Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

BACHELOR – PRÜFUNG

DATUM: **14. Februar 2025**

FACH: **Competitive Strategy**
Unternehmensstrategien im Wettbewerb

KLAUSURDAUER: 60 Min

PRÜFER: **Dr. Alexander Singer**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ERLÄUTERUNGEN:
Maximal number of points / Maximal erreichbare Punkte: 50
Please read careful before writing!
/ Lesen Sie die Aufgabenstellung vor dem Bearbeiten gründlich!
Write legibly, please! / Schreiben Sie, bitte, leserlich!
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PUNKTE:							**NOTE:**

Unterschrift des Prüfers/der Prüfer:

Exercise 1 (18 points)
Two firms, firm 1 and firm 2, compete sequentially in quantities. Firm 1 is the leader, firm 2 the follower. Inverse demand is given by

$$p(x_1 + x_2) = 16 - 2\,(x_1 + x_2)\,,$$

where x_1 denotes firm 1's output and x_2 denotes the output of firm 2. Firm 1's constant marginal and average costs are $c_1 = 3$. Firm 2's constant marginal and average costs are $c_2 = 6$.

a) Is entry of firm 2 blockaded?

b) Determine the limit quantity x_1^L.

c) Is entry of firm 2 deterred?

d) Is the strategy combination $(0, q_2)$ where firm 1 chooses the quantity 0 and firm 2 chooses the function q_2 with $q_2(x_1) = \begin{cases} 7, & x_1 > 0 \\ x_2^M, & x_1 = 0 \end{cases}$, where x_2^M is the monopoly output of firm 2, a Nash equilibrium?

Solution:

a) The profit function of firm 1 as a monopoly is given by

$$\Pi_1(x_1) = p(x_1)x_1 - c_1x_1 = (16 - 2x_1)x_1 - 3x_1 = (13 - 2x_1)x_1.$$

Solving the first-order condition

$$\frac{d\Pi_1(x_1)}{dx_1} = 13 - 4x_1 \overset{!}{=} 0$$

for x_1 yields the monopoly quantity

$$x_1^M = \frac{13}{4}.$$

Hence, we have $p_1^M = p(13/4) = 16 - 13/2 = 19/2 > 6 = c_2$, which implies that entry of firm 2 is not blockaded.

b) The profit function of firm 2 is given by

$$\Pi_2(x_1, x_2) = p(x_1 + x_2)x_2 - c_2x_2 = (10 - 2x_1 - 2x_2)x_2.$$

Solving the first-order condition

$$\frac{d\Pi_2(x_1, x_2)}{dx_2} = 10 - 2x_1 - 4x_2 \overset{!}{=} 0$$

for x_2 yields the reaction function

$$x_2^R(x_1) = 5/2 - x_1/2.$$

The limit quantity solves

$$x_2^R(x_1^L) = 5/2 - x_1^L/2 \overset{!}{=} 0$$

and is therefore given by $x_1^L = 5$.

c) Firm 1's reduced profit function is given by

$$\Pi_1^r(x_1) = p(x_1 + x_2^R(x_1))x_1 - c_1 x_1 = (16 - 2x_1 - 5 + x_1)x_1 - 3x_1 = (8 - x_1)x_1.$$

Since the marginal profit at the limit quantity satisfies

$$\left.\frac{d\Pi_1^r(x_1)}{dx_1}\right|_{x_1 = x_1^L} = 8 - 2x_1^L = 8 - 10 = -2 < 0,$$

firm 1 increases its profit by producing less than $x_1^L = 5$ units of output. Hence, entry of firm 2 is not deterred.

d) If firm 1 deviates to $x_1' > 0$, its profit satisfies

$$\Pi_1(x_1', 7) = (16 - 2x_1' - 14)x_1' - 3x_1' = (-1 - 2x_1')x_1' < 0 = \Pi_1(0, x_2^M).$$

Hence, firm 1 has no incentive to deviate. Firm 2 makes the monopoly profit $\Pi_2^M = \Pi_2(0, x_2^M)$ by playing q_2. Hence, firm 2 cannot do any better. Hence, $(0, q_2)$ is a Nash equilibrium. **Remark**: Note that $(0, q_2)$ is not subgame-perfect.

