

Exercise (Stackelberg)

- Find the equilibrium in a Stackelberg competition. Suppose that the demand function is given by p(X) = 24 - X and the costs per unit by $c_1 = 3$, $c_2 = 2$.

  S.: $\left(x_1^S = 10,\ x_2^R\right)$

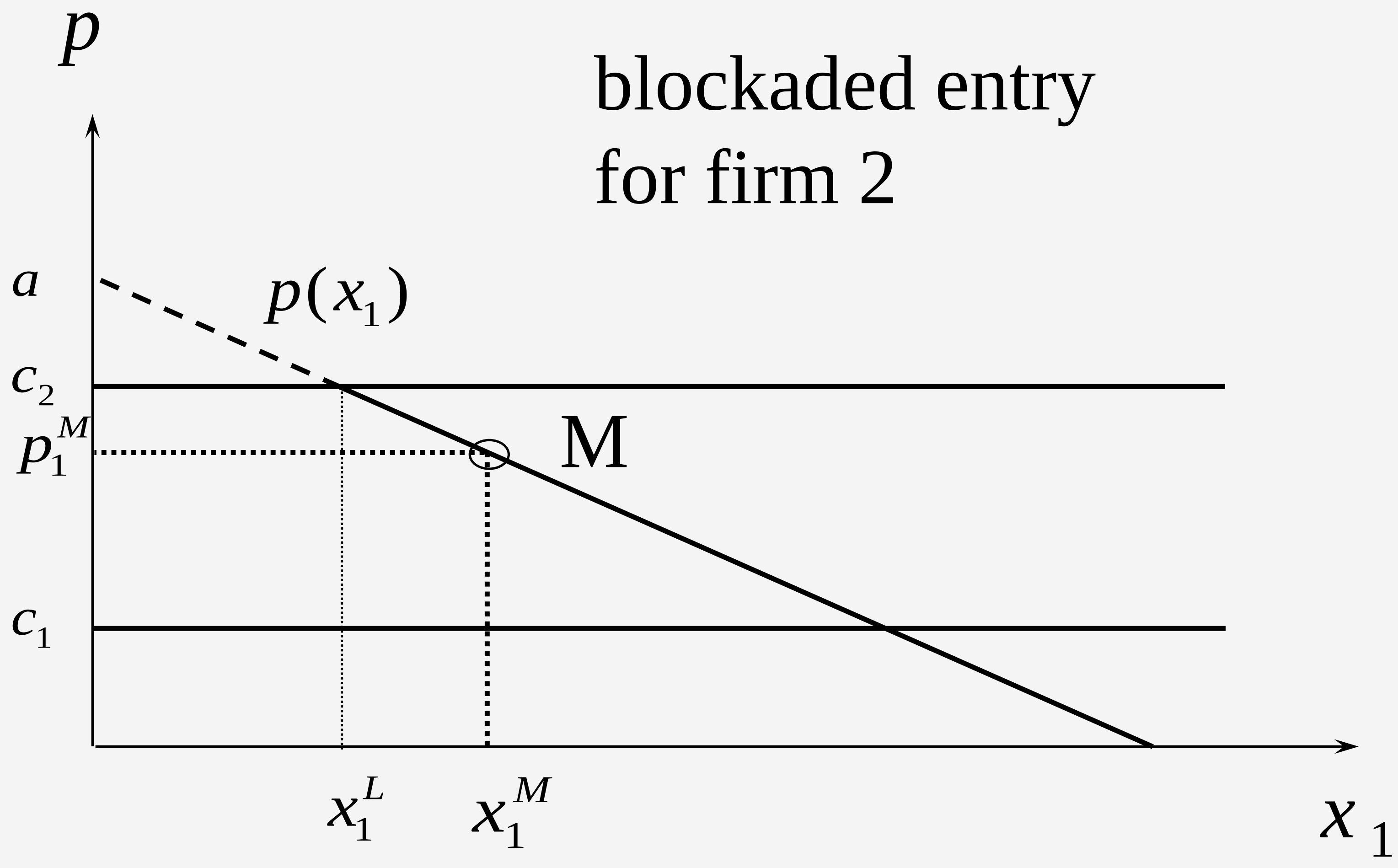
- Possible or not: $\Pi_1^C > \Pi_1^S$?

Exercise (three firms)

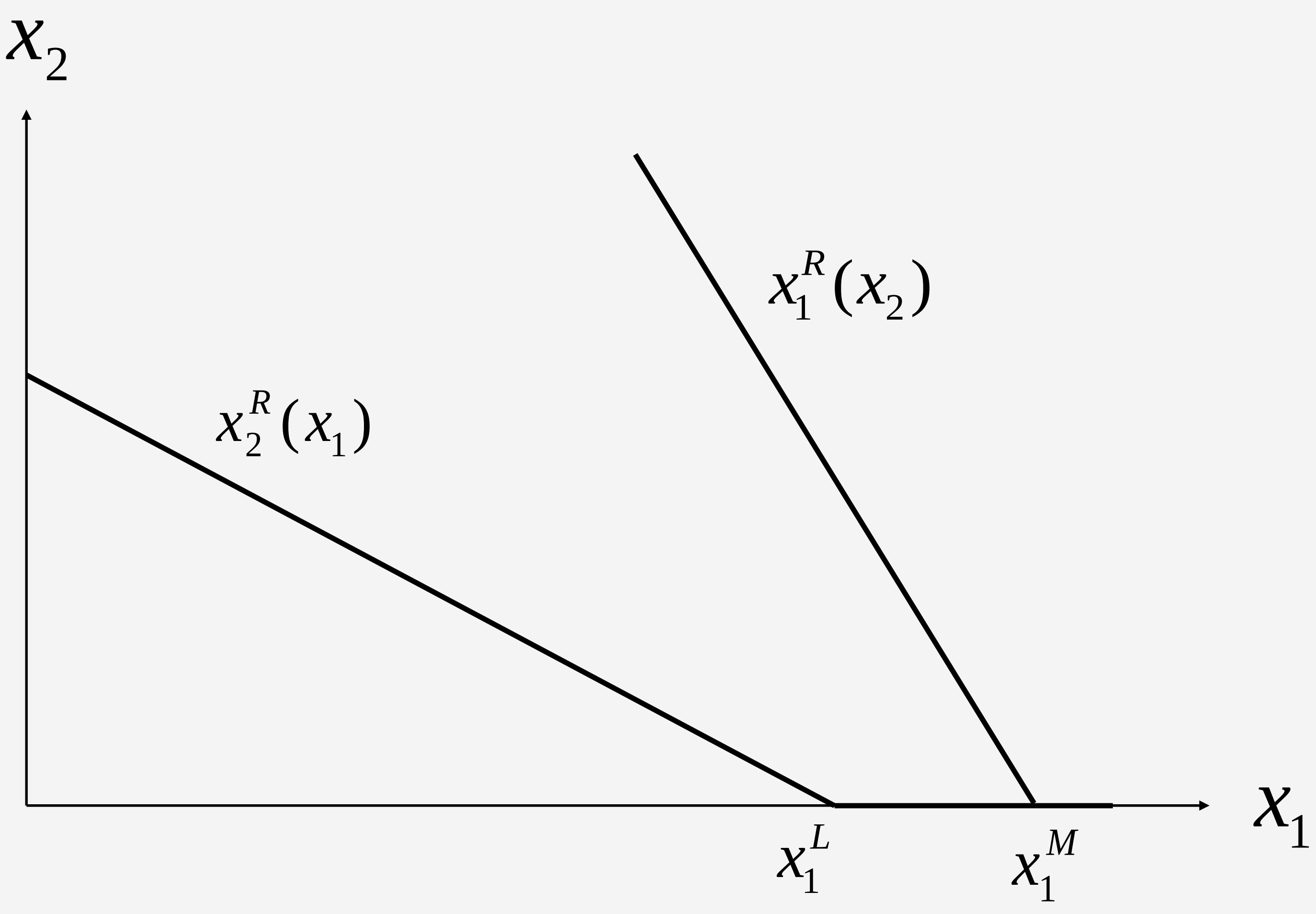
Three firms compete in a homogenous good market with $X(p)=100-p$. The costs are zero. At stage 1, firm 1 sets its quantity; at stage 2, firms 2 and 3 simultaneously decide on their quantities.

