Course outline I

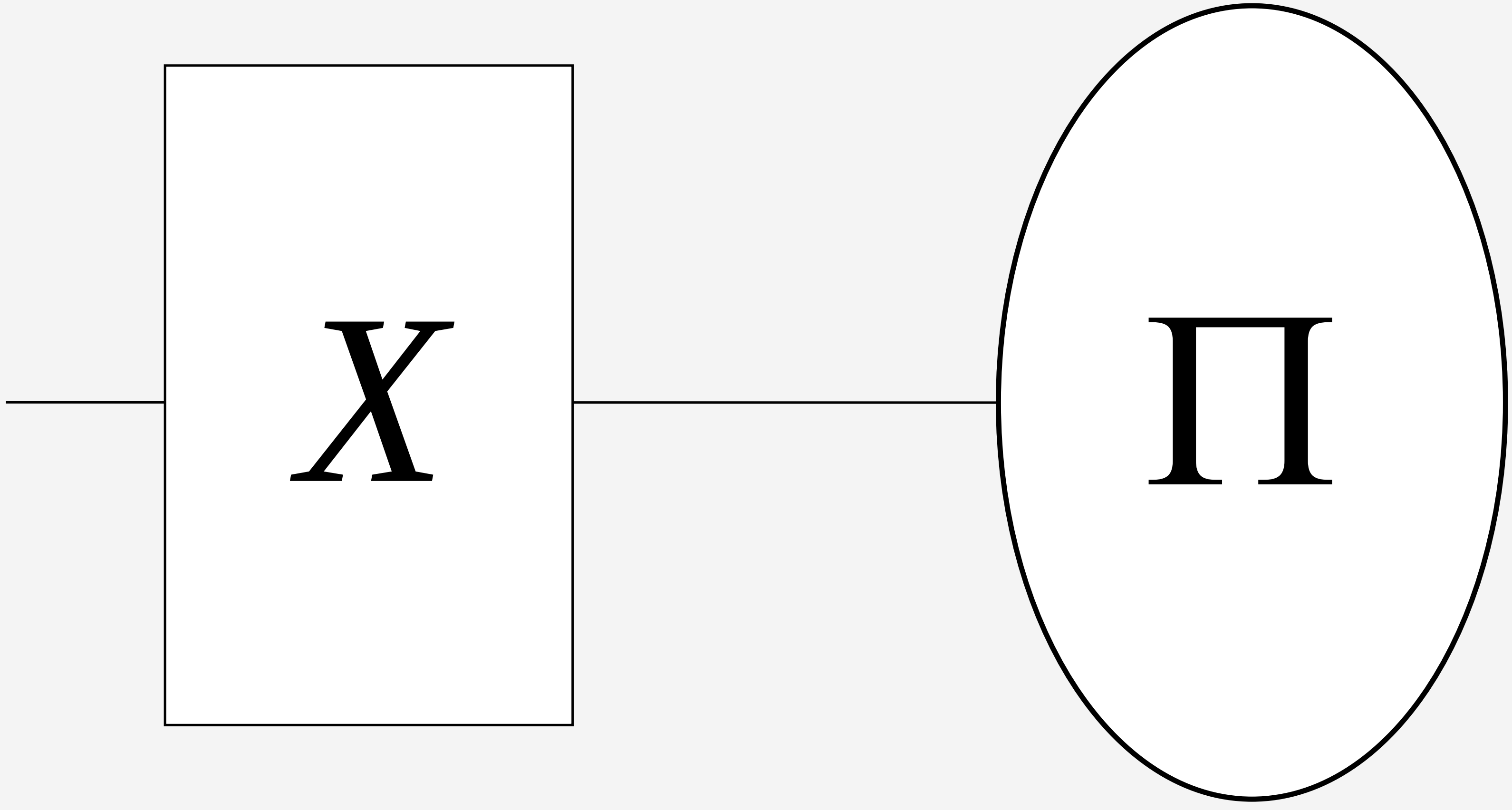
- Introduction
- Game theory
- Price setting
  - monopoly
  - oligopoly
- Quantity setting
  - monopoly
  - oligopoly
- Process innovation

Homogeneous goods

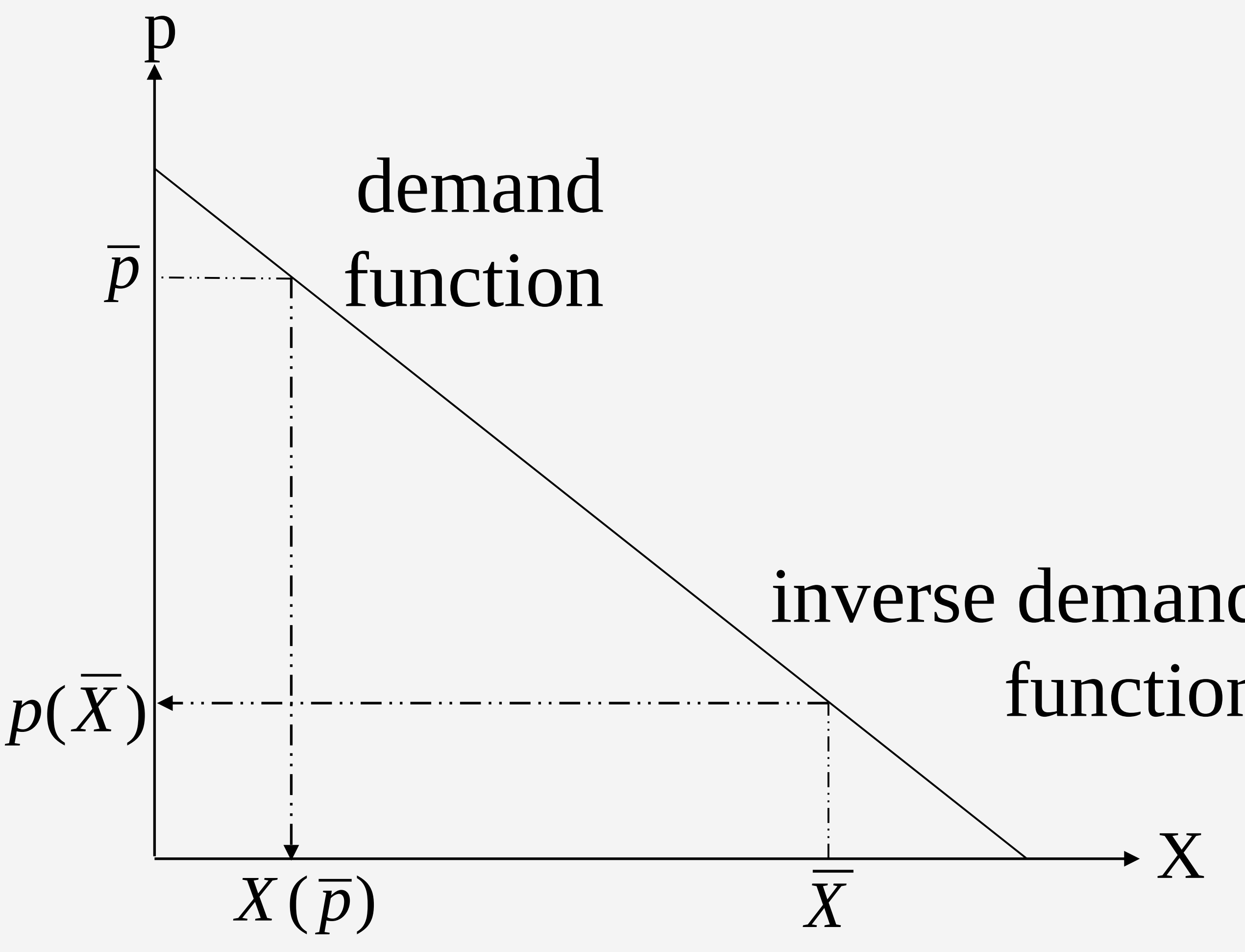
Monopoly (quantity setting)

- Inverse demand function
- Revenues, costs, profits
- Profit maximizing quantity
  - Basic model
  - Price discrimination
  - Several factories
- Double marginalization
- Welfare analysis
- Executive summary

Decision problem



Inverse demand function



Revenue, costs and profit

- Revenue: $R(X) = p(X) \cdot X$
- Costs: $C(X)$
- Profit: $\Pi(X) = R(X) - C(X)$

Marginal revenue with respect to quantity

$$\frac{dR}{dX} = MR_X = p(X) + \frac{dp(X)}{dX} X$$

When a firm increases the quantity by one unit, revenue

- goes up by p (the price of the last unit),
- but goes down by dp/dX X (the quantity increase diminishes the price and this price decrease is applied to all units)

Amoroso-Robinson relation:

$$MR_X = p(X)\left(1 + \frac{X}{p(X)}\frac{dp(X)}{dX}\right)$$

$$= p \cdot \left(1 + \frac{1}{\varepsilon_{X,p}}\right) = p \cdot \left(1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right)$$

Marginal revenue = price?

$$\frac{dR}{dX} = p(X) + \frac{dp(X)}{dX} X$$

1. $\frac{dp(X)}{dX} = 0 \Rightarrow$ i.e. horizontal demand curve, perfect competition

