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# Two puzzles of judicial wagers

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## Abstract:

This paper is about an Old Indian judicial institution called *paṇa* (“wager”). Within a court proceeding, a judicial wager is a certain sum of money that a party to a conflict offers to pay if he ends up losing his case. This paper explains the rationale of judicial wagers by showing that they may signal truthfulness. In so doing, the paper proposes a solution to (what I call) the incentive puzzle of wagers. In the conclusion, it is argued that wagers are of doubtful merit. If this argument is correct, the second wager puzzle of scarce evidence (discussed by Lariviere) is solved.

Keywords: judicial wager, evidence, ordeals, pooling equilibrium, separating equilibrium

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## I. Introduction

Consider a defendant in Ancient India who is accused of a misdeed. If defendant and accuser are not able sort out this disagreement between themselves, they resort to the king for a judgement. The usual procedure is this: The king considers the evidence presented to him and decides in favour of the defendant or of the accuser.

Apart from the “objective” evidence, the parties to a legal conflict may try to underline the trueness of their respective assertions by other means. In particular, and with special relevance for Old Indian law, they may resort to ordeals. Ordeals are a manner of saying: “I am speaking the truth; this will be revealed by God.”<sup>2</sup> Apparently, a second manner to insist on one’s truthfulness is the “judicial wager” called *paṇa* in the Old Indian law literature. Basically, a judicial wager amounts to proclaiming: “I am speaking the truth; if found otherwise by the king, I will pay the appropriate fine, and, on top, make a payment of size  $w$ .” In the conclusion, wager-type institutions in other legal traditions are briefly commented upon.

Lariviere (1981b) presents the scarce textual evidence. Here let it suffice to present a verse from a famous law text, the Yājñavalkya Smṛti:

*sapaṇas ced vivādaḥ syāt tatra hīnaṃ tu dāpayet |*  
*daṇḍaṃ ca svapaṇaṃ caiva dhanine dhanam eva ca ||*<sup>3</sup>

If the dispute should be with a wager, then he should make the defeated party pay the fine and his own wager as well, but only the contested amount to its owner.<sup>4</sup>

There is no need to repeat Lariviere’s inconclusive findings in detail. They can be summarised (for our purposes) in the following manner:

- The wager may have been placed by one or by both parties.
- The recipient might have been the king (the court), the opponent, or even both.
- The size of the wager seems not to have been fixed and was probably up to each party.

In this paper, it is assumed that the wager amount was determined by the king, but that the two parties to the legal conflict could decide on whether they chose this amount or the amount zero. The king is assumed to be the recipient of a

<sup>2</sup> According to *Manu* 8.114 (Olivelle (2005, p. 173)), a defendant is to “carry fire, stay submerged in water, or touch separately the heads of his sons and wife. When the blazing fire does not burn a man, the water does not push him up to the surface, and no misfortune quickly strikes him, he should be judged innocent by reason of his oath.” Ordeals would have been carried out in the context of formal trials but also as so-called restorative ordeals (see Brick (2015)). For economic analyses of ordeals, see Leeson (2012) and Wiese (2016a).

<sup>3</sup> Yājñavalkya Smṛti 2.18, see Olivelle (2019)

<sup>4</sup> Lariviere (1981b, p. 135)

party's wager, but only if he decided the case against that party. Thus, one or two parties might risk a wager. The wager of that party is lost against whom the king pronounces his verdict.

While one might be tempted to think that the king has an incentive to rule against a party with a positive wager, Lariviere (1981b, p. 143) does not entertain this possibility (nor the opposite one!) when he writes: "The *paṇa* seems ... not to be a factor at all in deciding the case ... ." Let us assume such a Lariviere king for a moment. Such a king would simply ignore the wagers placed by the parties and decide on the evidence available to him. In that case, the parties do not have any incentive to offer a non-zero wager. If the ruling goes in their favor, they do not have to pay the wager. If the ruling goes against them, they lose the case and have to pay the wager as an additional fine. So, wagers seem a puzzle from the perspective of a Lariviere king. Furthermore, if the king is tempted to rule against a party that has placed a wager, this party doubly loses. First, it increases the possibility of a negative ruling. Second, it loses the wager. I call this the incentive puzzle: Why might a party to a judicial conflict offer a positive wager?