Calculate the price on the market!

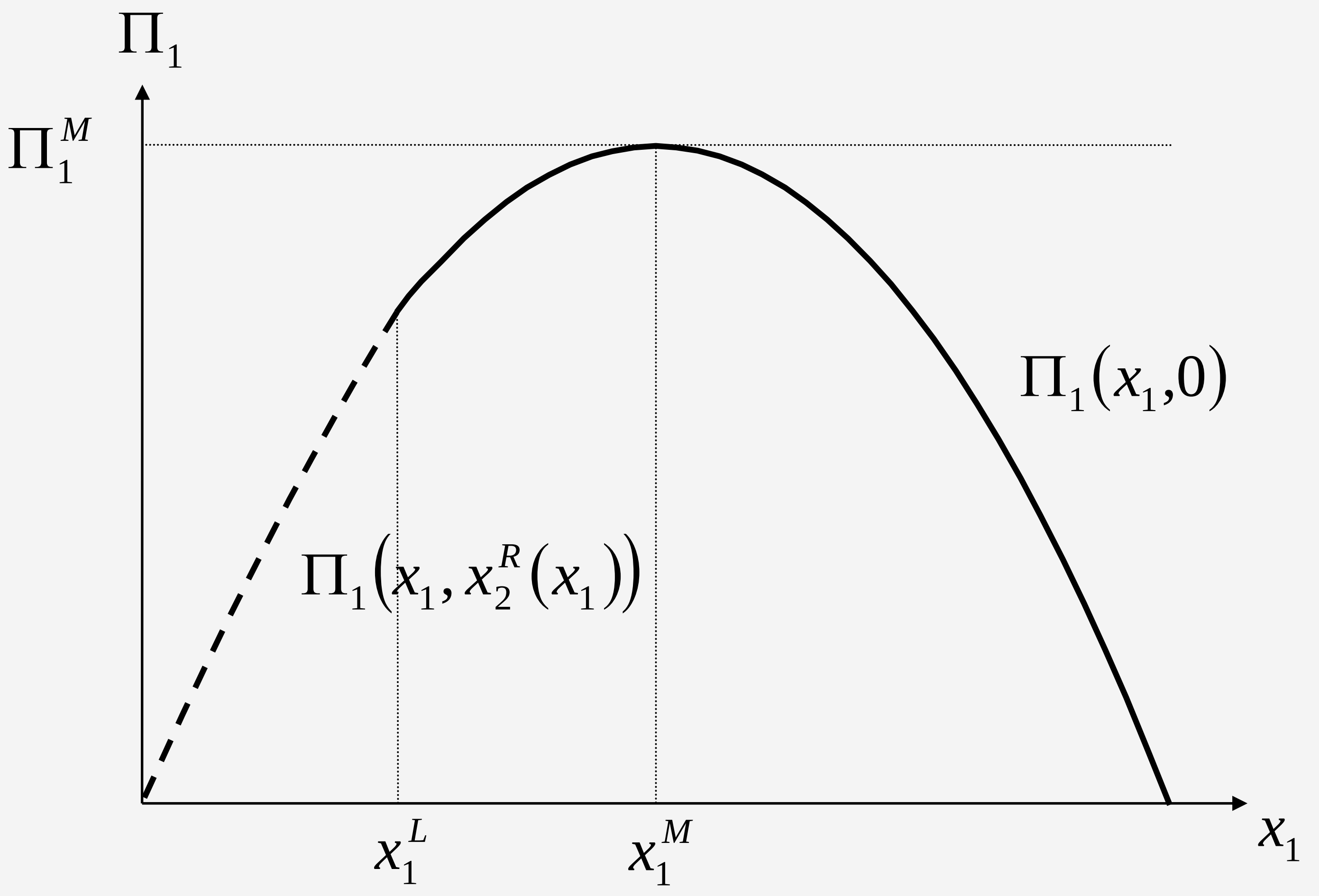
Blockaded entry



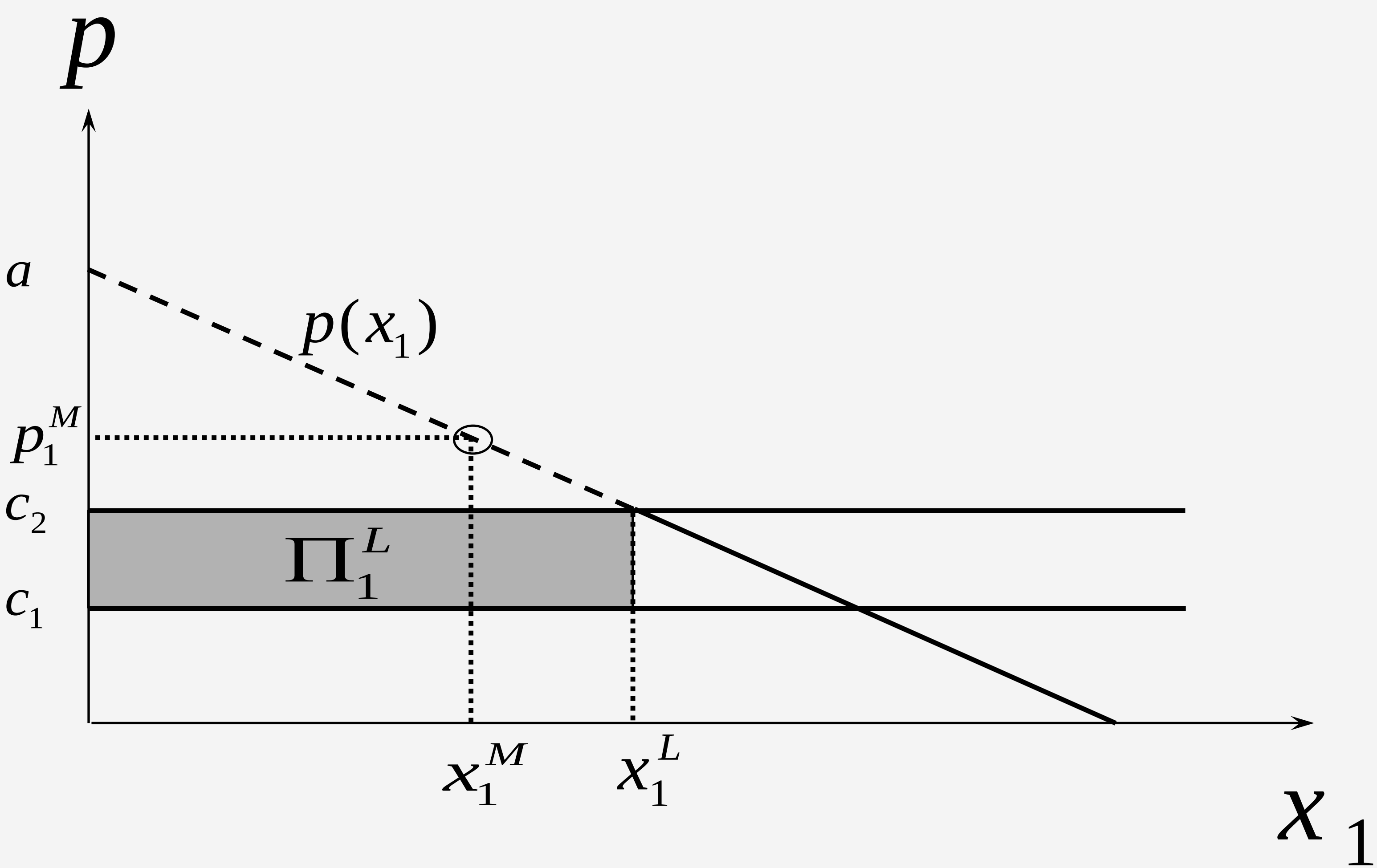
Reaction functions in the case of blockaded entry



