Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

BACHELOR – PRÜFUNG

DATUM:	**09. Februar 2024**
FACH:	**Competitive Strategy** **Unternehmensstrategien im Wettbewerb**
KLAUSURDAUER:	60 Min
PRÜFER:	**Dr. Alexander Singer**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ERLÄUTERUNGEN:
Maximal number of points / Maximal erreichbare Punkte: 50
Please read careful before writing!
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In case your need more space, please use the reverse side!
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PUNKTE:							**NOTE:**

Unterschrift des Prüfers/der Prüfer:

Exercise 1 (11 points)
Two **identical** firms are engaged in an innovation competition with spillover effects, in which firm 1 and firm 2 simultaneously set their R&D activities, F_1 and F_2, respectively. The profit function of firm 1 is given by

$$\Pi_1(F_1, F_2) = (10 - 2c_1 + c_2)^2 - \frac{7}{2}F_1^2$$

where $c_1 = 7 - F_1 - \frac{1}{4}F_2$ are firm 1's marginal and average production costs, and $c_2 = 7 - F_2 - \frac{1}{4}F_1$ are firm 2's marginal and average production costs. (Since the two firms are identical, the profit function of firm 2 is symmetric to the profit function of firm 1)

a) Determine the symmetric Nash equilibrium.

b) By looking at the externality imposed by firm 2 on firm 1, argue whether cooperation in R&D leads to less or more R&D activity compared to competition in R&D.

Solution:

a) The profit function of firm 1 can be written as

$$\begin{aligned}\Pi_1(F_1, F_2) &= \left(10 - 2\left(7 - F_1 - \frac{1}{4}F_2\right) + \left(7 - F_2 - \frac{1}{4}F_1\right)\right)^2 - \frac{7}{2}F_1^2 \\ &= \left(3 + \frac{7}{4}F_1 - \frac{1}{2}F_2\right)^2 - \frac{7}{2}F_1^2.\end{aligned}$$

The marginal profit of firm 1 is given by

$$\frac{d\Pi_1(F_1, F_2)}{dF_1} = \frac{7}{2}\left(3 + \frac{7}{4}F_1 - \frac{1}{2}F_2\right) - 7F_1.$$

By using symmetry, $F_1 = F_2$, firm 1's first-order condition yields

$$\begin{aligned}\frac{7}{2}\left(3 + \frac{7}{4}F_1 - \frac{1}{2}F_1\right) - 7F_1 &\overset{!}{=} 0 \\ \frac{21}{2} + \frac{35}{8}F_1 - 7F_1 &\overset{!}{=} 0 \\ \frac{21}{2} + \frac{35}{8}F_1 - \frac{56}{8}F_1 &\overset{!}{=} 0 \\ \frac{21}{8}F_1 &\overset{!}{=} \frac{21}{2}. \\ \Rightarrow F_1 &= 4.\end{aligned}$$

So the symmetric Nash equilibrium is given by $(F_1, F_2) = (4, 4)$.

b) The profit function of firm 1 is given by (see **a)**)

$$\Pi_1(F_1, F_2) = \left(3 + \frac{7}{4}F_1 - \frac{1}{2}F_2\right)^2 - \frac{7}{2}F_1^2.$$

Firm 2 imposes the externality

$$\begin{aligned}\left.\frac{d\Pi_1(F_1, F_2)}{dF_2}\right|_{F_1=F_2=4} &= -\left(3 + \frac{7}{4}F_1 - \frac{1}{2}F_2\right)_{F_1=F_2=4} \\ &= -(3 + 7 - 2) \\ &= -8 < 0\end{aligned}$$

on firm 1 in the symmetric equilirbium without cooperation. Since the externality is negative, two cooperating firms will reduce their R&D activities in order to increase their total profit.

Exercise 2 (14 points)
A firm is the only supplier in the market, whose inverse demand is given by

$$p(X) = 24 - X.$$

The firm **maximizes revenues**, its cost function is given by $C(X) = 18X$.

a) Determine the welfare loss.

b) From a welfare perspective, is revenue maximization better than profit maximization? Use a graph to answer this question.

Solution:

a) The firm's revenue is given by $R(X) = p(X)X = 24X - X^2$. The first-order condition yields

$$\frac{dR(X)}{dX} = 24 - 2X \stackrel{!}{=} 0$$
$$\Rightarrow X^R = 12, \quad p^R = p(12) = 12.$$

The welfare-maximizing output satisfies $p(X) = 24 - X \stackrel{!}{=} 18 = MC(X)$ and is therefore given by $X^W = 6$, implying $p^W = p(6) = 18$. Since both inverse demand and marginal cost are linear functions of output, the welfare loss given by

$$\begin{aligned} WL &= -\int_6^{12} (p(X) - MC(X))\, dX. \\ &= \left(p^W - p^R\right)\left(X^R - X^W\right)/2 \\ &= (18 - 12) \cdot (12 - 6)/2 \\ &= 18. \end{aligned}$$

b) The profit-maximizing output satisfies

$$\frac{dR(X)}{dX} = 24 - 2X \stackrel{!}{=} 18 = MC(X)$$

and is therefore given by $X^M = 3$, implying $p^M = p(3) = 21$. The graph should include at least: inverse demand, marginal revenue, marginal cost, and the two triangles representing the welfare losses for both revenue and profit maximization. Since the triangle representing the welfare loss due to revenue maximization is larger than that for profit maximization, profit maximization is preferable to revenue maximization from a welfare perspective.

Exercise 3 (14 points)
Two firms compete in quantities sequentially. Firm 1 is the leader, firm 2 the follower. Inverse demand is given by

$$p(X) = 16 - X.$$

Marginal and average costs of both firms are constant with $c_1 = 9$ and $c_2 = 6$.

a) Determine the Stackelberg quantities.

b) Now assume that firm 2 has to pay $F = 1$ in order to enter the market. Does firm 1 deter firm 2 from entering the market?