2. X=0 $\Rightarrow$ sale of first unit
   $\Rightarrow$ first degree price discrimination

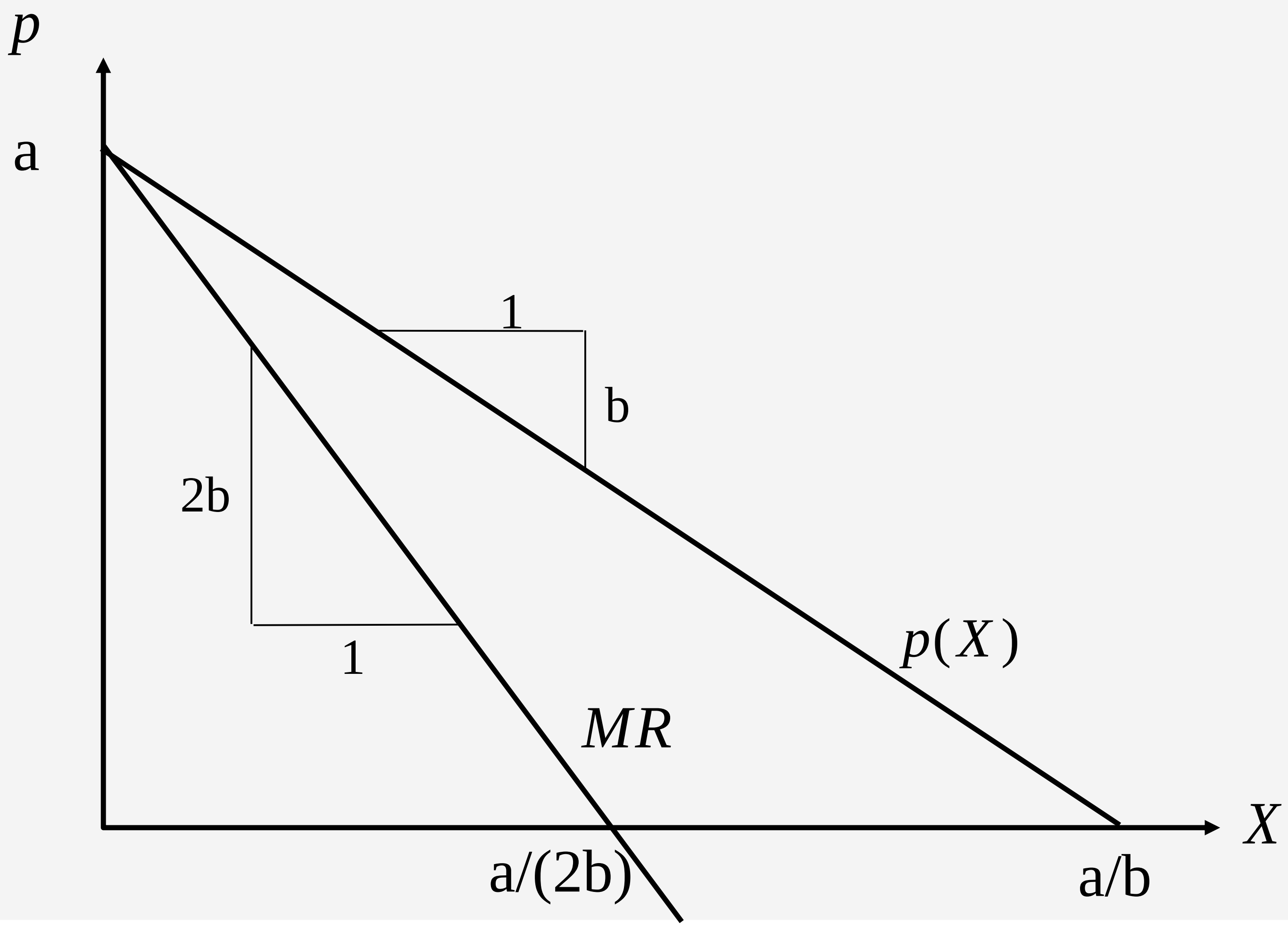
Linear demand curve in a monopoly

- Demand: $p(X) = a - bX$
- Revenue: $R(X) = aX - bX^2$
- Marginal revenue: $MR = a - 2bX$

Exercise (Depicting the linear demand curve $p(X) = a - bX$)

- Slope of demand curve: ....
- Slope of marginal revenue curve: ....
- The .... has the same vertical intercept, ..., as the demand curve.
- Economically,
  - the vertical intercept is ....,
  - the horizontal intercept is ....

Depicting demand and marginal revenue



First order condition

$$\frac{d\Pi(X)}{dX} = \underbrace{p(X) + X\frac{dp(X)}{dX}}_{MR_X} - \underbrace{\frac{dC(X)}{dX}}_{MC_X} \overset{!}{=} 0$$

- Notation: $MR := MR_X$

  $MC := MC_X$

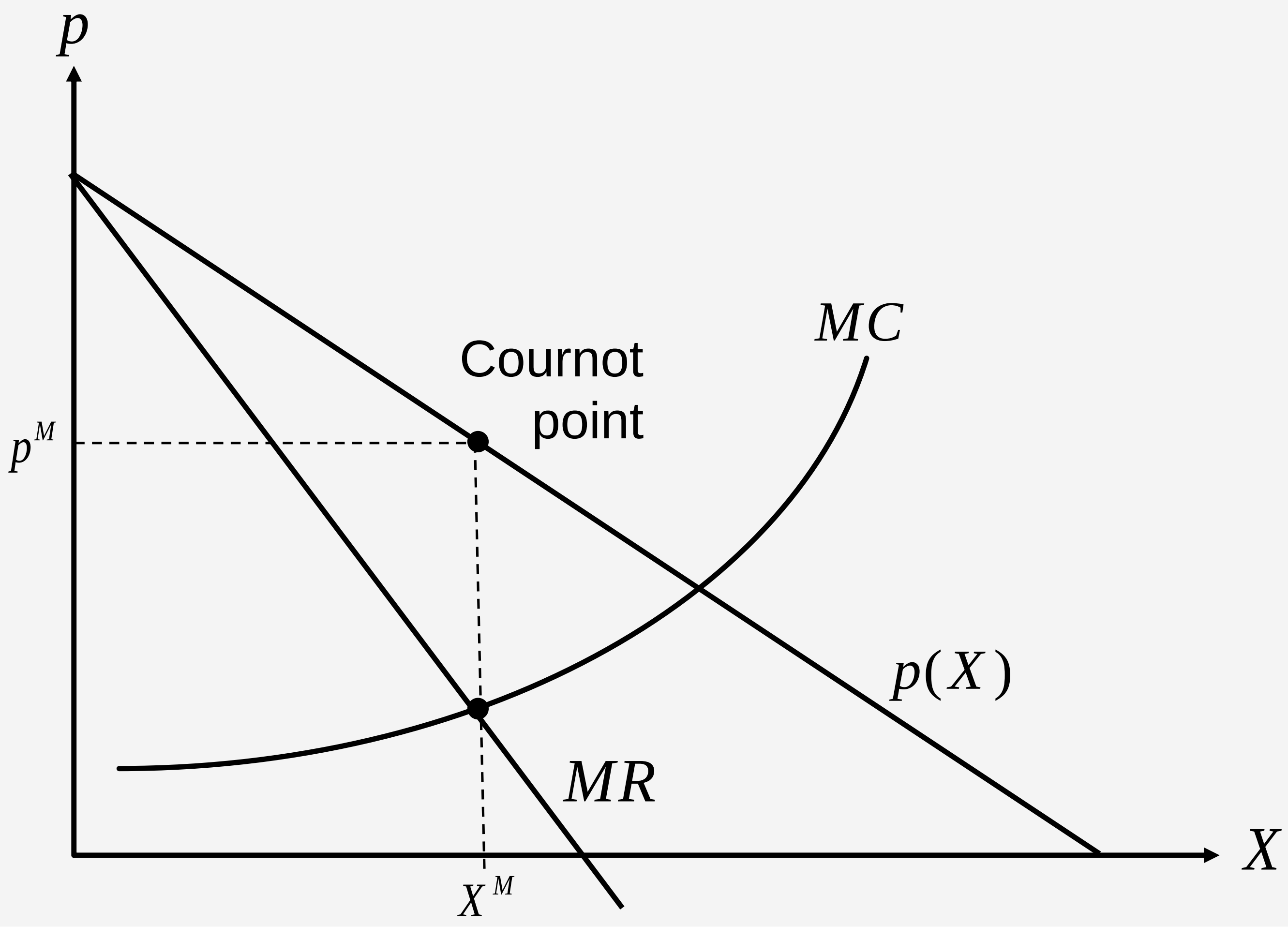
First order condition, alternative formulations

$$MC \overset{!}{=} MR = p \cdot \left(1 - \frac{1}{\left| \varepsilon_{X,p} \right|} \right)$$

$$p \overset{!}{=} \frac{1}{1 - \frac{1}{\left| \varepsilon_{X,p} \right|}} MC = \frac{\left| \varepsilon_{X,p} \right|}{\left| \varepsilon_{X,p} \right| - 1} MC$$

$$\frac{p - MC}{p} \overset{!}{=} \frac{1}{\left| \varepsilon_{X,p} \right|}$$ (price-cost margin)

Depicting the Cournot monopoly



Exercise (Quantity)

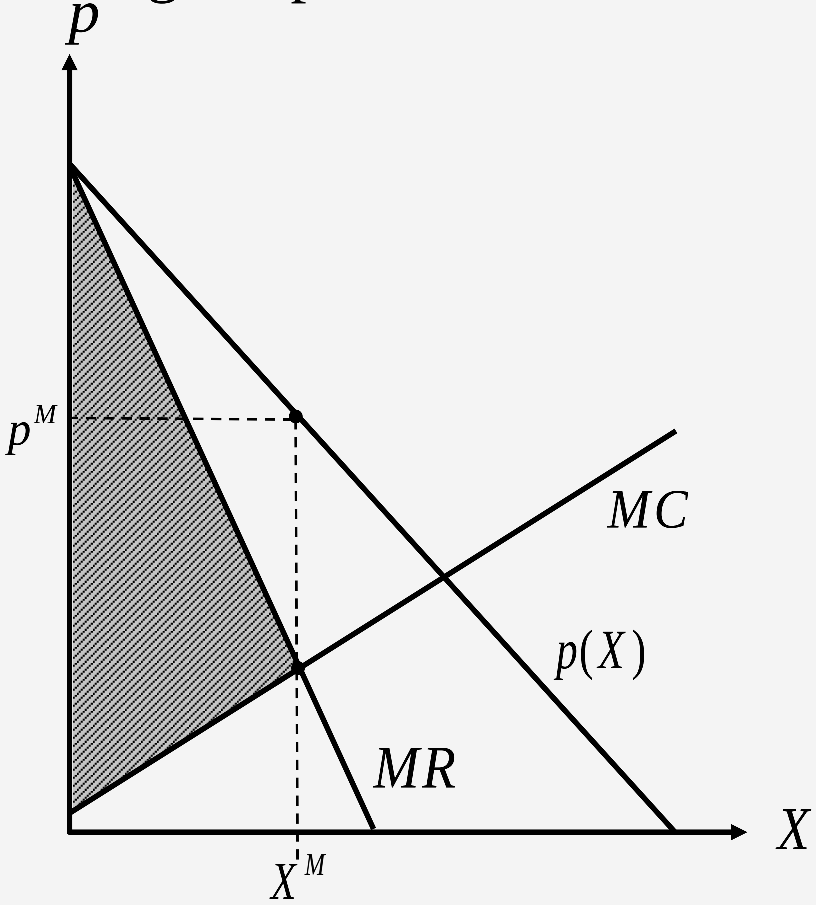
Consider a monopolist facing the inverse demand function $p(X)=24-X$.