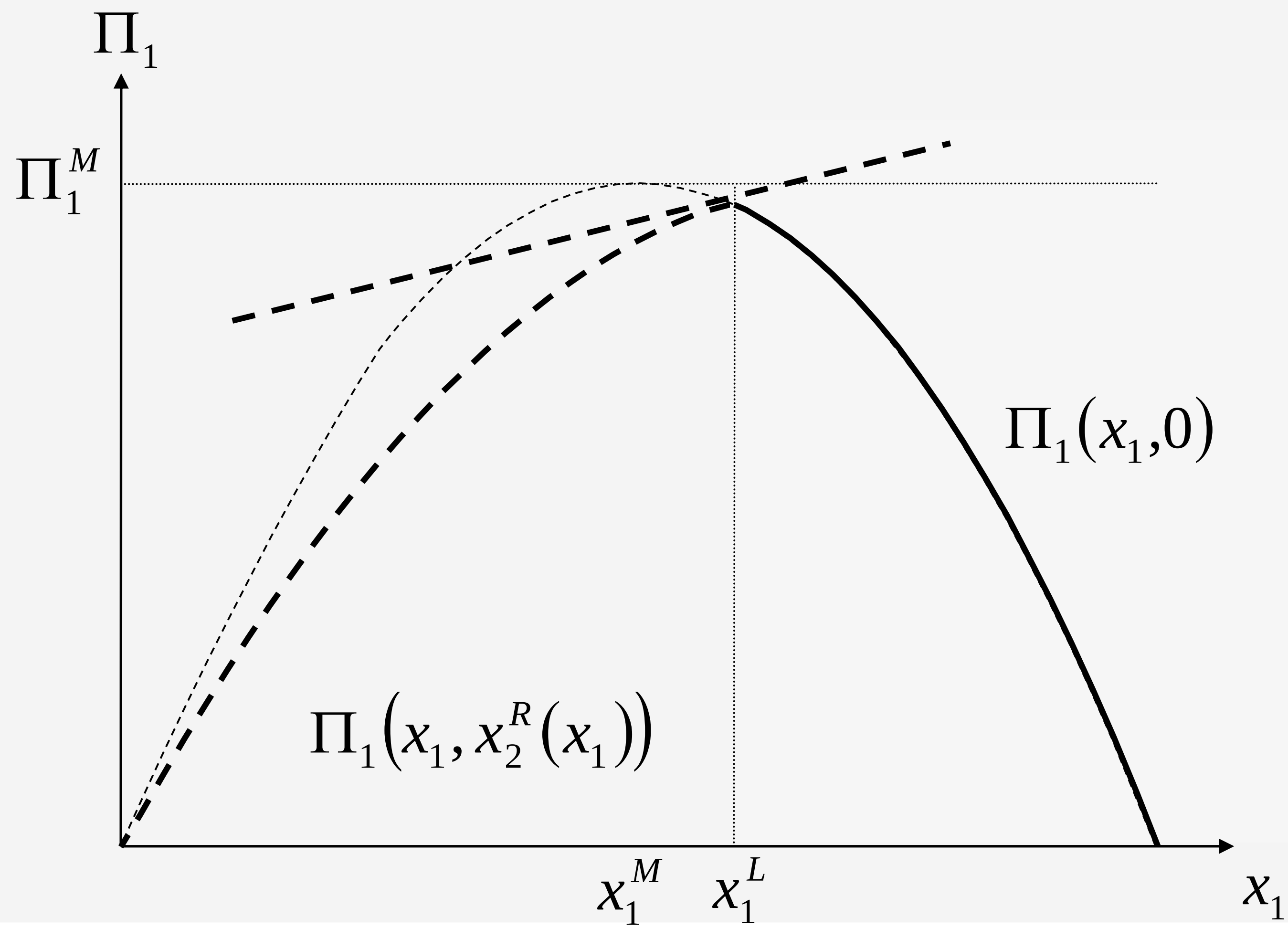
Profit function of firm 1 in the case of blockaded entry of firm 2