Solution:

a) The profit function of firm 2 is given by

$$\Pi_2(x_1, x_2) = (16 - x_1 - x_2 - 6)\, x_2 = (10 - x_1 - x_2)\, x_2.$$

Rearranging the first-order condition yields firm 2's reaction function

$$\frac{d\Pi_2(x_1, x_2)}{dx_2} = 10 - x_1 - 2x_2 \overset{!}{=} 0$$
$$\Rightarrow x_2^R(x) = 5 - \frac{x_1}{2}.$$

By substituting firm 2's reaction function into firm 1's profit function, we obtain firm 1's reduced profit function

$$\begin{aligned}\Pi_1^r(x_1) = \Pi_1(x_1, x_2^R(x_1)) &= \left(16 - x_1 - x_2^R(x_1) - 9\right) x_1 \\ &= \left(7 - x_1 - 5 + \frac{x_1}{2}\right) x_1 \\ &= \left(2 - \frac{x_1}{2}\right) x_1.\end{aligned}$$

The first-order condition, and the reaction function of firm 2, yield the two Stackelberg quantities

$$\frac{d\Pi_1^r(x_1)}{dx_1} = 2 - x_1 \overset{!}{=} 0$$
$$\Rightarrow x_1^S = 2, \quad x_2^S = x_2^R(x_1^S) = 4.$$

b) In order to deter firm 2 from entering the market firm 1's supply must be sufficiently large such that firm 2's profit is nonpositive for all $x_2 > 0$. From **a)**, we know that firm 2's profit-maximizing supply for $x_2 > 0$ is given by $x_2^R(x_1) = 5 - \frac{x_1}{2}$. In this case, firm 2's profit is nonpositiv if

$$\begin{aligned}\Pi_2(x_1, x_2^R(x_1)) &= \left(10 - x_1 - x_2^R(x_1)\right) x_2^R(x_1) - 1 \\ &= \left(10 - x_1 - 5 + \frac{x_1}{2}\right)\left(5 - \frac{x_1}{2}\right) - 1 \\ &= \left(5 - \frac{x_1}{2}\right)^2 - 1 \leq 0\end{aligned}$$

$$\begin{aligned}\Rightarrow \left(5 - \frac{x_1}{2}\right)^2 &\leq 1 \\ 5 - \frac{x_1}{2} &\leq 1 \\ x_1 &\geq 8 \equiv x_1^L\end{aligned}$$

If firm 1 supplies $x_1 = x_1^L$, its profit is given by

$$\Pi_1(x_1^L, 0) = (16 - 8 - 0 - 9)\, 8 = -8 < 2 = (16 - 2 - 4 - 9)2 = \Pi_1(x_1^S, x_2^S).$$

Hence, firm 1 does not deter firm 2 from entering the market.

Exercise 4 (7 points)
Two identical firms with constant average and marginal costs $c = 5$ are in perfect price competition. Market demand is given by

$$X(p) = 12 - p.$$

a) Is the price pair $(8, 8)$ a Nash equilibrium if firm 1 (but not firm 2) has offered a minimum price guarantee?

b) Is the price pair $(8, 8)$ a Nash equilibrium if both firms have offered a minimum price guarantee?

Solution:

a) At $(8, 8)$, the two firms share the total demand $X(8) = 4$ equally. So each firm makes the profit $(8-5)X(8)/2 = 6$. Firm 1 could increase its profit by lowering its price to 7, in which case it makes $(7-5)X(7) = 10>6$. Hence, $(8, 8)$ is no Nash equilibrium.

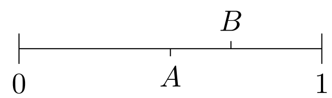
b) At $(8, 8)$, the two firms share the total demand $X(8) = 4$ equally. Each firm makes the profit $(8-5)X(8)/2 = 6$. If firm 1 increases its price to $p_1 > 8$, its best price guarantee applies, keeping firm 1's profit constant. If firm 1 lowers its price to $p_1 < 8$, firm 2's best price guarentee applies. Firm 1's profit is then given by $\Pi_1(p_1, 8) = (p_1 - 5)(6 - p_1/2)$. Since firm 1's marginal profit

$$\frac{d\Pi_1(p_1, 8)}{dp_1} = 6 + \frac{5}{2} - p_1 = 8.5 - p_1$$

is positive for all $p_1 < 8$, firm 1 cannot increase its profit by lowering its price to $p_1 < 8$. So firm 1 and firm 2 (by symmetry) cannot profitably deviate from $(8, 8)$. Hence, $(8, 8)$ is a Nash equilibrium.

Exercise 5 (4 points)
Consider two firms, A and B, that simultaneously choose their location on a Hotelling street of length one. Consumers are uniformly distributed on the Hotelling street and buy from the nearest firm. The price is set by the government and is above the firms' constant average and marginal costs. Draw a location combination that is not a Nash equilibrium in an appropriate diagram. Explain.

Solution:



In the diagram, the two firms' locations are $(x_A, x_B) = (0.5, 0.7)$. Firm B's consumer share is thus $X_B(0.5, 0.7) = 1 - \frac{0.5+0.7}{2} = 0.4$. By deviating to location $x'_B = 0.6$, firm B's share increases to $X_B(0.5, 0.6) = 1 - \frac{0.5+0.6}{2} = 0.45$. Since the price is above firm B's constant average and marginal cost, firm B increases its profit by deviating to x'_B . Hence, $(0.5, 0.7)$ is no Nash equilibrium.