Assume that the average and marginal costs are given by $AC=2$.

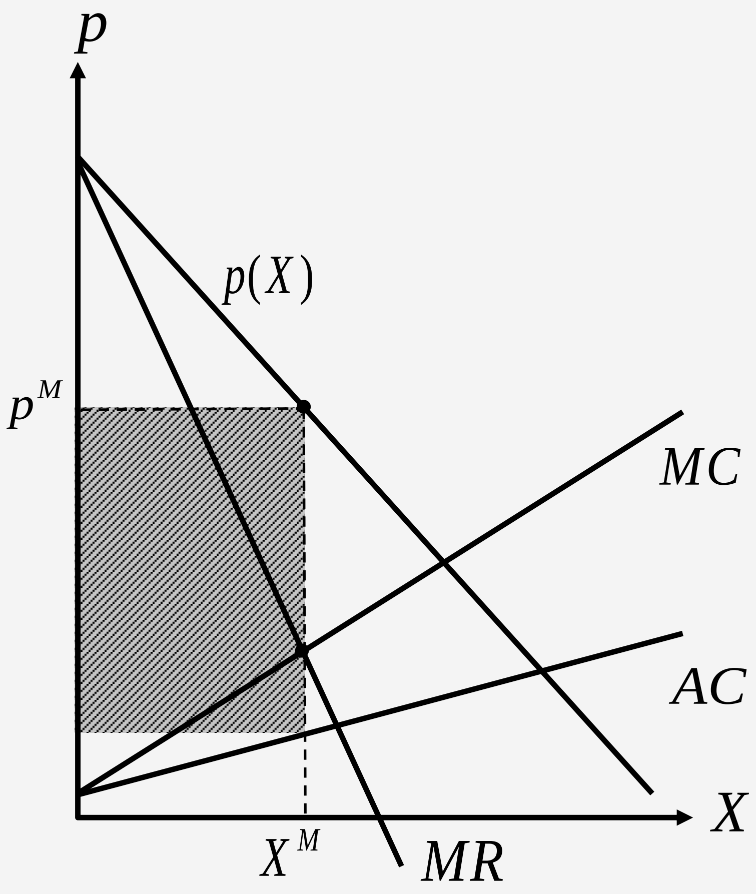
Find the profit-maximizing quantity!

Profit in a monopoly

Marginal point of view:



Average point of view:



Exercise (monopoly)

Consider a monopoly facing the inverse demand function $p(X)=40-X^2$.

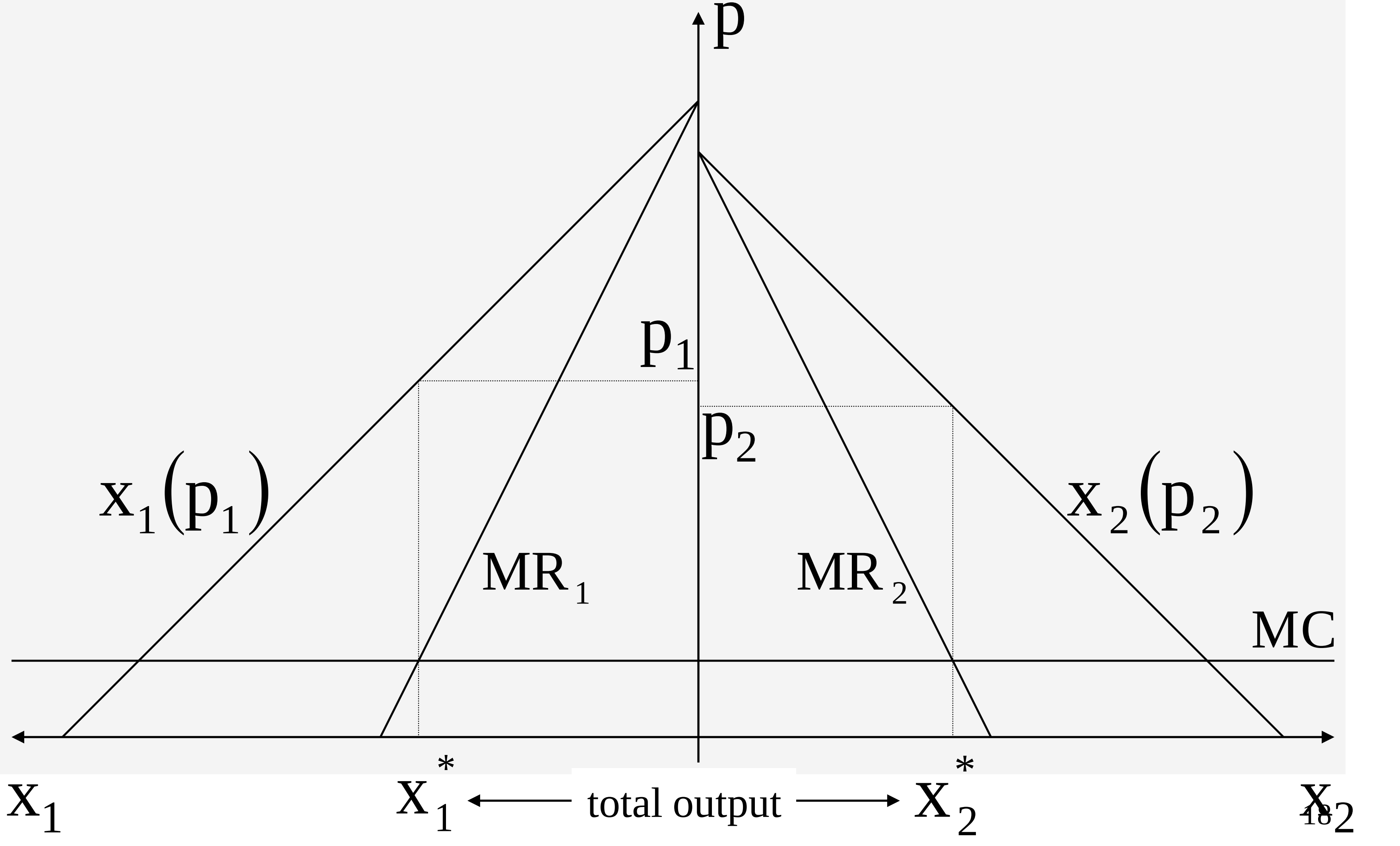
Assume that the cost function is given by $C(X)=13X+19$.

Find the profit-maximizing price and calculate the profit.

Price discrimination

- First degree price discrimination:
  Every consumer pays a different price which is equal to his or her willingness to pay.
- Second degree price discrimination:
  Prices differ according to the quantity demanded and sold (quantity rebate).
- Third degree price discrimination:
  Consumer groups (students, children, ...) are treated differently.

Monopolistic price discrimination (two markets)



Inverse elasticities rule for third degree price discrimination

Supplying a good X to two markets results in the inverse demand functions $p_1(x_1)$ and $p_2(x_2)$.

Profit function: $$\Pi(x_1,x_2) = p_1(x_1)\cdot x_1 + p_2(x_2)\cdot x_2 - C(x_1+x_2)$$

First order conditions:

$$\frac{\partial\Pi(x_1,x_2)}{\partial x_1} = MR_1(x_1) - MC(x_1+x_2) \overset{!}{=} 0$$

$$||$$

$$\frac{\partial\Pi(x_1,x_2)}{\partial x_2} = MR_2(x_2) - MC(x_1+x_2) \overset{!}{=} 0$$

Equating the marginal revenues (using the Amoroso-Robinson relation) leads to:

$$p_1(x_1)\cdot\left(1-\frac{1}{|\varepsilon_1(x_1)|}\right) \overset{!}{=} p_2(x_2)\cdot\left(1-\frac{1}{|\varepsilon_2(x_2)|}\right)$$

$$|\varepsilon_1(x_1)| < |\varepsilon_2(x_2)| \Rightarrow p_1(x_1) > p_2(x_2)$$

Exercise (two markets or one)

A monopoly sells in two markets:

$p_1(x_1)=100-x_1$ and $p_2(x_2)=80-x_2$.