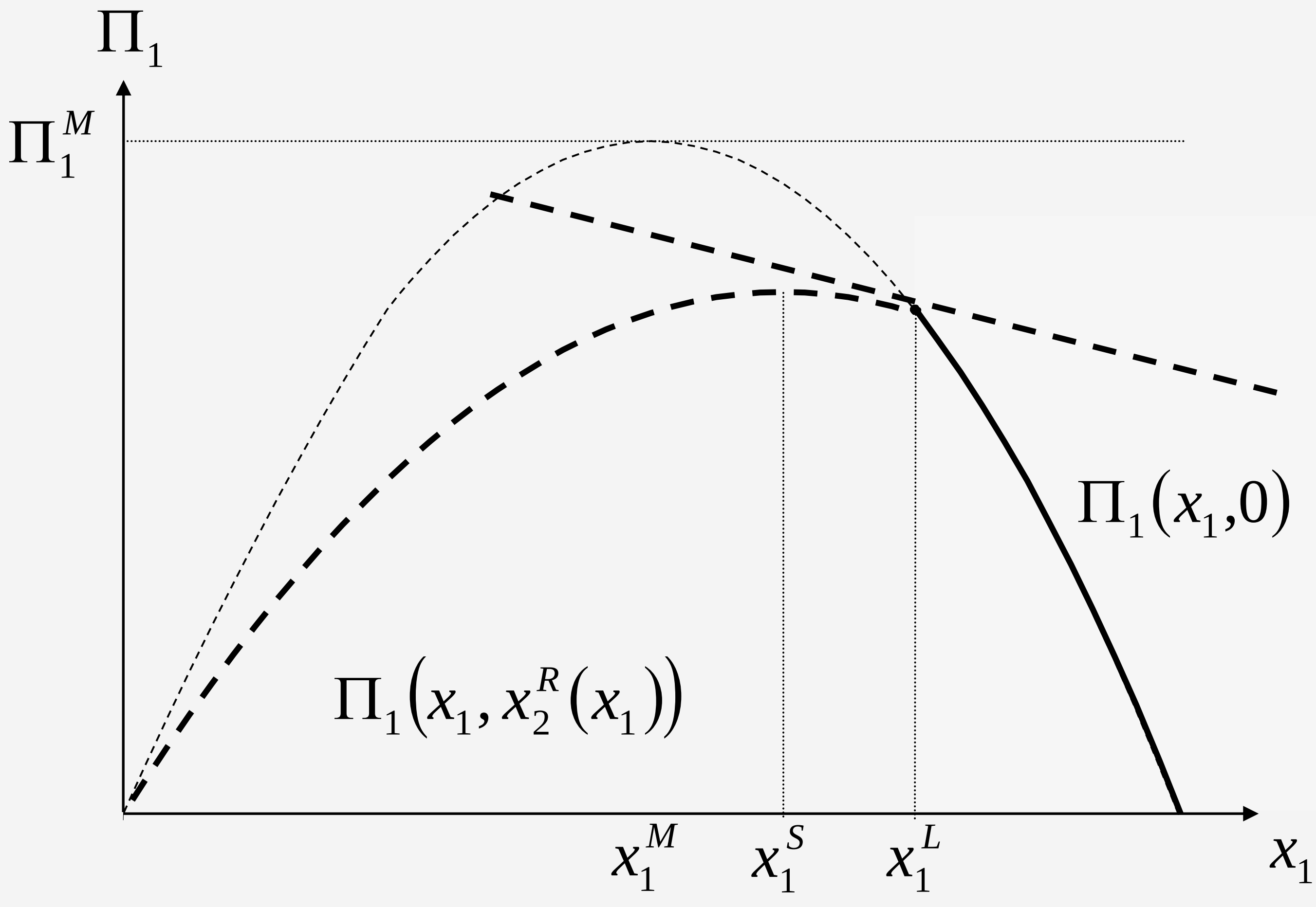
Deterring firm 2's entry



Deterrence pays, $\left.\frac{d\Pi_1\left(x_1, x_2^R(x_1)\right)}{dx_1}\right|_{x_1^L} > 0$



Deterrence does not pay $\left.\frac{d\Pi_1\left(x_1, x_2^R(x_1)\right)}{dx_1}\right|_{x_1^L} < 0$



Blockaded and deterred entry I

- Entry is blockaded for each firm:
  $$c_1 \geq a \; and \; c_2 \geq a$$
- Blockaded entry (firm 2):

$$\left[c_2 \geq p^M(c_1) \quad or \quad x_1^L \leq x_1^M\right] \quad and \quad c_1 < a$$

$$\Leftrightarrow c_2 \geq \frac{a + c_1}{2} \quad and \quad c_1 < a$$

Blockaded and deterred entry II

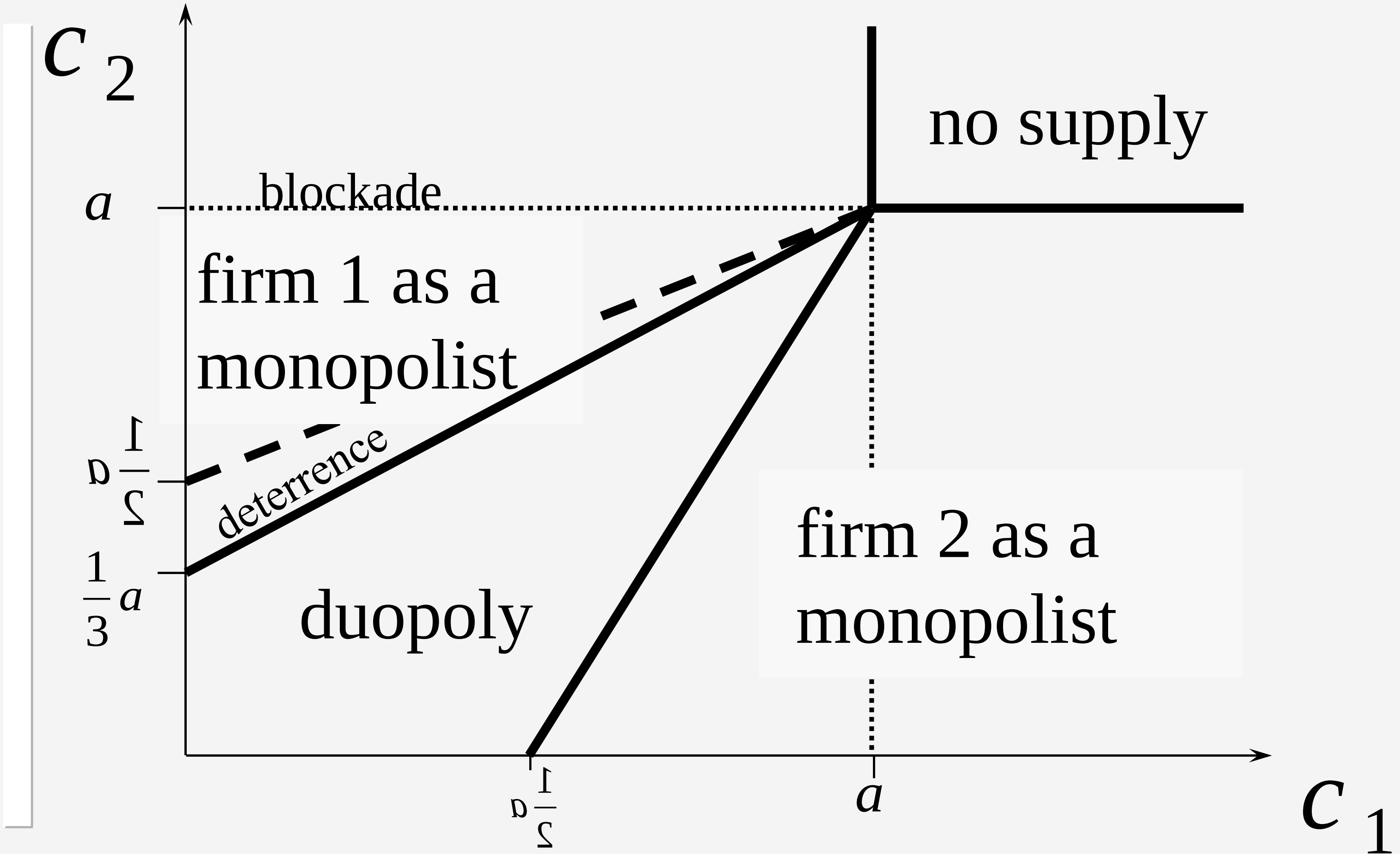
- Deterred entry (firm 2):
  - Entry is not blockaded if $c_2 < \frac{a+c_1}{2} = p_1^M$
  - Deterrence pays if

$$0 < \left.\frac{d\Pi_1\left(x_1, x_2^R(x_1)\right)}{dx_1}\right|_{x_1^L} = -bx_1^L + \frac{1}{2}a + \frac{1}{2}c_2 - c_1 = -\frac{1}{2}a + \frac{3}{2}c_2 - c_1$$

$$\Leftrightarrow \quad c_2 \geq \frac{1}{3}a + \frac{2}{3}c_1$$

- Deterrence if $\frac{1}{3}a + \frac{2}{3}c_1 \leq c_2 < \frac{a+c_1}{2}$

Blockade and deterrence



Exercise (entry and deterrence)

Suppose a monopolist faces a demand of the form $p(X)=4-0.25X$. The firm's unit costs are 2.

a) Find the profit-maximizing quantity and price. Is entry blockaded for a potential entrant (firm 2) with unit costs of 3.5?

b) How about unit costs of $c_2=1$?

c) Find firm 1's limit output level for $c_2=1$. Should the incumbent deter entry?

