Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

BACHELOR – PRÜFUNG

DATUM:	**30. März 2022**
FACH:	**Competitive Strategy** **Unternehmensstrategien im Wettbewerb**
KLAUSURDAUER:	60 Min
PRÜFER:	**Prof. Dr. Harald Wiese**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ERLÄUTERUNGEN:
Maximal number of points / Maximal erreichbare Punkte: 50
Please read careful before writing!
/ Lesen Sie die Aufgabenstellung vor dem Bearbeiten gründlich!
Write legibly, please! / Schreiben Sie, bitte, leserlich!
Jusity your answers! / Begründen Sie Ihre Antworten!
Make your calulations clear!
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In case your need more space, please use the reverse side!
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PUNKTE:							**NOTE:**

Unterschrift des Prüfers/der Prüfer:

		Firm 2	
		p_l	p_h
Firm 1	p_l	$(3; 2)$	$(10; 0)$
	p_h	$(0; 8)$	$(7; 6)$

Exercise 1 (8 points)
Two firms compete in prices sequentially. Firm 1 is time leader. They can choose between two prices only, p_l and p_h, where $p_h > p_l \geq 0$. They earn the profits $(\Pi_1; \Pi_2)$ (see Figure).

a) Use backward induction.

b) Firm 1 provides a best-price guarantee. Sketch the new profit matrix! Can firm 1 improve its situation by introducing the best-price guarantee?

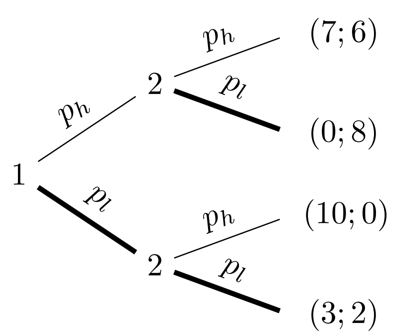
Aufgabe 1 (8 Punkte)
Am Markt konkurrieren zwei Unternehmen im sequentiellen Preiswettbewerb. Unternehmen 1 ist Zeitführer. Jedes Unternehmen kann entweder den hohen Preis p_h oder den niedrigen Preis p_l wählen, wobei $p_h > p_l \geq 0$ gilt. Dabei erzielen sie die Gewinne $(\Pi_1; \Pi_2)$ (siehe Abbildung).

a) Wenden Sie Rückwärtsinduktion an.

b) Unternehmen 1 führt nun eine Niedrigstpreisgarantie ein. Stellen Sie die neue Gewinnmatrix auf! Kann sich Unternehmen 1 durch die Garantie besser stellen?

Proposal of solution:

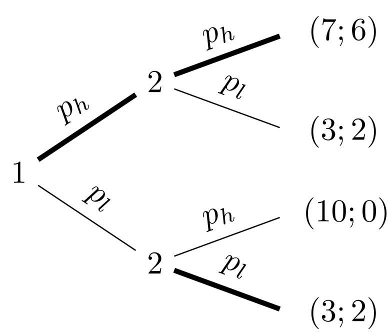
a) Applying backward induction, we first need to determine the reaction function of firm 2: If firm 1 sets a low price, firm 2 will also choose p_l; if firm 1 sets a high price, firm 2 will still choose p_l. Firm 2 thus has a strictly dominant strategy p_l. Anticipating the reaction of firm 2, firm 1 chooses p_l since $3 > 0$. (p_l, p_l) is thus the solution to the sequential game.



b) The profit of both firms changes for the price combination (p_h, p_l) because the best-price guarantee of firm 1 results in an effective price $p_1^{\text{eff}} = p_l$. The new profit matrix is thus given by

		Firm 2	
		p_l	p_h
Firm 1	p_l	$(3; 2)$	$(10; 0)$
	p_h	$(3; 2)$	$(7; 6)$

Applying backward induction again yields the reaction function of firm 2: If firm 1 sets a low price, firm 2 will also choose p_l; if firm 1 sets a high price, firm 2 will also choose p_h. Firm 1 can anticipate the reaction of firm 2, it thus compares the profit of 3 (resulting from (p_l, p_l)) to the profit of 7 (resulting from (p_h, p_h)). Since $3 < 7$, the optimal behavior of both firms is given by (p_h, p_h). Introducing the best-price guarantee allows firm 1 to improve by increasing its profit from 3 to 7.