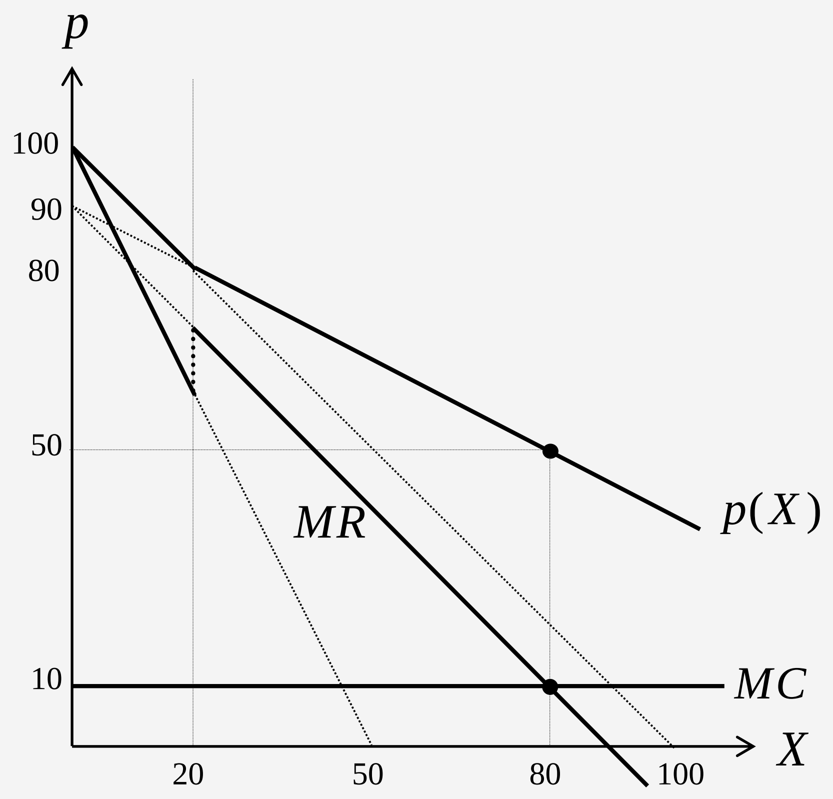
a) Calculate the profit-maximizing quantities and the profit at these quantities, if the cost function is given by $C(X)=X^2$.

b) Calculate the profit-maximizing quantities and the profit at these quantities, if the cost function is given by $C(X)=10X$.

c) What happens if price discrimination between the two markets is not possible anymore? Consider $C(X)=10X$.

Hint: Differentiate between quantities below and above 20.

Solution III (one market)



One market, two factories

- Profit function:

$$\Pi(x_1, x_2) = p(x_1, x_2)(x_1 + x_2) - C_1(x_1) - C_2(x_2)$$

- First order conditions:

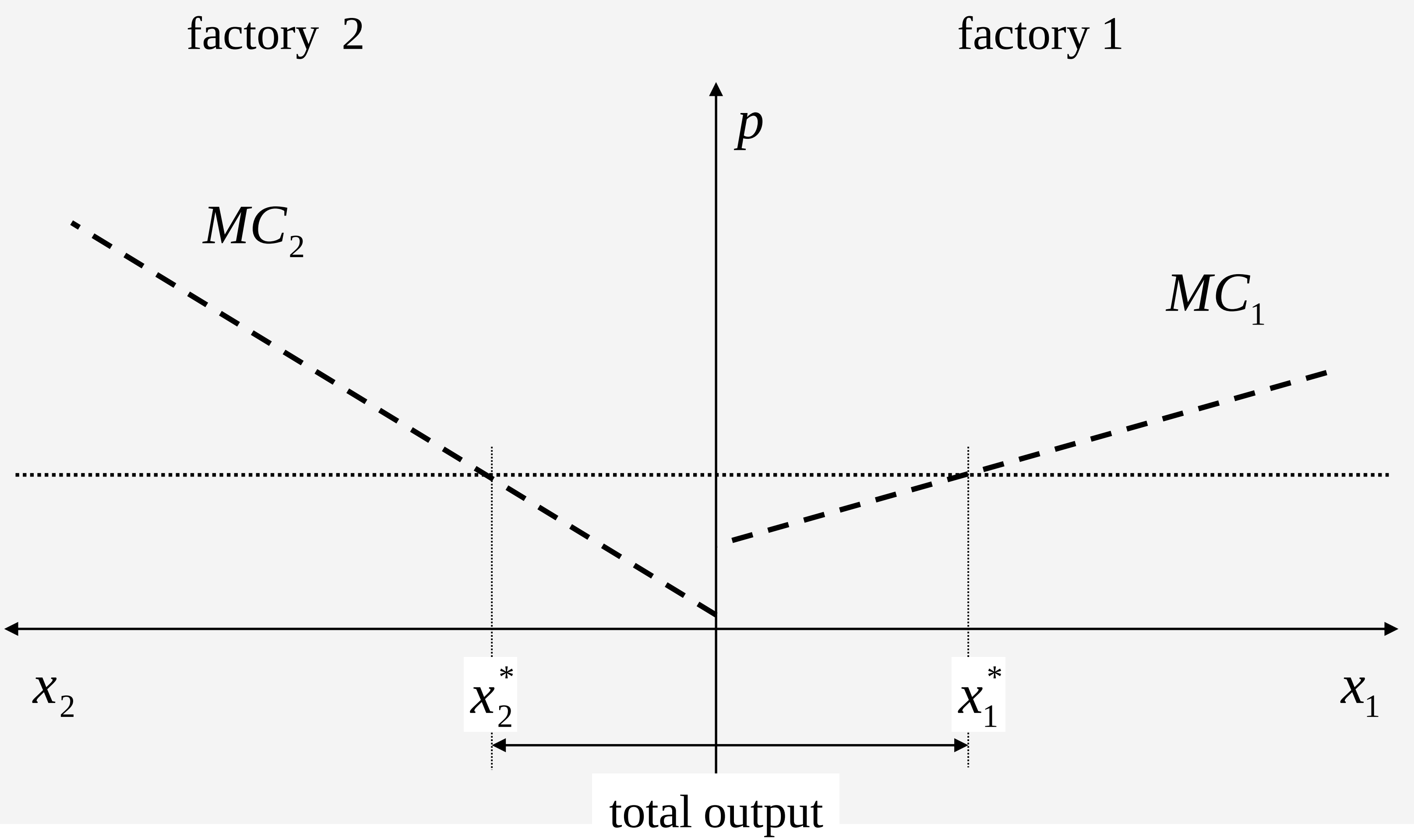
$$\frac{\partial \Pi(x_1, x_2)}{\partial x_1} = MR(x_1, x_2) - MC_1(x_1) \overset{!}{=} 0$$

$$||$$

$$\frac{\partial \Pi(x_1, x_2)}{\partial x_2} = MR(x_1, x_2) - MC_2(x_2) \overset{!}{=} 0$$

$$\Rightarrow \qquad MC_1(x_1) \overset{!}{=} MC_2(x_2)$$

One market, two factories II

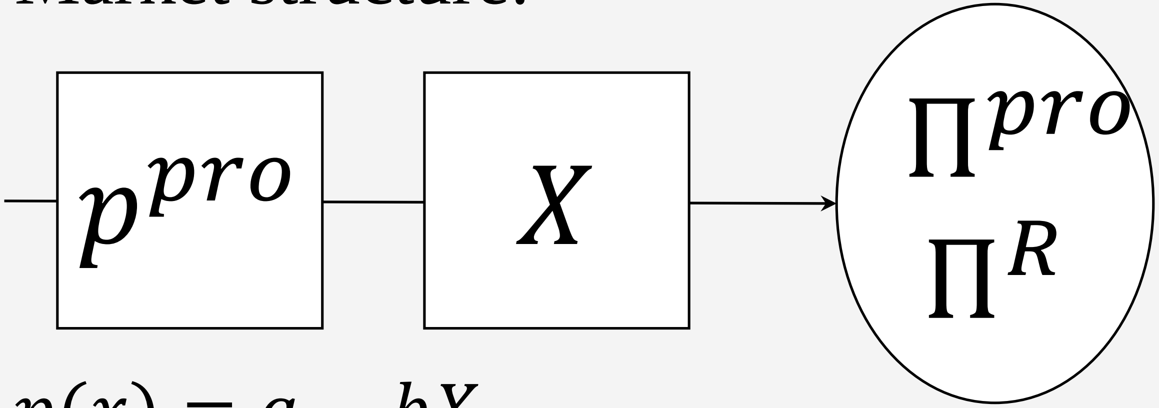


Double marginalization - idea

- Retailer, not producer sells to consumers.
- Assumptions:
  - Zero costs for retailing.
  - Producer decides on a quantity and charges a price p^{pro} to the retailer.
  - p^{pro} is the retailer‘s marginal cost.
  - The retailer‘s MR=MC condition defines the producer‘s demand function.

Double marginalization - Linear case I

- Market structure:



- $p(x) = a - bX$
- $MC^{pro} = AC^{pro} = c$

- Retailer (second stage):

$$MC = p_{pro} \overset{!}{=} a - 2bX = MR$$

Double marginalization – linear case II

- Producer (first stage):