Deterrence and sunk costs I

We now introduce quasifix costs of 3:

p(X)=4-0.25X

Leader's cost function

$$C_1(x_1) = \begin{cases} 3 + 2x_1, & x_1 > 0 \\ 0, & x_1 = 0 \end{cases}$$

Follower's cost function

$$C_2(x_2) = \begin{cases} 3 + x_2, & x_2 > 0 \\ 0, & x_2 = 0 \end{cases}$$

Deterrence and sunk costs II

- b) Entry blockaded ?

$$x_1^M \stackrel{?}{=} 4, \qquad p^M(4) \stackrel{?}{=} 3$$

$$\Pi_1(4,0) = 1 > 0 = \Pi_1(0,0) \rightarrow \underline{x_1^M = 4} \qquad \underline{p^M(4) = 3}$$

Comparison $p\left(x_1^M\right) \begin{matrix} \geq \\ < \end{matrix} c_2$ is not sufficient

$$x_2^R(x_1) = 6 - \frac{1}{2} x_1 \quad \rightarrow \quad x_2^R\left(x_1^M\right) = 4$$

$$\Pi_2(4,4) = (4 - 0{,}25 \cdot (4+4)) \cdot 4 - (3 + 1 \cdot 4) = 1 > 0$$

$\Rightarrow$ Entry not blockaded

Deterrence and sunk costs III

- c) Should firm 1 deter?

$x_1^{Lq} = \text{limit quantity with quasifixed costs}, \ x_1^{Lq} < x_1^L$ (why?)

$$0 = \Pi_2\left(x_1^{Lq}, x_2^R\left(x_1^{Lq}\right)\right) = p(X) \cdot x_2^R\left(x_1^{Lq}\right) - C_2\left(x_2^R\left(x_1^{Lq}\right)\right)$$

$$= \left[4 - \tfrac{1}{4} \cdot \left(x_1^{Lq} + x_2^R\left(x_1^{Lq}\right)\right)\right] \cdot x_2^R\left(x_1^{Lq}\right) - \left(3 + x_2^R\left(x_1^{Lq}\right)\right)$$

$$= \left[4 - \tfrac{1}{4} \cdot \left(x_1^{Lq} + 6 - \tfrac{1}{2} x_1^{Lq}\right)\right] \cdot \left(6 - \tfrac{1}{2} x_1^{Lq}\right) - \left(3 + \left(6 - \tfrac{1}{2} x_1^{Lq}\right)\right)$$

$$= \left[\tfrac{5}{2} - \tfrac{1}{8} x_1^{Lq}\right] \cdot \left(6 - \tfrac{1}{2} x_1^{Lq}\right) - 3 - \left(6 - \tfrac{1}{2} x_1^{Lq}\right)$$

$$= 15 - \tfrac{3}{4} x_1^{Lq} - \tfrac{5}{4} x_1^{Lq} + \tfrac{1}{16} {x_1^{Lq}}^2 - 3 - 6 + \tfrac{1}{2} x_1^{Lq}$$

$$= 6 - \tfrac{3}{2} x_1^{Lq} + \tfrac{1}{16} {x_1^{Lq}}^2$$

$${x_1^{Lq}}_1 = 12 + 4\sqrt{3}, \qquad x_1^{Lq} := {x_1^{Lq}}_2 = 12 - 4\sqrt{3} \quad \left(< 12 = x_1^L\right)$$

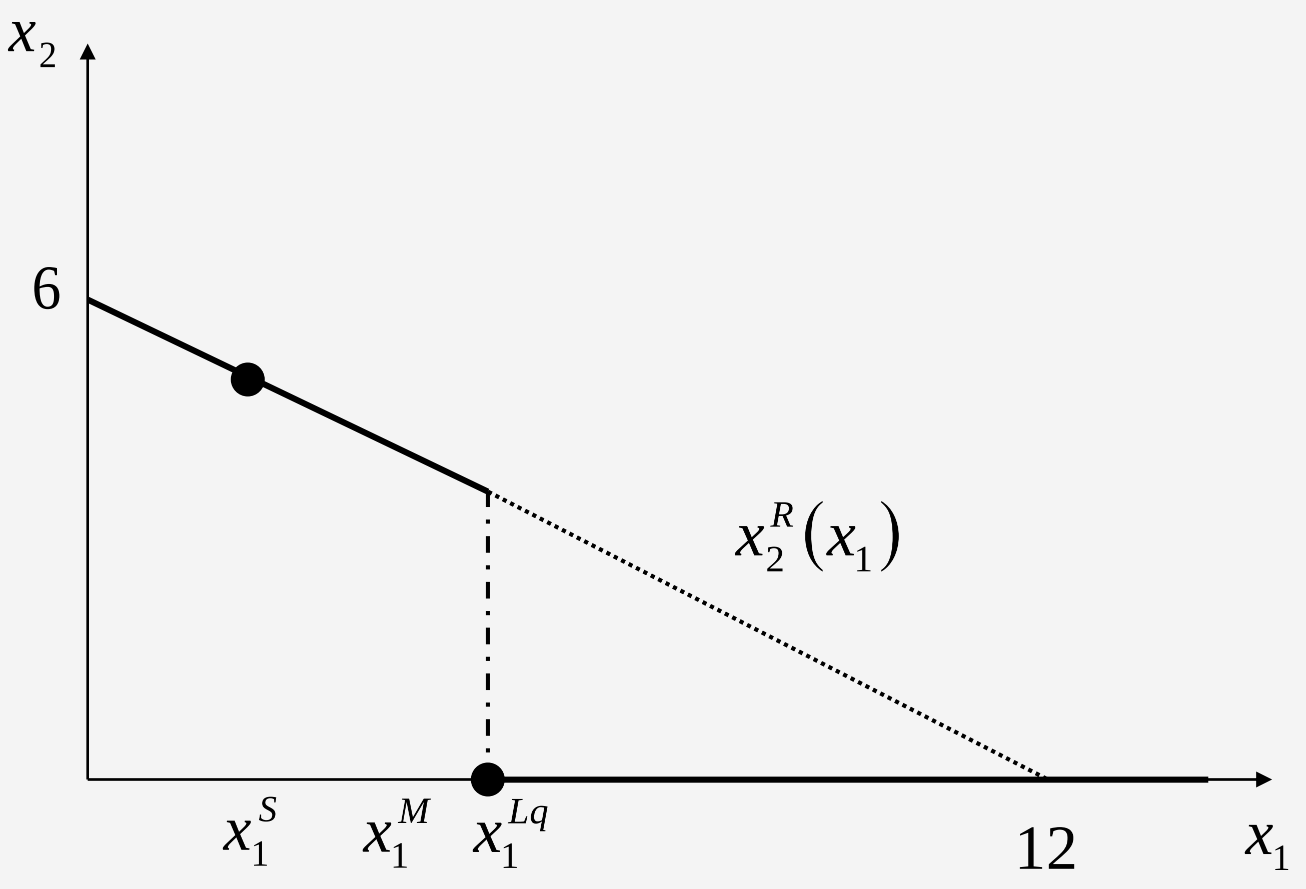
Deterrence and sunk costs IV

$$\Pi_1\left(x_1^{Lq},0\right)=\left(4-\tfrac{1}{4}\left(12-4\sqrt{3}+0\right)\right)\cdot\left(12-4\sqrt{3}+0\right)-\left(3+24-8\sqrt{3}\right)$$