Exercise 2 (13 points)
There are three firms competing in quantities. Constant marginal and average costs of all firms are $c = 0$. The inverse demand function is given by

$$p(X) = 120 - X.$$

First, firm 1 chooses its quantity x_1. Firm 2 can observe this quantity and chooses its own quantity x_2 afterwards. Finally, firm 3 can observe the quantities chosen by firm 1 and firm 2 and chooses its quantity x_3 afterwards. Apply backward induction.

Aufgabe 2 (13 Punkte)
Auf einem Markt befinden sich drei Unternehmen im Mengenwettbewerb. Die durchschnittlichen und marginalen Kosten für alle Unternehmen sind $c = 0$. Die inverse Nachfrage ist gegeben durch

$$p(X) = 120 - X.$$

Zuerst wählt Unternehmen 1 seine Mengen x_1. Unternehmen 2 kann diese Menge beobachten und wählt dann seine eigene Menge x_2. Schließlich kann Unternehmen 3 die von Unternehmen 1 und 2 gewählten Mengen beobachten und wählt dann seine eigene Menge x_3. Nutzen Sie Rückwärtsinduktion.

Proposal of solution:
The game is solved by backward induction. First, we determine the behavior of firm 3 in the third stage:

$$\Pi_3(x_1, x_2, x_3) = (120 - x_1 - x_2 - x_3)x_3$$

$$\begin{aligned}
\frac{\partial \Pi_3}{\partial x_3} = 120 - x_1 - x_2 - 2x_3 &\overset{!}{=} 0 \\
120 - x_1 - x_2 &= 2x_3 \\
x_3^R(x_1, x_2) &= 60 - \frac{1}{2}x_1 - \frac{1}{2}x_2
\end{aligned}$$

In the second stage, firm 2 anticipates how firm 3 will react in the third stage. The profit function is given by:

$$\begin{aligned}
\Pi_2\left(x_1, x_2, x_3^R(x_1, x_2)\right) &= \left(120 - x_1 - x_2 - x_3^R(x_1, x_2)\right) x_2 \\
&= \left(120 - x_1 - x_2 - (60 - \frac{1}{2}x_1 - \frac{1}{2}x_2)\right) x_2 \\
&= \left(60 - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) x_2
\end{aligned}$$

By taking the derivative with respect to x_2, the reaction function is found to be given by:

$$\begin{aligned}
\frac{\partial \Pi_2}{\partial x_2} = 60 - \frac{1}{2}x_1 - x_2 &\overset{!}{=} 0 \\
x_2^R(x_1) &= 60 - \frac{1}{2}x_1
\end{aligned}$$

In the first stage, firm 1 anticipates how firm 2 and firm 3 will react in the subsequent stages. The profit function is given by:

$$
\begin{aligned}
\Pi_1\left(x_1, x_2^R(x_1), x_3^R(x_1, x_2^R(x_1))\right) &= \left(120 - x_1 - x_2^R(x_1) - x_3^R(x_1, x_2^R(x_1))\right) x_1 \\
&= \left(120 - x_1 - \left[60 - \frac{1}{2}x_1\right] - \left[60 - \frac{1}{2}x_1 - \frac{1}{2}(60 - \frac{1}{2}x_1)\right]\right) x_1 \\
&= \left(120 - x_1 - 60 + \frac{1}{2}x_1 - \left[60 - \frac{1}{2}x_1 - 30 + \frac{1}{4}x_1\right]\right) x_1 \\
&= \left(120 - x_1 - 60 + \frac{1}{2}x_1 - 30 + \frac{1}{4}x_1\right) x_1 \\
&= \left(30 - \frac{1}{4}x_1\right) x_1
\end{aligned}
$$

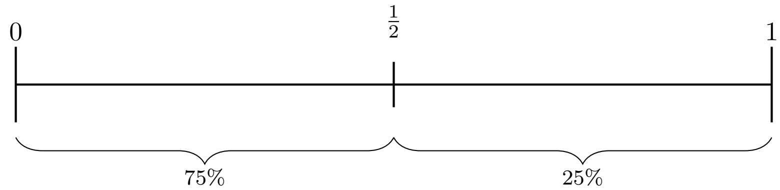
By taking the derivative with respect to x_1, the Stackelberg quantity of firm 1 can be determined:

$$
\begin{aligned}
\frac{\partial \Pi_1}{\partial x_1} = 30 - \frac{1}{2}x_1 &\overset{!}{=} 0 \\
x_1^S &= 60
\end{aligned}
$$