$$p^{pro}(X) = a - 2bX$$

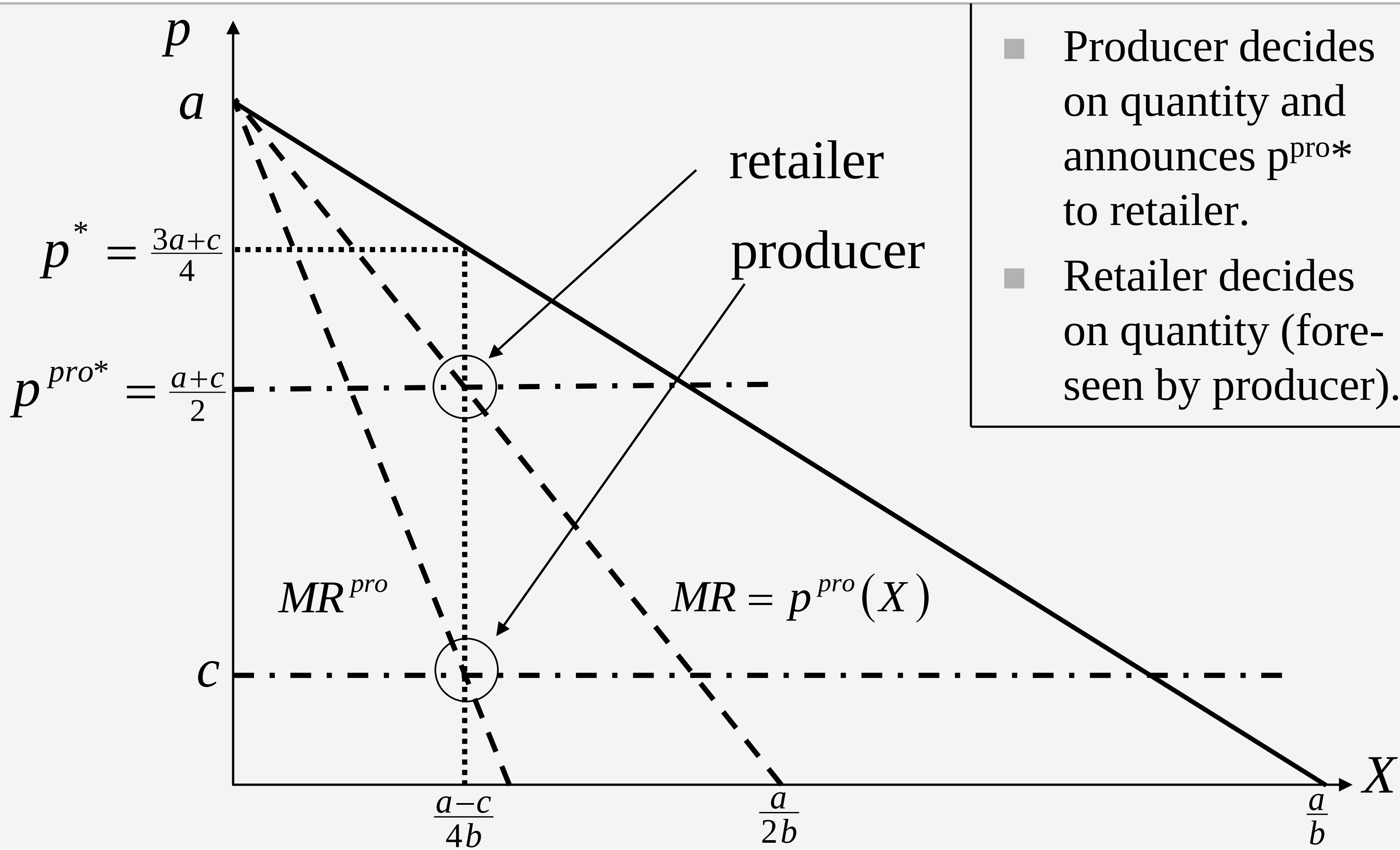
$$MC^{pro} = c \overset{!}{=} a - 4bX = MR^{pro}$$

- Results:

$$X = \frac{a-c}{4b} \quad \Rightarrow \quad p^{pro*} = \frac{a+c}{2}$$

$$\Rightarrow \quad p^{*} = \frac{3a+c}{4}$$

Double marginalization - depicting the solution



- Producer decides on quantity and announces p^{pro*} to retailer.
- Retailer decides on quantity (fore-seen by producer).

Double marginalization - exercise

$p(X) = 110 - X$

$c = 10$

a) Calculate the price the consumers have to pay!

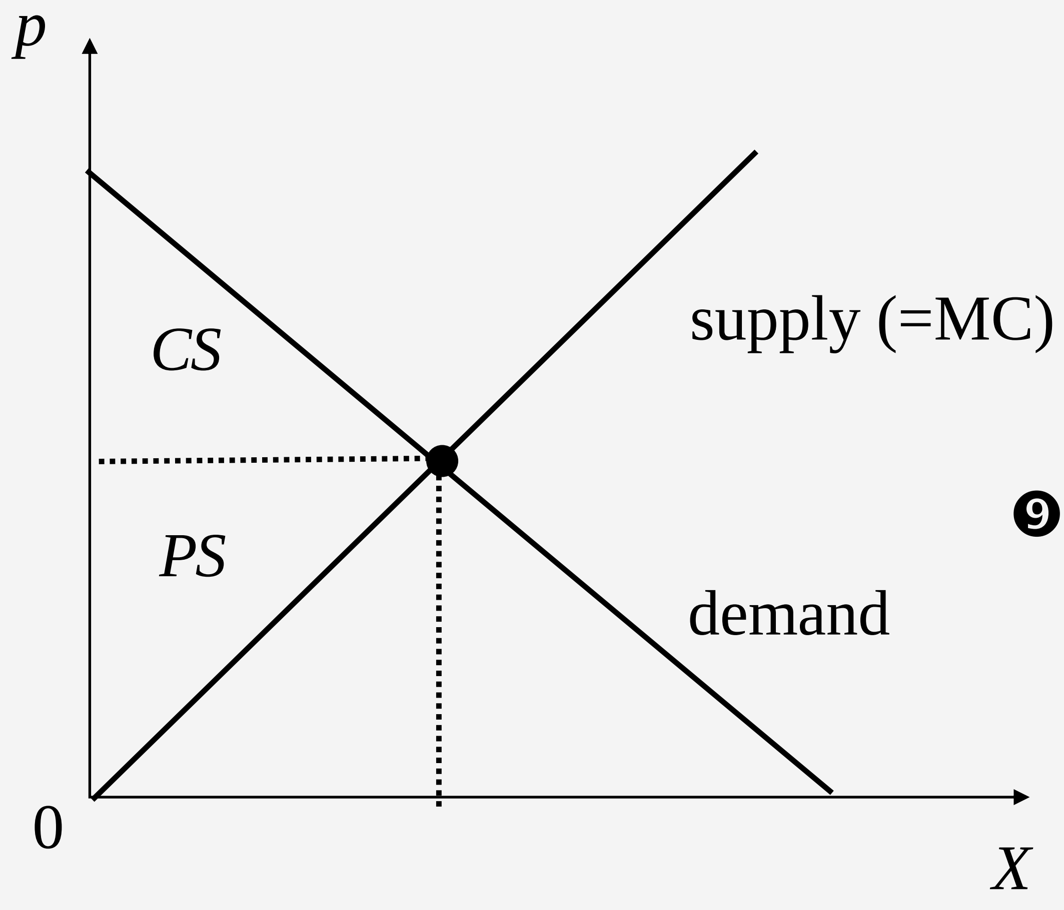
b) What price would they pay if the producer sold directly to the consumers?

Welfare Analysis

- Evaluation of economic policy measures
- Welfare = consumer surplus (CS)
  + producer surplus (PS)
  + taxes - subsidies
- CS = willingness to pay - price
- PS = revenue - variable costs

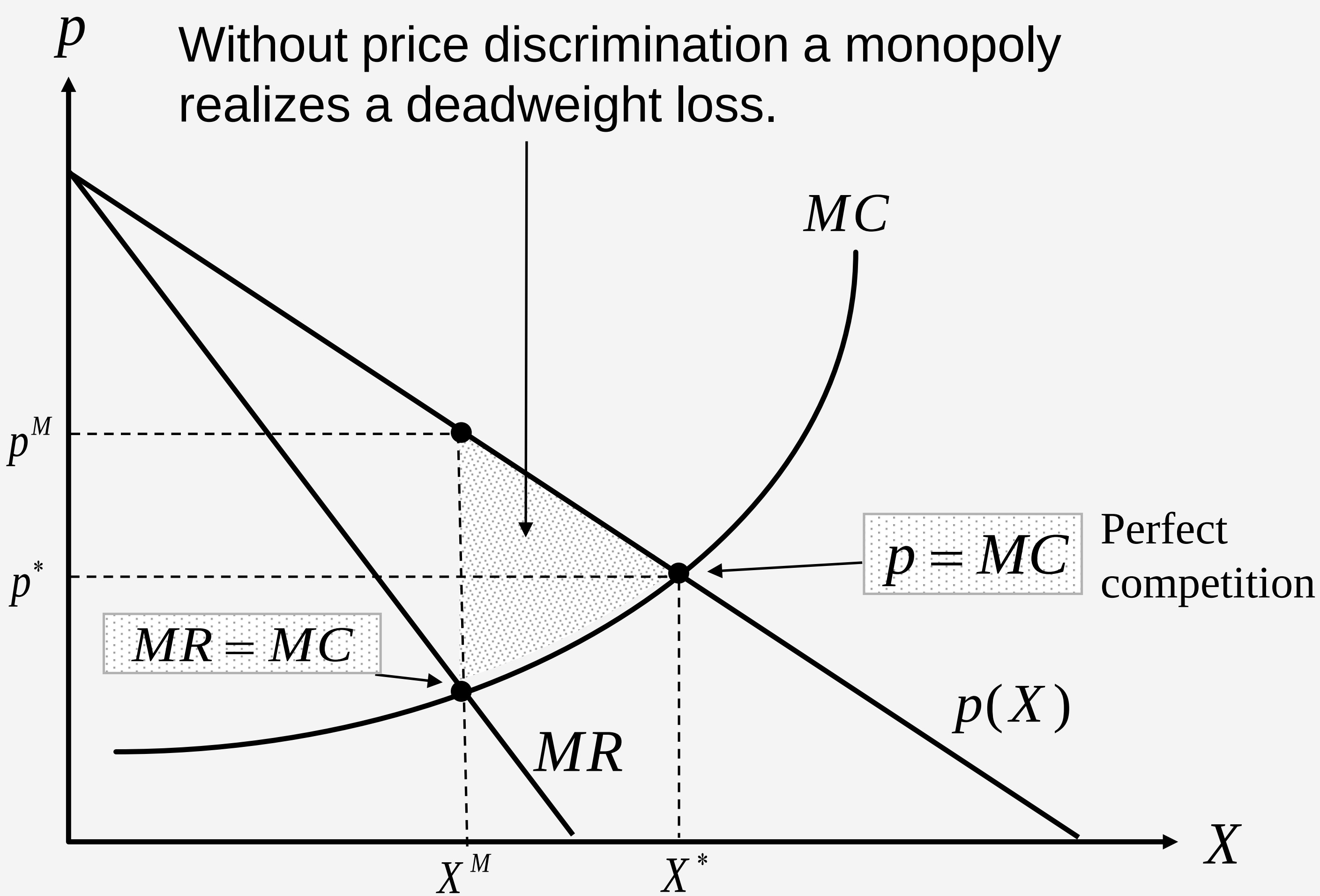
  = profit + fixed costs

CS, PS - graphically



❾ Welfare is maximized at the equilibrium.