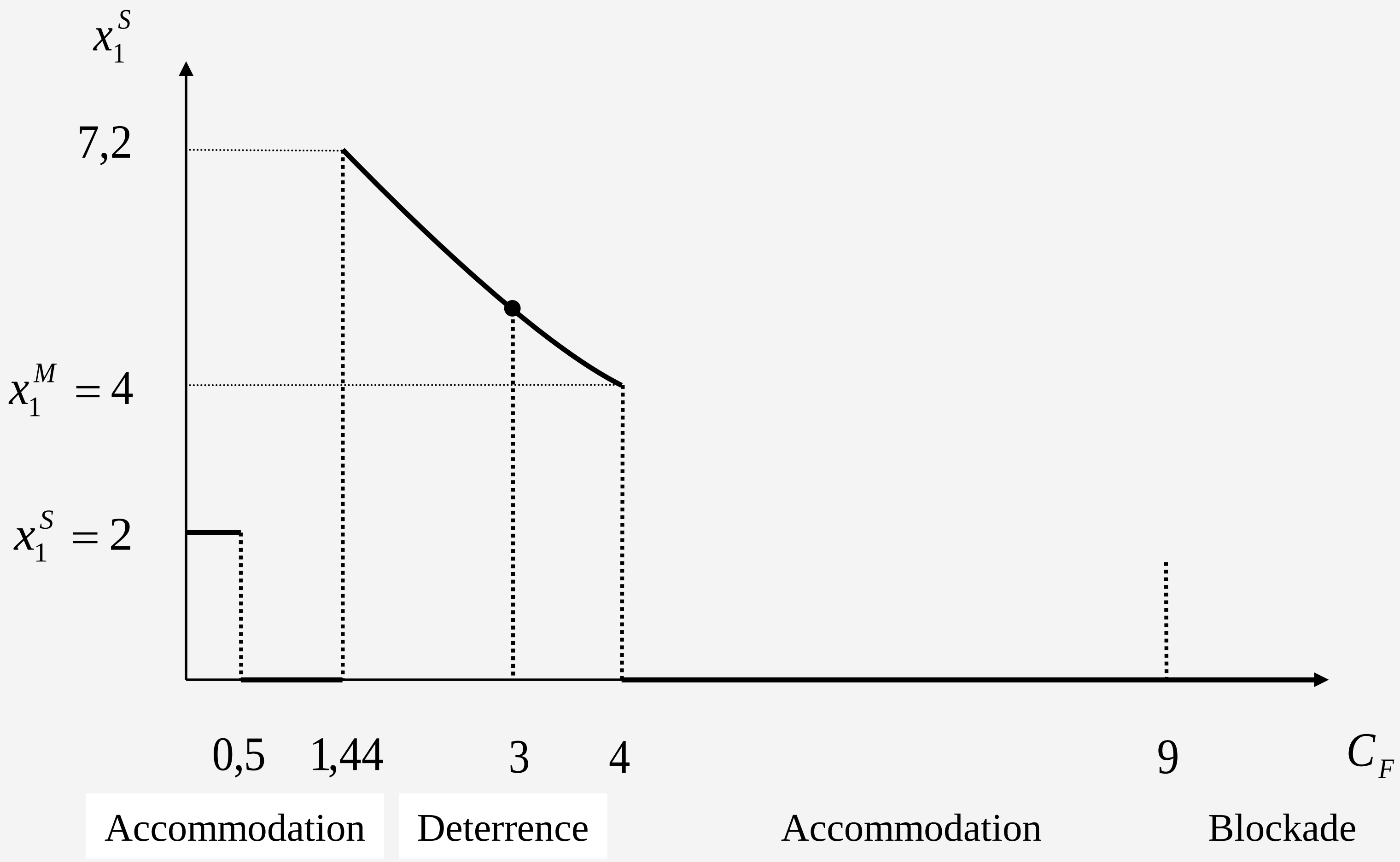
$$\approx 0{,}71>\frac{1}{2}=\Pi_1^S\left(x_1^S,x_2^R\left(x_1^S\right)\right)$$ (see exercise "entry and deterrence")

Result: deterrence pays

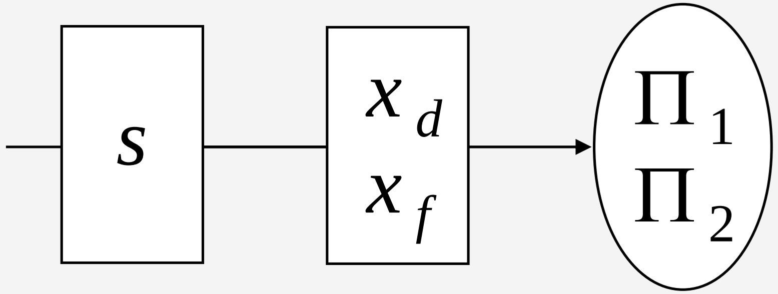
Deterrence and sunk costs V



Deterrence and sunk costs VI



Strategic trade policy



- Two firms, one domestic (*d*), the other foreign (*f*), compete on a market in a third country.
- The domestic government subsidizes its firm's exports using a unit subsidy *s*.
- The subsidy grants the domestic firm an advantage that is higher than the subsidy itself (Brander / Spencer (1981, 1983)).

Exercise (Strategic trade policy)

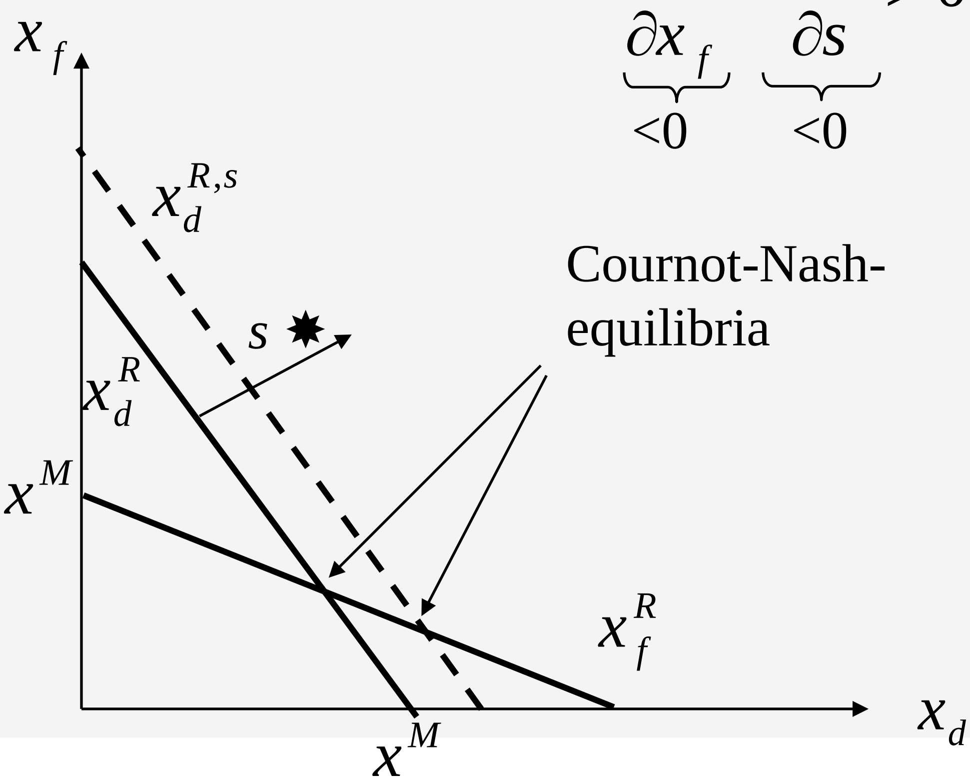
- In the setting just described, assume $c := c_1 = c_2$ and p(X)=a-bX.
- Since the two firms sell to a third country, the rent of the consumers is without relevance and domestic welfare given by

$$W(s) = \Pi_d^C(c-s,c) - sx_d^C(c-s,c)$$

- Which subsidy *s* maximizes domestic welfare?