Now, the quantities of firm 2 and firm 3 can be determined:

$$
\begin{aligned}
x_2^R(x_1^S = 60) = 60 - \frac{1}{2}x_1^S &= 30 \\
x_3^R(x_1^S = 60, x_2 = 30) = 60 - \frac{1}{2}x_1^S - \frac{1}{2}x_2 &= 15
\end{aligned}
$$

The associated price is $p^S = 120 - (60 + 30 + 15) = 15$. Firm 1 realizes a profit of $\Pi_1^S = 15 * 60 = 900$. Firm 2 realizes a profit of $\Pi_2^S = 15 * 30 = 450$. Firm 3 realizes a profit of $\Pi_3^S = 15 * 15 = 225$.



Exercise 3 (7 points)
Consider the following variant of the Hotelling model (see Figure). There are two profit-maximizing firms that choose their locations simultaneously. The price is fixed by the government and above the average and marginal costs of the firms. Each consumer consumes exactly one unit at the nearest firm. There are two districts. In district A, in the interval $(0, 0.5)$, 75% of the consumers are distributed equally. In district B, in the interval $(0.5, 1)$, 25% of the consumers are distributed equally. The two firms can freely choose their locations, a_1, a_2 respectively.

Determine all Nash equilibria.

Aufgabe 3 (7 Punkte)
Betrachten Sie folgende Variante des Hotelling-Modells (siehe Abbildung). Zwei gewinnmaximierende Unternehmen treffen ihre Standortentscheidungen simultan. Der Preis ist staatlich vorgegeben und liegt über den konstanten Grenz- und Durchschnittskosten der Unternehmen. Jeder Konsument kauft genau eine Einheit beim nächstgelegenen Unternehmen. Es gibt zwei Distrikte. In Distrikt A, im Intervall $(0, 0.5)$, sind 75% der Konsumenten gleichverteilt. In Distrikt B, im Intervall $(0.5, 1)$, sind 25% der Konsumenten gleichverteilt. Die beiden Firmen können ihre Standorte, a_1 bzw. a_2, frei wählen.

Bestimmen Sie alle Nash-Gleichgewichte.

Proposal of solution:
Firms do maximize profits by choosing the location where 50% of consumers are left, and 50% of consumers are right of their location. Since in the interval $(0, 0.5)$, 75 % of consumers are distributed equally, 50% of all consumers sit on the $\frac{2}{3}$ of this interval of length $\frac{1}{2}$. The location is thus $\frac{2}{3} * \frac{1}{2} = \frac{1}{3}$. Both firms will locate at $a_1 = a_2 = \frac{1}{3}$. This is indeed the unique Nash equilibrium. Deviating unilaterally is not profitable. If any firm locates further left, it will lose market share (<50%), if it deviates to the right, it also loses. All other candidates for Nash equilibria can be eliminated: If the two firms choose different locations ($a_1 \neq a_2$), each firm can improve by aproaching the other firm's location. If firms are located at the same location ($a_1 = a_2 \neq \frac{1}{3}$), each firm can profitably deviate by choosing a location just right (left) of the initial location if $a_1 = a_2 < \frac{1}{3}$ ($a_1 = a_2 > \frac{1}{3}$).

Exercise 4 (10 points)
A market is characterized by its demand function

$$X(p) = 12 - p.$$

A monopolistic producer sells his good via two retailers that compete in prices. First, the producer chooses the retail price p_P. Then, the retailers choose their prices p_1 and p_2, respectively. Production and delivery take place after prices are determined. The producer faces constant marginal and average costs of $c_P = 2$. Solve by backward induction! Determine the price that is paid by the consumers.

Aufgabe 4 (10 Punkte)
Gegeben sei ein Markt, der durch die Nachfragefunktion

$$X(p) = 12 - p$$

charakterisiert ist. Ein monopolistischer Produzent vertreibt das produzierte Konsumgut über zwei Händler, die sich im Preiswettbewerb befinden. Zunächst wählt der Produzent den Preis p_P, den er von den Händlern verlangt. Danach wählen die Händler ihre jeweiligen Preise p_1, bzw. p_2. Produktion und Lieferung finden erst statt, nachdem die Preise festgelegt wurden. Die konstanten Grenz- und Durchschnittskosten des Produzenten betragen $c_P = 2$. Lösen Sie per Rückwärtsinduktion! Wie lautet der Preis, den die Konsumenten zahlen?