There is a second puzzle that becomes transparent from Lariviere's article. The verse cited above and two verses cited from the Nārada Smṛti (Lariviere (1981b, p. 135)) "point out what should be an important point in the general description of legal procedure since it divides all legal procedure into two categories. This is just the sort of thing which one would expect to find often repeated (or at least alluded to) in other basic *smṛtis*, but these three verses are the only ones that we find in the whole corpus of *dharma-śāstra*. This is unusual. It might not be so unusual if the verses gave a thorough and complete description of the *paṇa*, but that is hardly the case, and the context in which they occur does not shed any further light on the procedure. In both texts, the verses occur early in the discussion of legal procedure and are found with a hodge-podge of more or less unconnected and general statements about legal procedure."<sup>5</sup> I propose to call this the scarce-evidence puzzle.

In this paper, judicial wagers are analysed in game-theoretic terms. The king is assumed to act on two motivations. While he enjoys receiving the wager, he is also interested in passing just judgements. After all, if he is not considered a just king, he might risk losing his people's support. The importance of loyalty<sup>6</sup> is clearly spelled out in the *Arthaśāstra*:

*viraktā yānty amitraṃ vā bhartāraṃ ghnanti vā svayam* |<sup>7</sup>

<sup>5</sup> Lariviere (1981b, pp. 135–136).

<sup>6</sup> Wiese (2016b) argues that the so-called Varuṇa rule is a solution to the king's problem of wanting to be considered a just king.

<sup>7</sup> Arthaśāstra 7.5.27cd, see Kangle (1969).

when they are disloyal, they either go over to the enemy or kill their lord themselves.<sup>8</sup>

Now, while the king has some evidence for deciding a case, this evidence will often be far from conclusive. Then, so I like to argue, the wagers may help the king to arrive at a just verdict. Indeed, the wager risked by a party may indicate that party's confidence of winning the case. And this confidence in turn may be based on that party's knowledge about her innocence and the other party's dishonest dealings. Thus, the king might think that an accuser who files a correct complaint or an innocent defendant tend to risk the positive wager while dishonest accusers or defendants might not.

So far, these are speculations that need to be borne out by a more rigorous analysis. The methods to do so are provided by game theory. In order to arrive at theoretical predictions of how agents will behave, game theory uses the so-called equilibrium concepts. Roughly speaking, equilibria are situations where no-one has an incentive to choose otherwise. Thus, equilibria provide some sort of stability condition.

In the problem at hand, we need to turn to so-called signalling games.<sup>9</sup> The idea is that the king is confronted with differing signals where one party risks a wager while the other does not. The king might then rule in favour of the wager-risking party. However, given that the parties know the king's incentives, would they indeed be willing to give these differing signals? Why should we not expect the outcome where no party or both parties risk a wager?

In the game-theoretic signalling literature so-called pooling equilibria are distinguished from signalling equilibria. In our context a pooling equilibrium is characterized by both parties not risking a wager or by both parties risking a wager. In contrast, in a separating equilibrium, the two parties behave differently and hence the king—if so inclined—can infer the truthfulness of the agents from that different behaviour.

It turns out that one need to distinguish between a just king and an unjust king. For an unjust king, the importance of passing a correct judgement is smaller than the payoff he obtains from a positive wager. Such a king cannot use wagers as signals in a separating equilibrium. The parties will foresee that an unjust king prefers to cashing in on the wager rather than delivering a correct verdict. In contrast, the just king's payoff and his beliefs make at least one party choose a positive wager. A superjust king (for whom the justice payment is significantly above the wager payment) will always achieve a separating equilibrium. However, a king who is just, but not superjust, will enjoy a pooling equilibrium

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<sup>8</sup> Olivelle (2013).