Solution (Strategic trade policy) - interpretation

- Direct effect of subsidy for domestic welfare is zero.
- Strategic effect: $\underbrace{\frac{\partial \Pi_d}{\partial x_f}}_{<0} \cdot \underbrace{\frac{\partial x_f^C}{\partial s}}_{<0} > 0$



$$x_d^C(c-s,c) = x_d^S(c,c)!!$$

(firm *d* Stackelberg leader)

Strategic trade policy - problems

- The recommendation depends on whether there is price or quantity competition.
- „One can always do better than free trade, but the optimal tariffs or subsidies seem to be small, the potential gains tiny, and there is plenty of room for policy errors that may lead to eventual losses rather than gains.“

Trade Policy and Market Structure; Helpman/Krugman, S. 186

Stackelberg – Executive Summary

- Time leadership is worthwhile: in a Stackelberg equilibrium the leader realizes a profit that is higher
  - than the follower's and
  - his own in a Cournot equilibrium.
- Costs of entry (even in the form of identical quasifix costs) make the follower's deterrence easier.
- Strategic trade policy may conceivably pay.

Example: The OPEC Cartel I

- The best known cartel is the OPEC, which was formed in 1960 by Saudi Arabia, Venezuela, Kuwait, Iraq and Iran. Each member nation must agree to an individual output quota, except for Saudi Arabia, which adjusts its production as necessary to maintained high prices.
- In 1982, OPEC set an overall output limit of 18 million barrels per day (before 31 million).
- Production quota at 28 million barrels per day effective July 1, 2005.

The quantity cartel

- The firms seek to maximize joint profits

$$\Pi_1(x_1, x_2) + \Pi_2(x_1, x_2)$$
$$= p(X)(x_1 + x_2) - C_1(x_1) - C_2(x_2)$$

- Optimization conditions

$$\frac{\partial(\Pi_1 + \Pi_2)}{\partial x_1} = p(X) + (x_1 + x_2)\frac{dp}{dX} - MC_1(x_1) \overset{!}{=} 0$$

$$\frac{\partial(\Pi_1 + \Pi_2)}{\partial x_2} = p(X) + (x_1 + x_2)\frac{dp}{dX} - MC_2(x_2) \overset{!}{=} 0$$

- Compare monopoly with two factories.

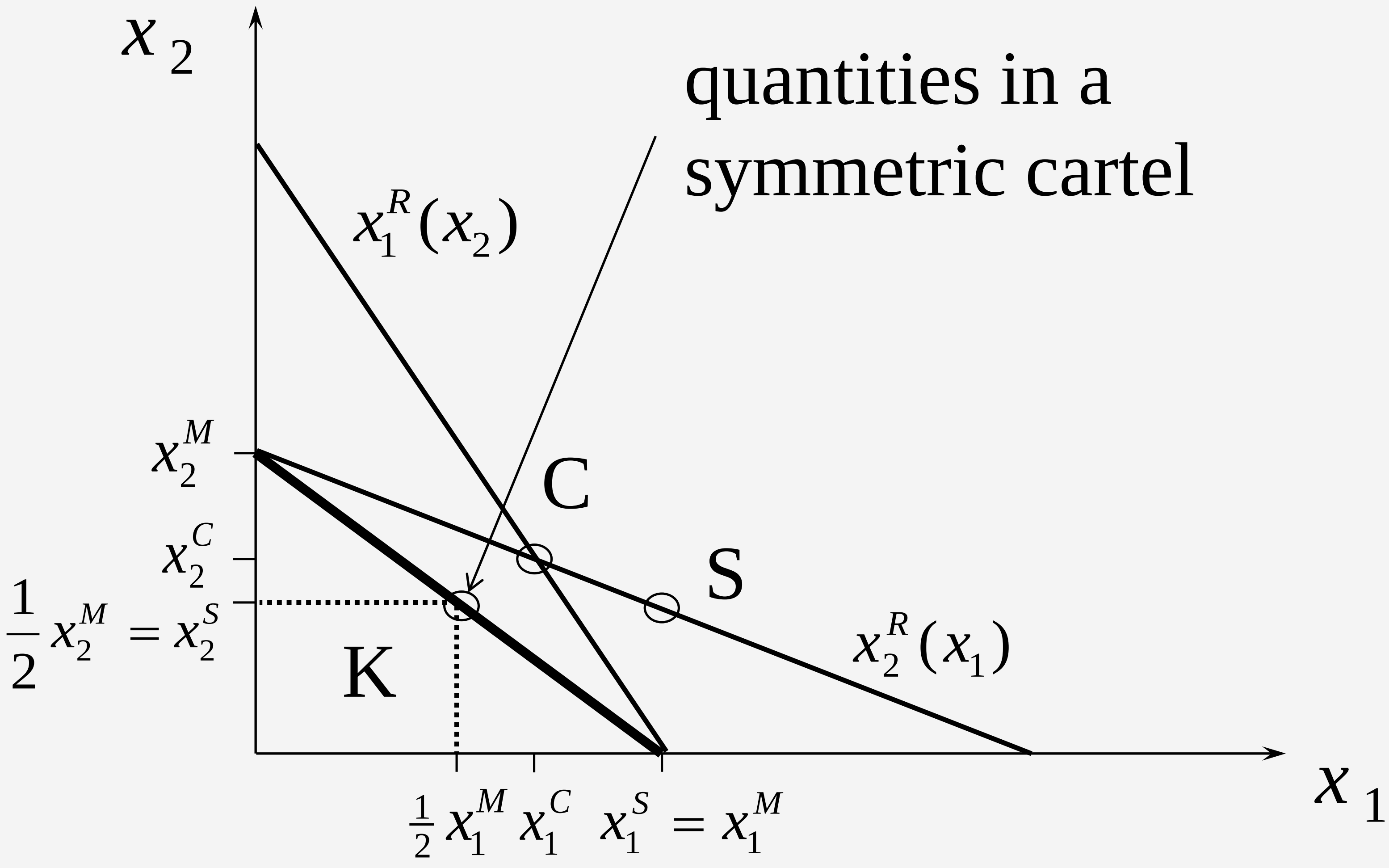
The cartel agreement

- The optimization condition is given by

$$\frac{d\Pi_1}{dx_1} = p(X) + x_1 \frac{dp}{dX} - MC_1(x_1) \overset{!}{=} - x_2 \frac{dp}{dX} > 0$$

- Each firm will be tempted to increase its profits by unilaterally expanding its output.
- In order to maintain a cartel, the firms need a way to detect and punish cheating, otherwise the temptation to cheat may break the cartel.

Cartel quantities



Exercise (cartel quantities)

- Consider a cartel in which each firm has identical and constant marginal costs. If the cartel maximizes total industry profits, what does this imply about the division of output between the firms?