Proposal of solution:
The two-stage game is solved by backward induction. The profit functions of the producer and the retailers are given by

$$\Pi_P = [p_P - c_P]\, X = [p_P - 2] \cdot [12 - p]$$

$$\Pi_{R,1} = \begin{cases} [p_1 - p_P]\,[12 - p_1]\,, & p_1 < p_2 \\ [p_1 - p_P]\,[6 - 0.5p_1] & p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

$$\Pi_{R,2} = \begin{cases} 0, & p_1 < p_2 \\ [p_2 - p_P]\,[6 - 0.5p_2] & p_1 = p_2 \\ [p_2 - p_P]\,[12 - p_2] & p_1 > p_2 \end{cases}$$

Starting at the second stage, retailers are in price competition. The retailer offering the good at the lower price, will get all the demand. This triggers a behavior of undercutting prices. The Bertrand paradox arises where both retailers choose

$$(p_1, p_2) = (p_P, p_P).$$

At the first stage, the producer can foresee the result of the price competition: $p = p_P$. His profit function can be adjusted to $\Pi_P = [p_P - 2] \cdot [12 - p_P]$. The FOC yields the optimal price.

$$\frac{\partial \Pi_P}{\partial p_P} = 12 - p_P - p_P + 2 \stackrel{!}{=} 0$$

$$p_P = 7$$

Since retailers sell to consumers at the same price at which they buy from producers, the price consumers pay is $p = p_P = 7$. The effect of double marginalization is revoked by the price competition at the second stage. So that consumers pay the same price as in the absence of the retailers, that is the monopolistic price of the producer $p_P^M = 7$.

Exercise 5 (12 points)
Two firms 1 and 2 compete in prices. The demand function is given by

$$X(p) = 26 - 2p.$$

Constant average and marginal costs are $c_1 = 7$ and $c_2 = 9$, respectively.

a) Show that the entry of firm 2 is not blockaded.

b) Firm 2 has the opportunity to realize an innovation which reduces its marginal costs to $c_2^n = 3$. Is it a drastic or a non-drastic innovation?

c) How much is firm 2 maximally willing to invest in research and development to realize the innovation?

Aufgabe 5 (12 Punkte)
Die Unternehmen 1 und 2 agieren an einem Markt im Preiswettbewerb. Die Nachfragefunktion ist gegeben durch

$$X(p) = 26 - 2p.$$

Die Grenz- und Durchschnittskosten sind konstant mit $c_1 = 7$ und $c_2 = 9$.

a) Zeigen Sie, dass der Eintritt für Unternehmen 2 nicht blockiert ist.

b) Unternehmen 2 hat die Möglichkeit zu innovieren, wodurch sich seine Grenzkosten auf $c_2^n = 3$ reduzieren. Handelt es sich hierbei um eine drastische oder nicht-drastische Innovation?

c) Wie hoch sind die Forschungs- und Entwicklungsausgaben, die Unternehmen 2 maximal bereit ist aufzubringen, um die Innovation zu realisieren?

Proposal of solution:

a) If $p^M(c_1) \geq c_2$, the market entry for firm 2 is not blockaded.

$$\Pi_1 = (p - 7)(26 - 2p)$$

$$\frac{\partial \Pi_1}{\partial p} = 26 - 2p - 2p + 14 \stackrel{!}{=} 0$$
$$40 = 4p$$
$$p^M(c_1) = 10$$

Since $p^M(c_1) = 10 > 9 = c_2$, the market entry of firm 2 is not blockaded.

b) The innovation is drastic if the market participation is blockaded for firm 1 that is $p^M(c_2^n) < c_1$.

$$\Pi_2 = (p - 3)(26 - 2p)$$

$$\frac{\partial \Pi_1}{\partial p} = 26 - 2p - 2p + 6 \stackrel{!}{=} 0$$
$$32 = 4p$$
$$p^M(c_2^n) = 8$$

The innovation is thus non-drastic.

c) Firm 2 can realize a positive profit by undercutting the constant average and marginal costs of firm 1 by a tiny bit: $p_2 = 7 - \varepsilon$. This results in a quantity $X(p_2) \approx 26 - 2 \cdot 7 = 12$. Firm 2 realizes a profit of

$$\begin{aligned} \Pi_2 \approx (7-3) \cdot (12) - F &\geq 0 \\ 48 &\geq F \end{aligned}$$

which should be non-negative. It is therefore willing to invest up to 48 to realize the innovation.