<sup>9</sup> A suitable textbook for our purposes is Rasmusen (2009), in particular the signalling chapter.

where both parties place a positive wager. This king does not use the wagers as signals, but, somewhat maliciously, makes both parties place a positive wager. Thus, the king's payoff includes the wager (of one of the two parties), but obtains the justice payment only if his evidence is of sufficiently high quality.

The main part of this paper tackles the incentive puzzle by presenting a signalling game along the lines sketched above. The scarce-evidence puzzle is briefly dealt with in the conclusion.

## II. A model of judicial wagers

### A. *Wagers without signalling*

Assume a person  $d$  (defendant) who is accused by some other person  $a$  (accuser) of not paying back a loan  $x$ . Both  $d$  and  $a$  are free to place wagers  $w_d$  and  $w_a$  from  $\{0, w\}$ , respectively. Therefore, we have four wager combinations:  $(0,0)$ ,  $(w, 0)$ ,  $(0, w)$ , and  $(w, w)$ , where the first entry is the defendant's wager and the second the accuser's wager. That is, the wager combination  $(w, 0)$  means that the defendant places the positive wager  $w$ , while the defendant choose the zero wager. Now, let  $(w_d, w_a)$  be the prevalent wager combination. If party  $i$  ( $i \in \{d, a\}$ ) loses his case, he has to pay  $w_i$  (which might be zero) to the king.

If the king rules in favour of the defendant,  $d$  obtains zero payoff. However, if the defendant is found guilty, he has to pay (back) both the loan  $x$  and his wager  $w_d$ . In this latter case, the accuser obtains the amount  $x$  that he claims. Party  $i$ 's payoff is denoted by  $p_i$  and the payoffs for the defendant and the accuser (in that order) are expressed by

$$(1) \quad (p_d, p_a) = \begin{cases} (0, -w_a), & \text{verdict in favour of defendant} \\ (-x - w_d, x), & \text{verdict in favour of accuser} \end{cases}$$

This specific example is symmetric in the sense that the winning defendant avoids paying  $x$ , while the winning accuser wins  $x$ . Thus,  $x$  measures the contested amount that is at stake. By this symmetry, it does not matter who of the two parties is assumed to be innocent. In order to avoid repetitious arguments and calculations, we assume that the defendant is innocent.

The king decides on the basis of some evidence given to him and possibly also on the basis of the wagers risked by the two parties. The evidence  $e$  is binary and points the king to an innocent defendant ( $e = d$ ) or a truthtelling accuser ( $e = a$ ). We express  $e = d$  in an alternative manner by  $e = -a$ . The latter means that the evidence does not point favourably to the accuser, i.e., the evidence declares the defendant innocent. Inversely,  $e = a$  is equivalent to  $e = -d$ .

At the end of the trial, the king pronounces his verdict  $v$  which is again binary, i.e., the verdict clears the defendant ( $v = d$ ) or it pronounces the defendant guilty ( $v = a$ ). The king might pass his verdict in line with the evidence ( $v = e$ ) or, otherwise, against the evidence ( $v = -e$ ). His payoff depends on whether he takes the evidence into account. The evidence is correct with some probability  $q$ . Assuming  $\frac{1}{2} < q < 1$  we call  $q$  the quality of evidence. If the king thinks that the defendant is innocent by the strength of the evidence available to him, he is correct with probability  $q$ .