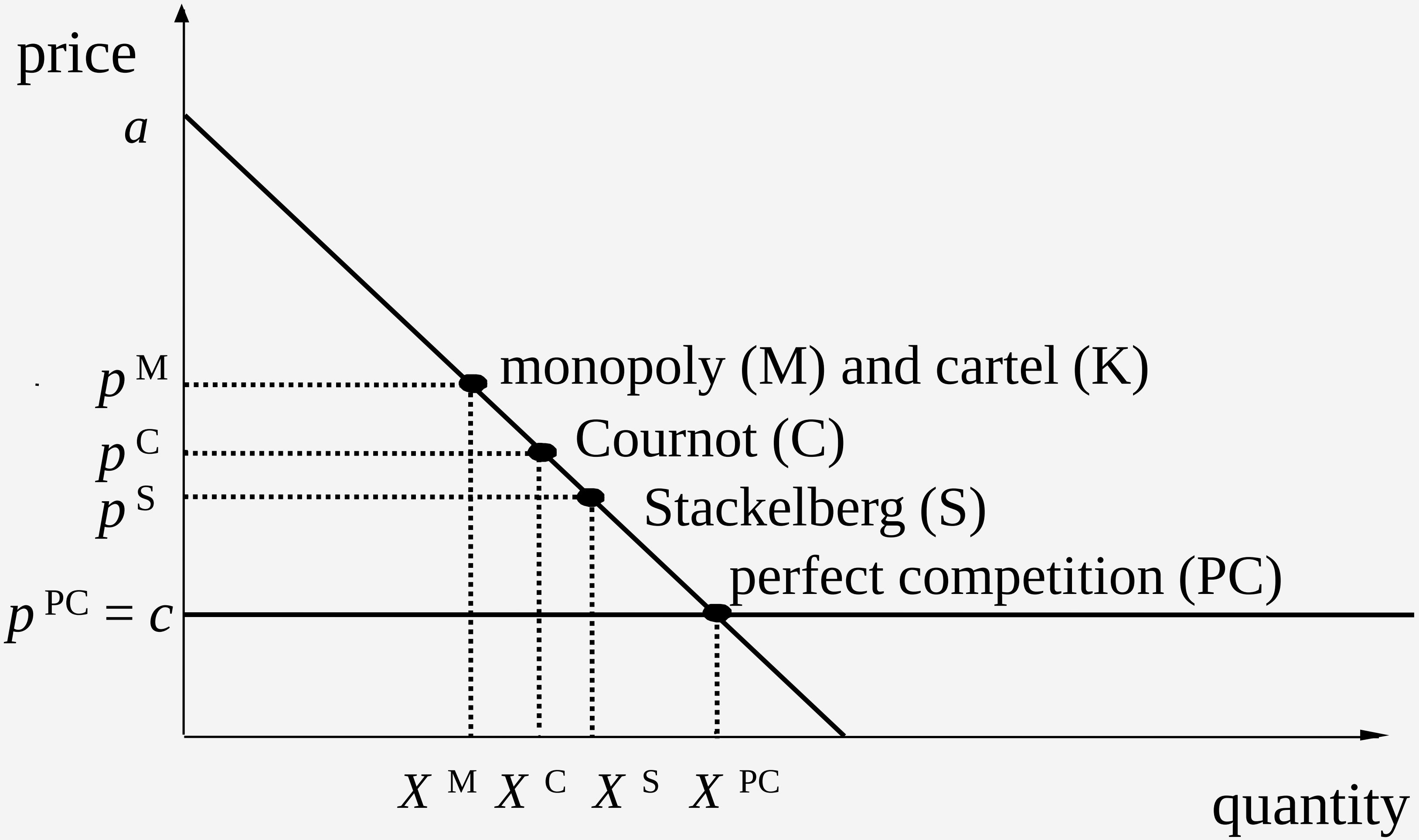
Cartel – Executive Summary

- If all firms keep the cartel agreement, they can increase their profits compared to Cournot competition.
- Nevertheless cartels are unstable from a static point of view.
- However, cartel agreements may be stable from the point of view of repeated games.

Example: The OPEC Cartel II

- 2014-2016: oversupply of oil
  - OPEC members exceed conveying limits on a regular basis
  - Slowdown of economic growth in China
  - Doubled oil production in the US (by fracking) in comparison to 2008
- Saudi Arabia is blockading claims from smaller OPEC members regarding stricter conveying limits.

The outcomes of our models



Antitrust laws and enforcement, Germany

- laws
  - Gesetz gegen unlauteren Wettbewerb (1896)
  - **Gesetz gegen Wettbewerbsbeschränkungen** (GWB), (1957)
- enforcement
  - Bundeskartellamt

C_k concentration ratio

Setup: n firms, $s_i = \frac{x_i}{X}$ and $s_1 \geq s_2 \geq \ldots \geq s_k \geq \ldots \geq s_n$

Definition: $C_k = \sum_{i=1}^{k} s_i$, for n identical firms: $C_k = \frac{k}{n}, k \leq n$

monopoly: $n = k = 1 \rightarrow \frac{k}{n} = 1$

perfect comp.: $\lim_{n \to \infty} \frac{k}{n} = 0$ for identical firms

Exercise: Calculate C_2 for

- 2 firms with equal market shares,
- 3 firms with shares of 0.1, 0.1 and 0.8 or
- 3 firms with shares of 0.2, 0.6 and 0.2 ?

GWB, §19 (3)

- One firm is called „market dominating“ if $C_1 > 1/3$.
- A group of firms is called „market dominating“ if

$$C_k \geq 1/2, \quad k \leq 3$$

or

$$C_k \geq 2/3, \quad k \leq 5.$$

The Herfindahl (Hirschman) index

- Definition:

$$H = \sum_{i=1}^{n} \left(\frac{x_i}{X} \right)^2 = \sum_{i=1}^{n} s_i^2$$

monopoly : $H = 1$

n identical firms : $H = \frac{1}{n}$

perfect competitio n $(n \to \infty) : H \to 0$

- **Exercise:** Calculate H for
  - 2 firms with equal market shares,
  - 3 firms with shares of 0.8, 0.1 and 0.1 or
  - 3 firms with shares of 0.6, 0.2 and 0.2 ?

n firms in Cournot competition

- Total industry output: $X = x_1 + x_2 + ... + x_n$
- Firm i's profit function:

$$\Pi_i(x_1,...,x_n) = p(x_1 + ... + x_n)x_i - C_i(x_i)$$

- Firm i's marginal revenue:

$$MR(x_i) = p + x_i \frac{dp}{dx_i} = p + x_i \frac{dp}{dX} \frac{d(x_1 + \cdots + x_n)}{dx_i} = p + x_i \frac{dp}{dX} \cdot 1$$

$$= p\left(1 + \frac{x_i}{X} \frac{X}{p} \frac{dp}{dX}\right) = p\left(1 - \frac{s_i}{\left|\varepsilon_{X,p}\right|}\right)$$

Lerner index of market power

- First order condition:

$$MR(x_i) = p(X) + x_i \frac{dp}{dX}\frac{dX}{dx_i} \overset{!}{=} MC(x_i)$$

- Lerner index for one firm:

$$\frac{p - MC_i}{p} \overset{!}{=} \frac{p - p\left(1 - \frac{s_i}{\left|\varepsilon_{X,p}\right|}\right)}{p} = \frac{s_i}{\left|\varepsilon_{X,p}\right|}$$

- Lerner index for the industry:

$$\sum_{i=1}^{n} s_i \frac{p - MC_i}{p} \overset{!}{=} \sum_{i=1}^{n} s_i \frac{s_i}{\left|\varepsilon_{X,p}\right|} = \frac{H}{\left|\varepsilon_{X,p}\right|}$$

Exercise (Replication)

In a homogenous good market there are *m* identical costumers and *n* identical firms. Every costumer demands the quantity *1-p* at price *p*. The cost function of firm *j* is given by $C_j\left(x_j\right) = 0{,}5x_j^2$.

a) Calculate the inverse market demand function!

b) Calculate the reaction function of firm j and the total market output $X^C = x_1^C + x_2^C + \ldots + x_n^C$ and p^C in the symmetric Cournot-equilibrium! Hint: Use $X_{-j} = x_1 + \ldots + x_{j-1} + x_{j+1} + .. + x_n$

c) Now the number of firms and costumers is multiplied by λ. Calculate again p^C and MC_j! Prove that for $\lambda \to \infty$ the gap between price and marginal costs converges to zero!