Exercise 2 (10 points)
Inverse demand is given by

$$p(X) = 24 - X.$$

A monopolistic producer sells its good via a monopolistic retailer. First, the producer chooses the retail price p_P. Then, the retailer chooses the quantity X that he buys at the price p_P and subsequently sells to the consumers. The producer faces constant marginal and average costs of $c_P = 8$, the retailer does not incur any costs due to his trading activity. Solve by backward induction! Determine the price that is paid by the consumers.

Solution:

The profit function of the retailer is given by

$$\Pi_r(X, p_p) = p(X)X - p_p X = (24 - p_p - X)X.$$

Solving the first-order condition

$$\frac{d\Pi_r(X, p_p)}{dX} = 24 - p_p - 2X \stackrel{!}{=} 0$$

for p_p yields the inverse demand function $p_p(X) = 24 - 2X$. The reduced profit function of the producer is then given by

$$\Pi_p^r(X) = p_p(X)X - c_p X = (24 - 2X)X - 8X = (16 - 2X)X.$$

Solving the first-order condition

$$\frac{d\Pi_p^r(X)}{dX} = 16 - 4X \stackrel{!}{=} 0$$

for X yields the profit-maximizing quantity $X^* = 4$, the retailer price $p_p^* = p_p(X^*) = 16$, and the consumer price $p^* = p(X^*) = 20$.

Exercise 3 (5 points)
Two inverse demand functions are given by

$$p_1(x_1) = 60 - x_1,$$
$$p_2(x_2) = 80 - 2x_2.$$

Determine the aggregate demand function!

Solution:

The two demand functions are given by

$$x_1(p) = 60 - p,$$
$$x_2(p) = 40 - \frac{p}{2}.$$

The prohibitive price in market 1 is 60, the prohibitive price in market 2 is 80. Aggregate demand is given by

$$X(p) = \begin{cases} 0, & p > 80 \\ 40 - \frac{p}{2}, & 60 < p \leq 80 \\ 100 - \frac{3p}{2}, & p \leq 60. \end{cases}$$

Exercise 4 (4 points)
The market shares of four firms are given by

$$s_4 = 0.1, \quad s_3 = 0.2, \quad s_2 = 0.3, \quad s_1 = 0.4.$$

Determine the concentration ratio C_2 and the Herfindahl index.
Solution:
The concentration ratio is $C_2 = 0.4 + 0.3 = 0.7$. The Herfindahl index is

$$H = 0.1^2 + 0.2^2 + 0.3^2 + 0.4^2 = 0.01 + 0.04 + 0.09 + 0.16 = 0.3$$

Exercise 5 (13 points)
Consider two firms, firm 1 and firm 2, in the following two-stage game variant of the Hotelling model. Consumers (of mass one) are uniformily distributed along the Hotelling street $[0,1]$. Firm 2 is located at $a_2 = 1$. Firm 1 **first** chooses its location $a_1 \in [0, 1/4]$ at stage one. **Then,** the two firms set prices simultaneously at stage two. Average and marginal costs of both firms are zero. Effective prices for a consumer at location $h \in [0,1]$ are given by

$$p_1^{eff} = p_1 + |h - a_1|, \quad p_2^{eff} = p_2 + |1 - h|.$$

a) Show that the indifferent consumer is located at $h^* = \frac{p_2 - p_1}{2} + \frac{1+a_1}{2}$.

b) Determine the equilibrium location of firm 1 by applying backward induction! Verify that stage-two equilibrium prices are given by $p_1(a_1) = 1 + \frac{a_1}{3}$ and $p_2(a_1) = 1 - \frac{a_1}{3}$.

