Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

BACHELOR – PRÜFUNG

DATUM:	**08. Februar 2023**
FACH:	**Competitive Strategy**
	Unternehmensstrategien im Wettbewerb
KLAUSURDAUER:	60 Min
PRÜFER:	**Prof. Dr. Harald Wiese**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ERLÄUTERUNGEN:
Maximal number of points / Maximal erreichbare Punkte: 50
Please read careful before writing!
/ Lesen Sie die Aufgabenstellung vor dem Bearbeiten gründlich!
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PUNKTE:						**NOTE:**

Unterschrift des Prüfers/der Prüfer:

Exercise 1 (8 points)
Two firms compete in prices. The demand function of firm 1 is given by

$$x_1(p_1, p_2) = 3 + p_2 - p_1.$$

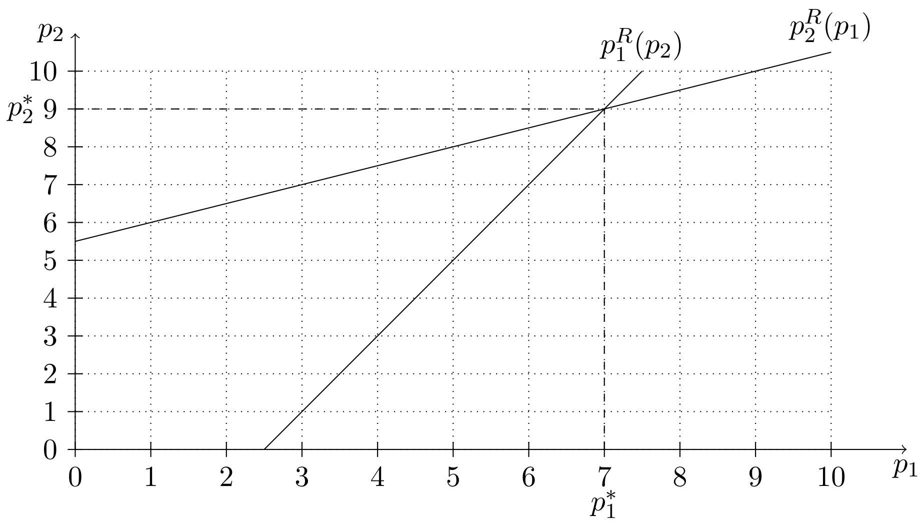
The demand function of firm 2 is given by

$$x_2(p_1, p_2) = 6 + p_1 - p_2.$$

Firm 1's marginal and average costs are given by $c_1 = 2$. Firm 2's marginal and average costs are given by $c_2 = 5$.

a) Determine the reaction function of each firm.

b) Plot the two reaction functions in the diagram below and determine graphically the Nash equilibrium.



Solution:

a) The profit function of firm 1 is given by

$$\Pi_1(p_1, p_2) = (p_1 - 2) \cdot (3 + p_2 - p_1).$$

Solving the first-order condition for p_1 yields firm 1's reaction function:

$$\frac{d\Pi_1(p_1, p_2)}{dp_1} = 3 + p_2 - p_1 - p_1 + 2 \overset{!}{=} 0$$
$$5 + p_2 - 2p_1 = 0$$
$$\Rightarrow p_1^R(p_2) = \frac{5}{2} + \frac{p_2}{2}.$$

The profit function of firm 2 is given by

$$\Pi_2(p_1, p_2) = (p_2 - 5) \cdot (6 + p_1 - p_2).$$

Solving the first-order condition for p_2 yields firm 2's reaction function:

$$\frac{d\Pi_2(p_1, p_2)}{dp_2} = 6 + p_1 - p_2 - p_2 + 5 \overset{!}{=} 0$$
$$11 + p_1 - 2p_2 = 0$$
$$\Rightarrow p_2^R(p_1) = \frac{11}{2} + \frac{p_1}{2}.$$

b) See diagram above where $(p_1^*, p_2^*) = (7, 9)$ indicates the Nash equilibrium.

Exercise 2 (16 points)
Two firms compete in quantities sequentially. Firm 1 is the leader, firm 2 is the follower. Firm 1's marginal and average costs are given by $c_1 = 2$. Firm 2's marginal and average costs are given by $c_2 = 6$. Inverse demand is given by

$$p(x_1, x_2) = 12 - x_1 - x_2.$$

a) Determine whether entry of firm 2 is blockaded.

b) Determine the reaction function of firm 2.

c) Determine firm 1's limit output level.

d) Determine whether entry of firm 2 is deterred.

e) State the Stackelberg quantities.