The deadweight loss of a monopoly

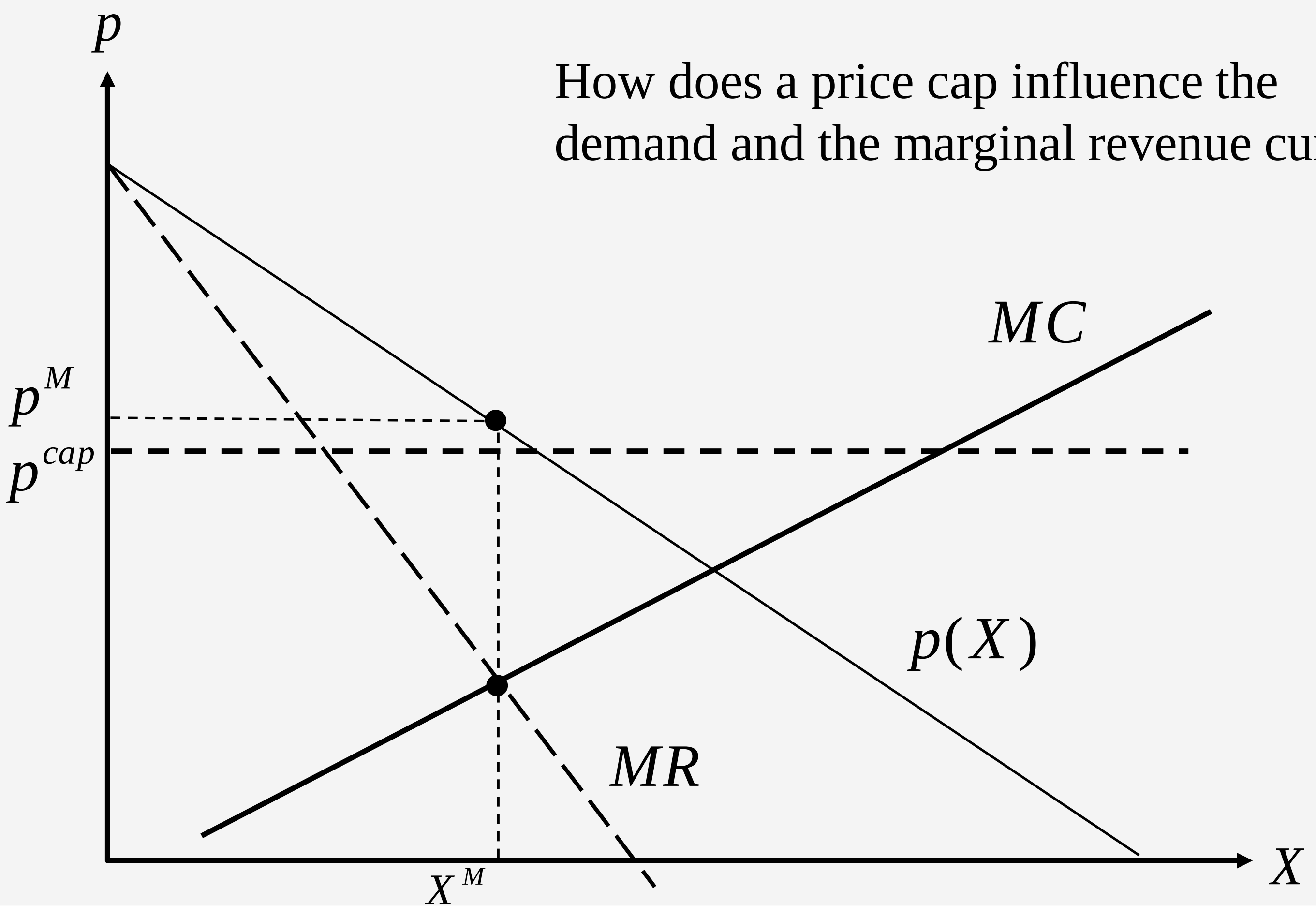


Exercise (deadweight loss)

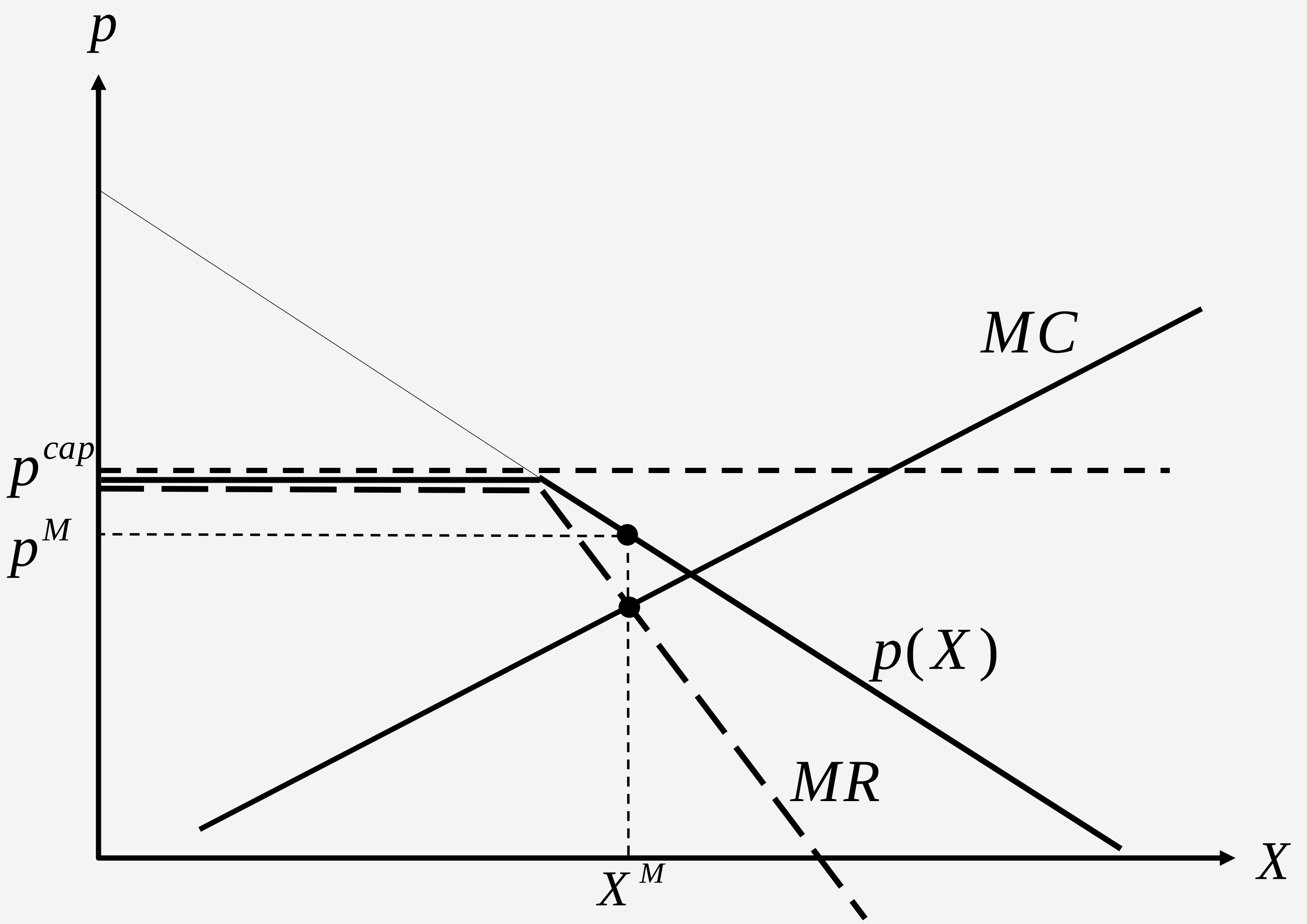
- Consider a monopoly where the demand is given by $p(X)=-2X+12$. Suppose that the marginal costs are given by $MC=2X$.
- Calculate the deadweight loss
  - without price discrimination,
  - with perfect price discrimination.