Solution:

a) The indifferent consumer is located at $h^* \in (a_1, 1)$ where $p_1^{eff} = p_2^{eff}$. We thus have

$$\begin{aligned} p_1 + h - a_1 &= p_2 + 1 - h \\ \Rightarrow 2h &= p_2 - p_1 + 1 + a_1 \\ \Rightarrow h^* &= \frac{p_2 - p_1}{2} + \frac{1 + a_1}{2}. \end{aligned}$$

b) The profit function of firm 1 is given by

$$\Pi_1(a_1, p_1, p_2) = h^* p_1 = \left(\frac{p_2 - p_1}{2} + \frac{1 + a_1}{2} \right) p_1,$$

the profit function of firm 2 by

$$\Pi_2(a_1, p_1, p_2) = (1 - h^*) p_1 = \left(1 - \frac{p_2 - p_1}{2} - \frac{1 + a_1}{2} \right) p_2 = \left(\frac{1 - a_1}{2} + \frac{p_1 - p_2}{2} \right) p_2.$$

The two first order conditions are given by

$$\begin{aligned} \frac{\partial \Pi_1}{\partial p_1} &= \frac{p_2 - p_1}{2} + \frac{1 + a_1}{2} - \frac{p_1}{2} = \frac{p_2}{2} + \frac{1 + a_1}{2} - p_1 \overset{!}{=} 0, \\ \frac{\partial \Pi_2}{\partial p_2} &= \frac{1 - a_1}{2} + \frac{p_1 - p_2}{2} - \frac{p_2}{2} = \frac{1 - a_1}{2} + \frac{p_1}{2} - p_2 \overset{!}{=} 0. \end{aligned}$$

Multiplying the first FOC by 2 and adding this equation to the second FOC yields

$$\begin{aligned} p_2 + 1 + a_1 - 2p_1 + \left(\frac{1 - a_1}{2} + \frac{p_1}{2} - p_2 \right) &= 0 \\ \frac{3}{2} + \frac{a_1}{2} - \frac{3p_1}{2} &= 0 \\ \frac{3}{2} + \frac{a_1}{2} &= \frac{3p_1}{2} \\ \Rightarrow p_1(a_1) &= 1 + \frac{a_1}{3}. \end{aligned}$$

Substituting $p_1(a_1)$ into the second FOC yields

$$\begin{aligned} \frac{1-a_1}{2} + \frac{1}{2}\left(1+\frac{a_1}{3}\right) - p_2 &= 0 \\ \frac{1-a_1}{2} + \frac{1}{2} + \frac{a_1}{6} &= p_2 \\ 1 - \frac{2a_1}{6} &= p_2 \\ \Rightarrow p_2(a_1) &= 1 - \frac{a_1}{3}. \end{aligned}$$

So the reduced profit function of firm 1 is given by

$$\begin{aligned} \Pi_1^r(a_1) &= \left(\frac{p_2(a_1) - p_1(a_1)}{2} + \frac{1+a_1}{2}\right) p_1(a_1) \\ &= \left(\frac{1 - \frac{a_1}{3} - \left(1+\frac{a_1}{3}\right)}{2} + \frac{1+a_1}{2}\right)\left(1+\frac{a_1}{3}\right) \\ &= \left(-\frac{a_1}{3} + \frac{1}{2} + \frac{a_1}{2}\right)\left(1+\frac{a_1}{3}\right) \\ &= \left(\frac{1}{2} + \frac{a_1}{6}\right)\left(1+\frac{a_1}{3}\right) \\ &= \frac{1}{2}\left(1+\frac{a_1}{3}\right)^2. \end{aligned}$$

Since $\Pi_1^r(a_1)$ is increasing in a_1, firm 1 chooses $a_1 = \frac{1}{4}$ in equilibrium. **Remark:** We did not check that, for all $a_1 \in [0, 1/4]$, price undercutting is unprofitable for both firms. Firm 2 could undercut firm 1's price $p_1(a_1) = 1 + \frac{a_1}{3}$ by offering $p_2^c(a_1) = p_1(a_1) - (1 - a_1) = \frac{4a_1}{3}$ in order to supply all consumers, leading to the profit $\Pi_2^c(a_1) = \frac{4a_1}{3}$. Since $\Pi_2^c(a_1) = \frac{4a_1}{3} < \frac{1}{2}\left(1 - \frac{a_1}{3}\right)^2 = \Pi_2^r(a_2)$ if $a_1 < 15 - \sqrt{216} \approx 0.303$ holds, undercutting is unprofitable and $p_1(a_1)$ and $p_2(a_1)$ are indeed subgame-perfect equilibrium prices.