Solution:

a) The monopoly profit function of firm 1 is given by

$$\Pi_1^M(x_1) = (12 - x_1 - 2)x_1 = (10 - x_1)x_1.$$

The profit-maximizing quantity satisfies

$$\frac{d\Pi_1^M(x_1)}{dx_1} = 10 - 2x_1 \stackrel{!}{=} 0$$
$$\Rightarrow x_1^M = 5.$$

The monopoly price is then given by $p_1^M = 12 - 5 = 7 > 6 = c_2$. Hence, entry of firm 2 is not blockaded.

b) The profit function of firm 2 is given by

$$\Pi_2(x_1, x_2) = (12 - x_1 - x_2 - 6)\, x_2.$$

Solving the first-order condition for x_2 yields firm 2's reaction function:

$$\frac{d\Pi_2(x_1, x_2)}{dx_2} = 6 - x_1 - 2x_2 \stackrel{!}{=} 0$$
$$\Rightarrow x_2^R(x_1) = 3 - \frac{x_1}{2}.$$

c) Firm 1's limit output level x_1^L satisfies

$$x_2^R(x_1^L) = 3 - \frac{x_1}{2} \stackrel{!}{=} 0$$
$$\Rightarrow x_1^L = 6.$$

d) Firm 1 anticipates firm 2's reaction function. The reduced profit function of firm 1 is given by

$$\begin{aligned}\Pi_1^r(x_1) = \Pi_1(x_1, x_2^R(x_1)) &= \left(12 - x_1 - x_2^R(x_1) - 2\right) x_1 \\ &= \left(10 - x_1 - \left(3 - \frac{x_1}{2}\right)\right) x_1 \\ &= \left(7 - \frac{x_1}{2}\right) x_1.\end{aligned}$$

Deterrence pays for firm 1 if $\frac{d\Pi_1^r(x_1)}{dx_1}|_{x_1=x_1^L} > 0$ (as this condition implies that, for all quantities $x_1 \in [0, x_1^L)$, firm 1's marginal profits are positive). We have

$$\begin{aligned}\frac{d\Pi_1^r(x_1)}{dx_1}|_{x_1=x_1^L} &= 7 - x_1^L \\ &= 1 > 0.\end{aligned}$$

Hence, entry of firm 2 is deterred.

e) Since entry of firm 2 is deterred (and not blockaded), firm 1 offers the Stackelberg quantity $x_1^S = x_1^L = 6$, while firm 2 offers the Stackelberg quantity $x_2^S = x_2^R(x_1^S) = 0$.

Exercise 3 (18 points)
There are two markets, one for computers and one for software. Firm C produces x_C computers at zero costs. Firm S produces x_S software at zero costs. Inverse demand for computers is given by

$$p_C(x_C, x_S) = 3 + \frac{x_S}{2} - x_C.$$

Inverse demand for software is given by

$$p_S(x_C, x_S) = 18 + \frac{x_C}{2} - x_S.$$

a) Assuming quantity competition, determine the equilibrium quantities and prices, x_C^*, x_S^*, p_C^*, p_S^*.

b) Determine welfare in the computer market. *Hint: Use* $p_C(x_C, x_S^*)$.

c) Does a regulator seeking to maximize welfare in the computer market permit the merger of firm C and S?

Solution:

a) The two firms' profit functions are given by

$$\begin{aligned}\Pi_C(x_C, x_S) &= \left(3 + \frac{x_S}{2} - x_C\right) x_C,\\ \Pi_S(x_C, x_S) &= \left(18 + \frac{x_C}{2} - x_S\right) x_S.\end{aligned}$$

The two first-order conditions are given by

$$\begin{aligned}\frac{d\Pi_C(x_C, x_S)}{dx_C} &= 3 + \frac{x_S}{2} - 2x_C \stackrel{!}{=} 0,\\ \frac{d\Pi_S(x_C, x_S)}{dx_S} &= 18 + \frac{x_C}{2} - 2x_S \stackrel{!}{=} 0.\end{aligned}$$

Multiplying the first first-order condition by four and adding the second one yields

$$\begin{aligned}30 - \frac{15}{2}x_C &= 0\\ \Rightarrow x_C^* &= 4.\end{aligned}$$

Inserting x_C^* in the first first-order condition yields

$$\begin{aligned}3 + \frac{x_S}{2} - 2 \cdot 4 &= 0\\ \Rightarrow x_S^* &= 10.\end{aligned}$$