Exercise (price cap in a monopoly)

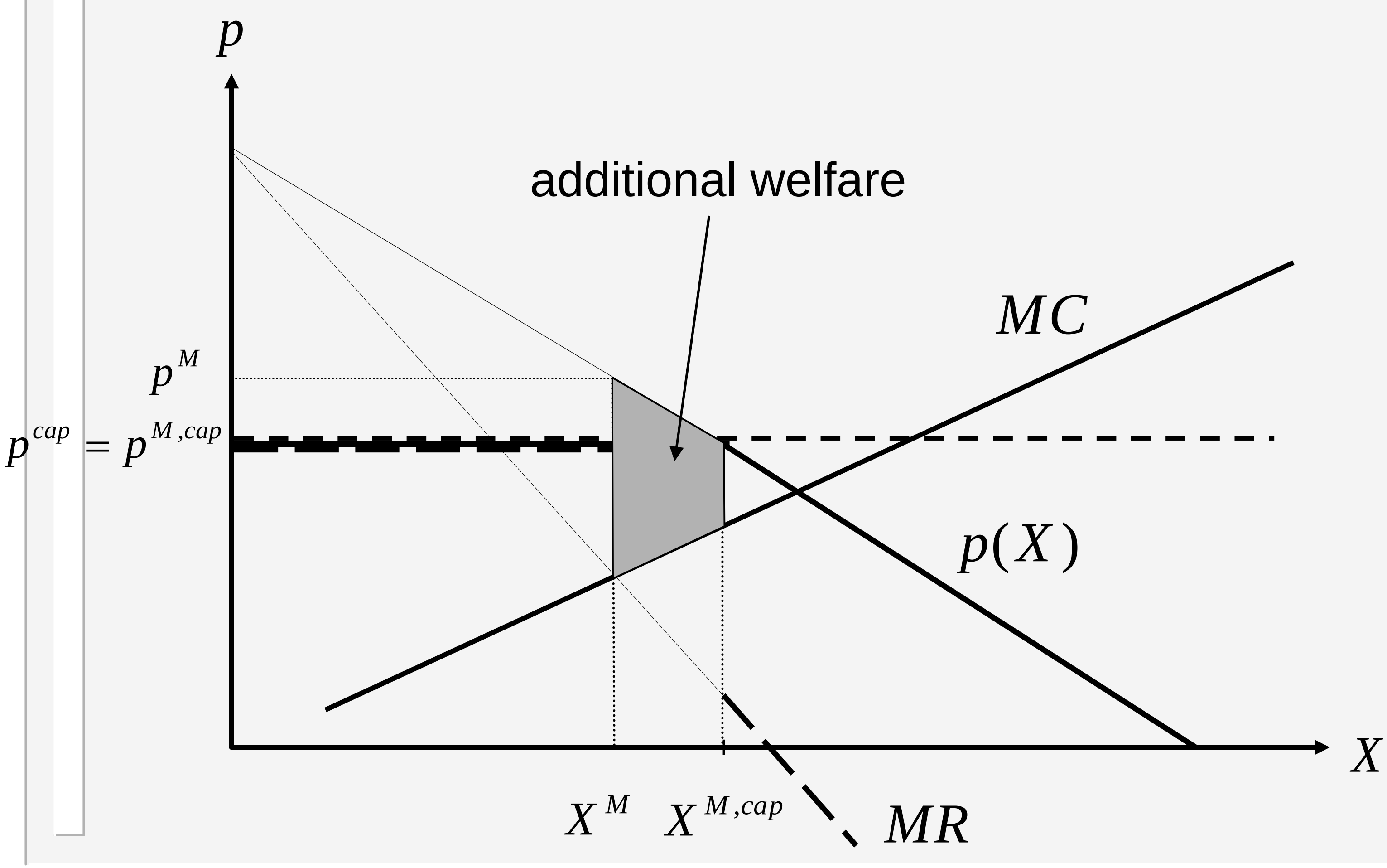
How does a price cap influence the demand and the marginal revenue curves?



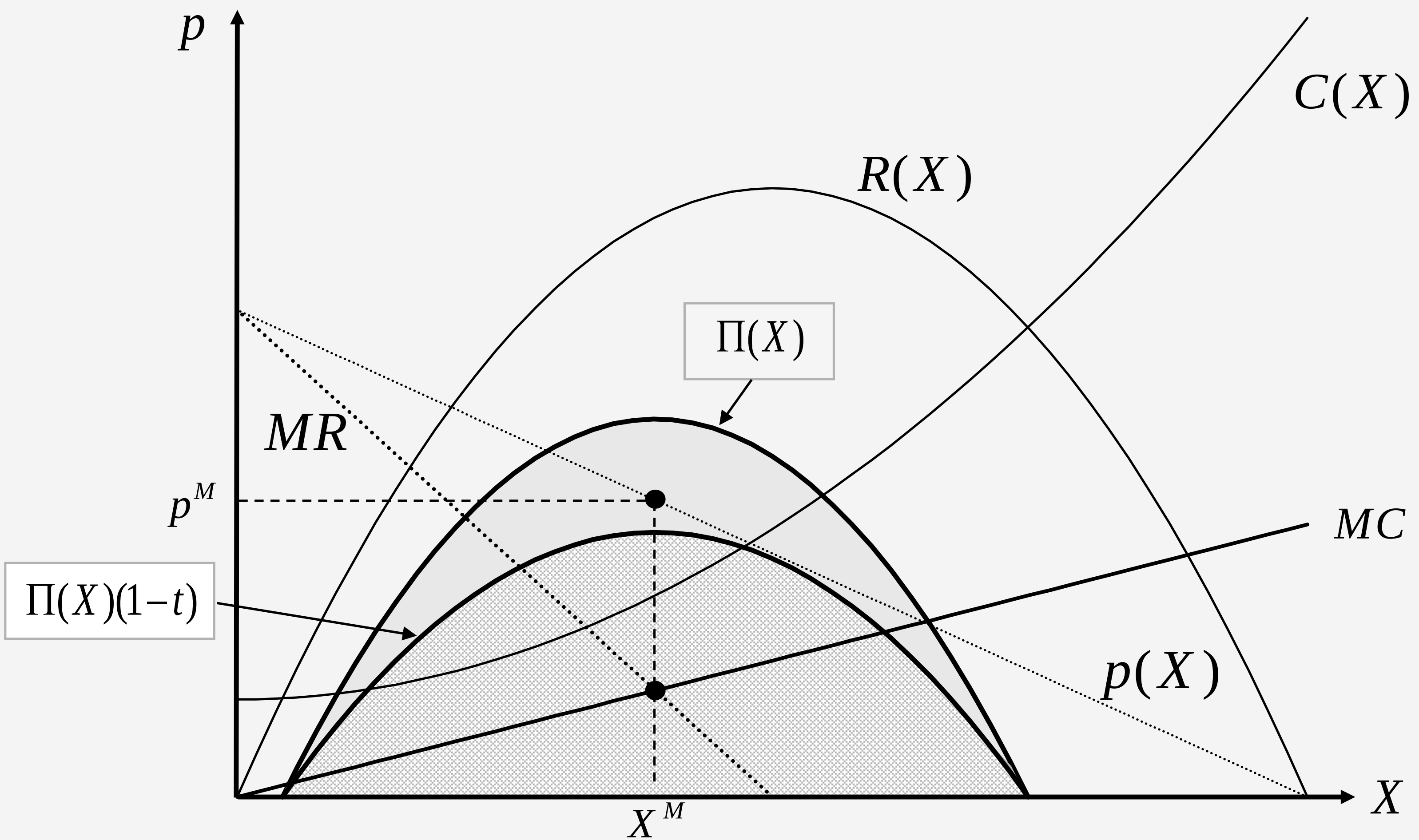
Right or wrong? Why?



Price cap and welfare



Taxes on profits



Taxes on profits and welfare

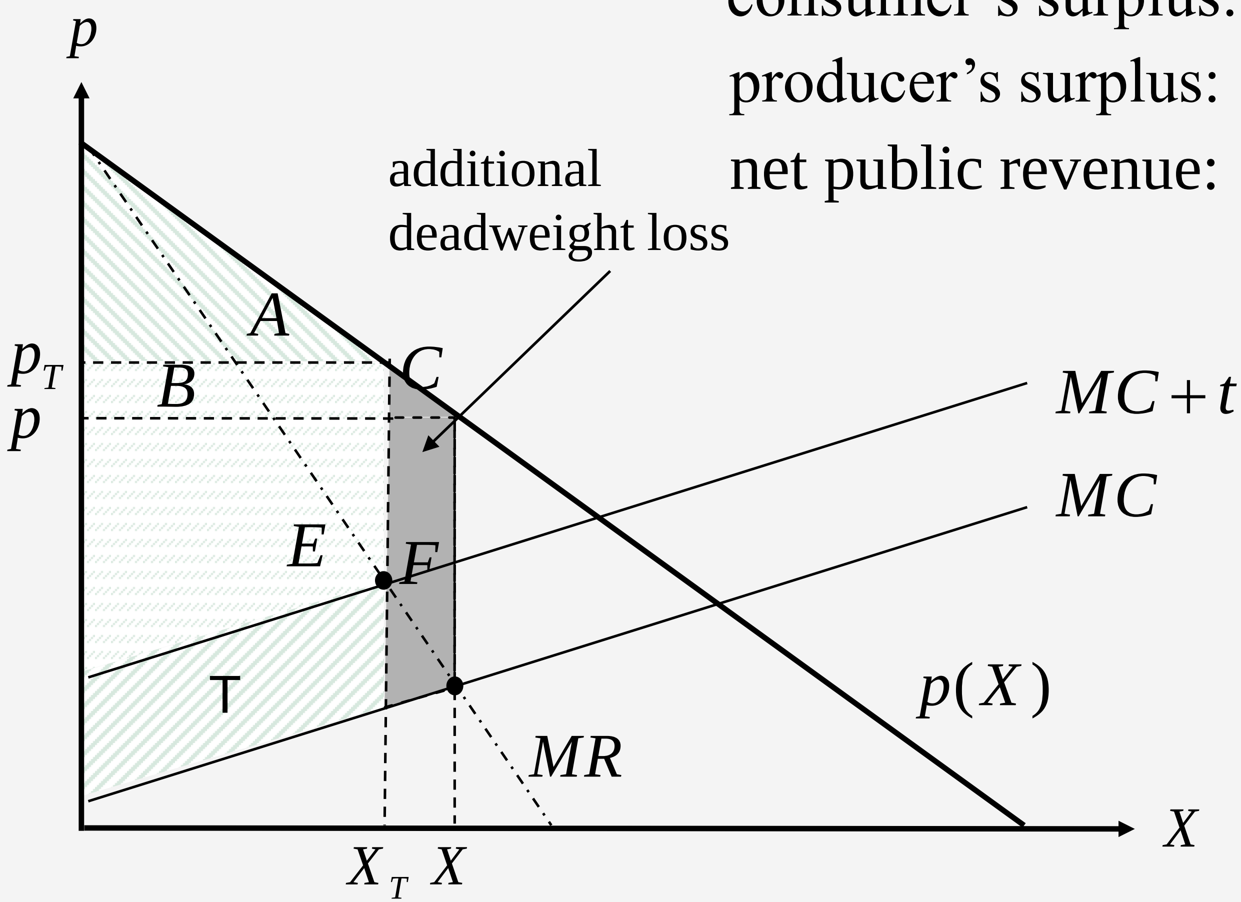
- Quantity / price unchanged
- CS is constant
- PS decrease; in the same extent net public revenues increase
- No changes in deadweight loss