The equilibrium quantities are thus given by $x_C^* = 4$ and $x_S^* = 10$, yielding the equilibrium prices

$$\begin{aligned}p_C^* =& p_C(x_C^*, x_S^*) = 3 + 5 - 4 = 4,\\ p_S^* =& p_S(x_C^*, x_S^*) = 18 + 2 - 10 = 10.\end{aligned}$$

b) Welfare in the computer market is given by

$$\begin{aligned} W_C(x_C^*, x_S^*) &= \int_0^{x_C^*} p_C(x_C, x_S^*) dx_C \\ &= \int_0^4 (8 - x_C)\, dx_C \\ &= \left(8x_C - \frac{x_C^2}{2}\right)_0^4 \\ &= 8 \cdot 4 - 2 \cdot 4 = 6 \cdot 4 = 24. \end{aligned}$$

c) After the merger, the profit function is given by

$$\Pi(x_C, x_S) = \Pi_C(x_C, x_S) + \Pi_S(x_C, x_S).$$

The two first-order conditions are given by

$$\begin{aligned} \frac{d\Pi(x_C, x_S)}{dx_C} &= 3 + \frac{x_S}{2} - 2x_C + \frac{x_S}{2} \overset{!}{=} 0, \\ \frac{d\Pi(x_C, x_S)}{dx_S} &= 18 + \frac{x_C}{2} - 2x_S + \frac{x_C}{2} \overset{!}{=} 0. \end{aligned}$$

Multiplying the first first-order condition by two and adding the second one yields

$$\begin{aligned} 24 - 3x_C &= 0 \\ \Rightarrow x_C^M &= 8. \end{aligned}$$

Inserting x_C^M in the first first-order condition yields

$$\begin{aligned} 3 + x_S - 2 \cdot 8 &= 0 \\ \Rightarrow x_S^M &= 13. \end{aligned}$$

The equilibrium quantities after the merger are thus given by $x_C^M = 8$ and $x_S^M = 13$. The equilibrium prices are given by

$$\begin{aligned} p_C^M &= p_C(x_C^M, x_S^M) = 3 + 6.5 - 8 = 1.5, \\ p_S^M &= p_S(x_C^M, x_S^M) = 18 + 4 - 13 = 9. \end{aligned}$$

Since (i) the number of computers sold rises, $x_C^M = 8 > 4 = x_C^*$, (ii) the prohibitive price rises, $9.5 > 8$, but the market price decreases, $p_C^M = 1.5 < 3 = p_C^*$, and (iii) costs of production are zero, the merger increases welfare in the computer market. Hence, the regulator should permit the merger. Alternatively, we can calculate welfare in the computer market after the merger to prove that welfare increases:

$$\begin{aligned} W_C(x_C^M, x_S^M) &= \int_0^{x_C^M} p_C(x_C, x_S^M) dx_C \\ &= \int_0^8 (9.5 - x_C)\, dx_C \\ &= \left(9.5x_C - \frac{x_C^2}{2}\right)_0^8 \\ &= 9.5 \cdot 8 - 4 \cdot 8 = 5.5 \cdot 8 = 44 > 24 = W_C(x_C^*, x_S^*). \end{aligned}$$

Exercise 4 (8 points)
A monopoly facing the inverse demand function

$$p(y) = 14 - y$$

produces its output $y = y_1 + y_2$ in two factories, 1 and 2. The costs of producing y_1 units in factory 1 are given by $C_1(y_1) = \frac{y_1^2}{2}$, the costs of producing y_2 units in factory 2 are given by $C_2(y_2) = 4y_2$. Determine the profit-maximizing quantities.

Solution:

The monopoly produces each marginal unit in the factory where marginal costs are lowest. Marginal costs are given by

$$MC_1(y_1) = y_1,$$
$$MC_2(y_2) = 4.$$

Hence, the first four units are produced in factory 1, while all remaining units are produced in factory 2. Revenue is given by $R(y) = p(y)y = 14y - y^2$. Marginal revenue is then given by

$$MR(y) = 14 - 2y.$$

Since $MR(4) = 14 - 8 = 6 > 4 = MC_1(4)$, the monopoly quantity must satisfy $y > 4$, leading to $y_1^M = 4$ and

$$MR(y) = 14 - 2y \overset{!}{=} 4 = MC_2(y - 4)$$
$$\Rightarrow y^M = 5,$$
$$\Rightarrow y_2^M = 5 - 4 = 1.$$