Additional deadweight loss due to quantity tax

consumer's surplus: A+B+C→A

producer's surplus: T+E+F →E+B

net public revenue: 0 →T



Exercise (quantity taxes)

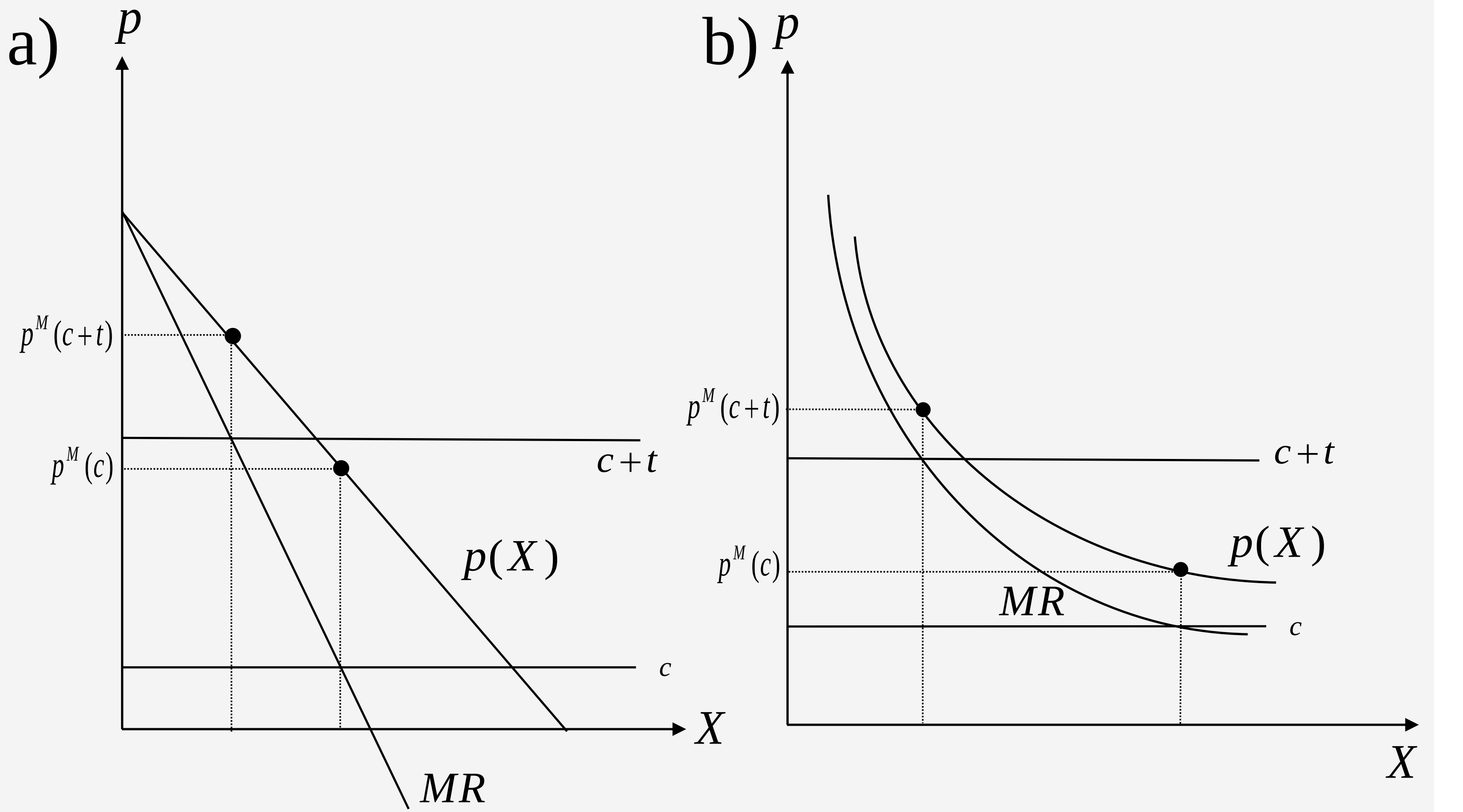
A monopolist is facing a demand curve given by $p(X)=a-X$. The monopoly's unit production cost is given by $c>0$. Now, suppose that the government imposes a specific tax of *t* dollars per unit sold.

a) Show that this tax would raise the price paid by consumers by less than *t*.

b) Would your answer change if the market inverse demand curve is given by $p(X)=-\ln(X)+5$.

c) If the demand curve is given by $p(X)=X^{-1/2}$, what is the influence on price?

Illustrating the solutions



Lerner index of monopoly power

- First order condition:

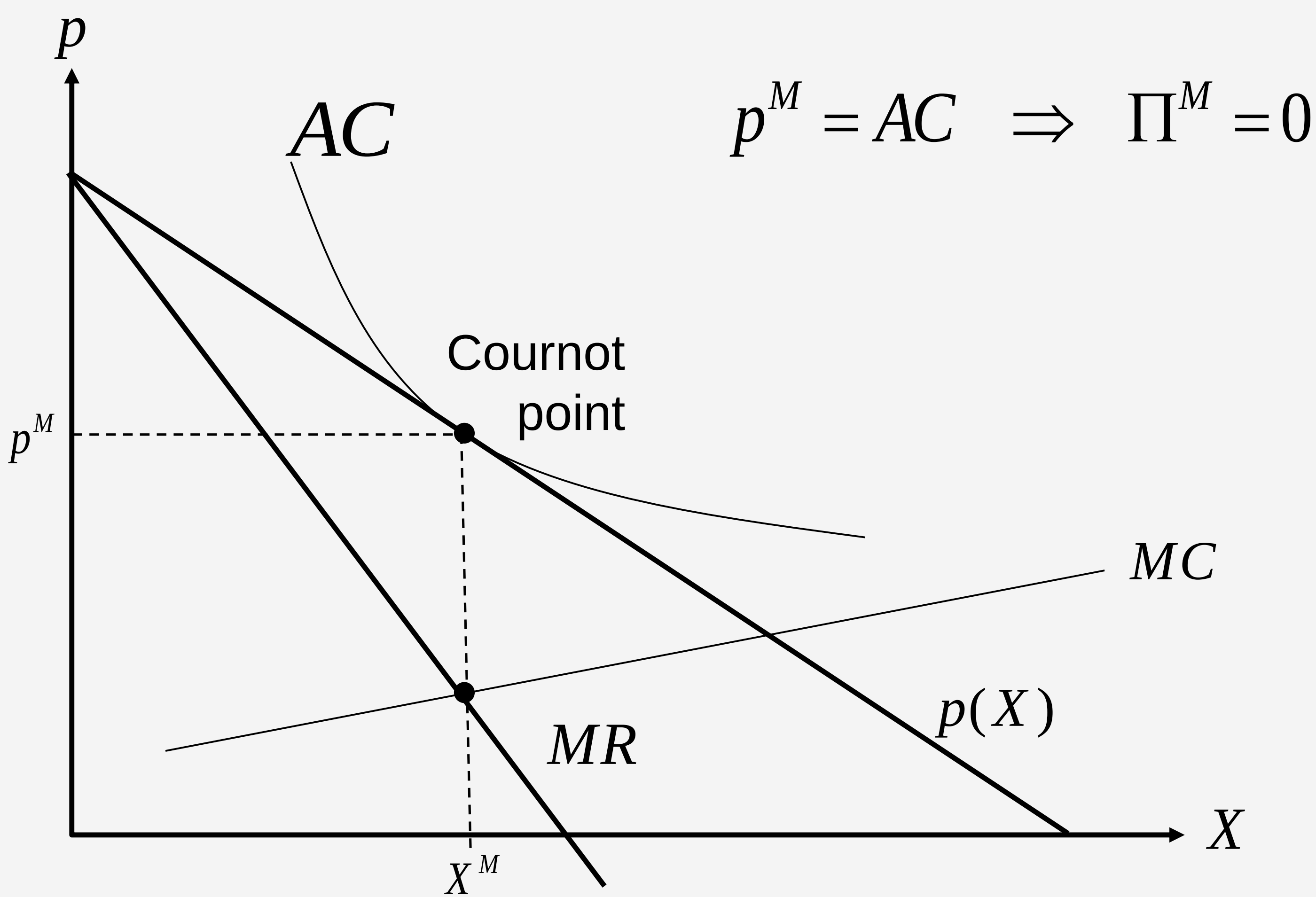
$$MC(X) \overset{!}{=} MR(X) = p(X) + X\frac{dp}{dX} = p\left(1 - \frac{1}{|\varepsilon_{X,p}|}\right)$$

- Lerner index:

$$\downarrow$$

$$\frac{p - MC}{p} \overset{!}{=} \frac{p - p\left(1 - \frac{1}{|\varepsilon_{X,p}|}\right)}{p} = \frac{1}{|\varepsilon_{X,p}|}$$

Monopoly profits and monopoly power



Executive summary

- A profit-maximizing monopolist always sets the quantity in the elastic region of the demand curve.
- Monopolistic quantities without price discrimination (!) lead to a welfare loss.
- A quantity tax leads to a welfare loss, a tax on profits does not.

Executive summary

- Distinction between monopolistic power and monopoly profit:
  - Monopolistic power: price will be set above the marginal cost by a profit maximize.
  - Monopoly profit: If the demand curve is tangent to the average cost curve, the profit-maximizing price is set above marginal cost and equal to average cost.
    $\Rightarrow$ monopolistic power and zero profits