We assume that the king obtains a justice payoff  $J$  if his ruling is correct, i.e., if he decides against the guilty defendant or in favour of the defendant if the accusal is unjustified. The justice payment may represent the loyalty that the just king enjoys among his people. In the present section we assume that the king does not take differing wagers as a signal for truth-telling. One may say that the king does not entertain the belief that the wagers tell him anything about the truthfulness of the two parties involved. His expected payoff from passing a just judgement is  $qJ$  for a verdict following the evidence, but only  $(1 - q)J$  if he passes the verdict against the evidence. A Lariviere king would simply ignore any wagers and follow the evidence by  $q > 1/2$ . However, our king will enjoy the wager obtained by him if he rules against a wager-risking party. Summarizing, the king's payoff is given by

$$(2) \quad p_K(v) = \begin{cases} qJ + w_{-v}, & v = e \\ (1 - q)J + w_{-v}, & v = -e \end{cases}$$

The reader is invited to consult the appendix I where eq. 2<sup>App</sup> is a rewriting of eq. 2, with the king's payoff given as a function of the two possible verdicts  $v = d$  and  $v = a$ , the two possible evidences  $e = d$  and  $e = a$ , and the four wager combinations.

We will now consider two-stage models. At the first stage, the political parties decide on their wagers. At the second stage, the king pronounces a verdict. It is assumed that the parties correctly foresee the king's verdict based on the king's belief about the possible signalling potential of wagers (absent for now), based on the wagers placed by themselves, and based on the model's parameters ( $x$ ,  $q$ ,  $w$ , and  $J$ ). Generally, we are not interested in very specific parameter combinations. Thus, we might consider  $x < qJ$  and  $x > qJ$ , but feel free to disregard  $x = qJ$ . In some sense, the probability of encountering this equality is zero.

We now employ a game-theoretic equilibrium concept called backward induction (see Gibbons (1992, pp. 57–61)). In line with backward induction, we start with the last stage, the king's verdict. On the one hand, the king likes to follow the evidence ( $v = e$ ) by  $q > 1/2$ . On the other hand, the king is tempted to rule against a wager-risking party ( $v = -e$ ). Consider, for example,  $w_D = w$ ,  $w_A =$

0. After learning  $e = a$ , the king is happy to rule against the defendant by  $qJ + w > (1 - q)J + 0$ . If the evidence suggests that the defendant is innocent ( $e = d$ ), the king still proclaims the defendant guilty if  $(1 - q)J + w > qJ + 0$ , i.e., if  $w > (2q - 1)J$ . The king's optimal verdict is summarized in proposition A below.

After having ascertained the king's behaviour who has observed one of the four possible wager combinations, we try to make a theoretical prediction of the wager combination. Here, the theoretical predictions are typically made on the basis of so-called dominant actions and/or Nash equilibria (for these concepts, consult Gibbons (1992, pp. 1–12)). A dominant action is a best action irrespective of the other party's action. A Nash equilibrium is a condition for stability. Each party can choose the action "zero wager" or "positive wager". Thus, we arrive at four action combinations. An action combination consists of an action for each party, for example (defendant chooses zero wager, accuser chooses positive wager). A particular action combination is called a Nash equilibrium if no party can profit from deviating unilaterally. Thus, the action combination (defendant chooses zero wager, accuser chooses positive wager) is a Nash equilibrium if the defendant is not better off at the action combination (defendant chooses positive wager, accuser chooses positive wager) and if the accuser is not better off at the strategy combination (defendant zero wager, accuser chooses zero wager).

Revisit the action combination  $(w, 0)$  where the king might proclaim the defendant guilty. In that case, the (innocent!) defendant would have been cleared if he had chosen the zero wager as did his opponent. Indeed, a zero wager is always better than the positive wager in the present case. After all, the king likes to obtain the wager, but does not attribute any signal quality to the wager. Thus, we obtain

**Proposition A:**

Assume that the defendant is innocent. If the king does not entertain the belief that the wagers convey any signal about the parties' truthfulness, his verdict is given by

$$v = \begin{cases} d, & (w_d, w_a) \in \{(0,0), (0, w), (w, w)\} \\ d, & (w_d, w_a) = (w, 0) \text{ and } w < (2q - 1)J \\ a, & (w_d, w_a) = (w, 0) \text{ and } w > (2q - 1)J \end{cases}$$

Foreseeing these verdicts, each of the two parties has the dominant strategy 0 and the parties's equilibrium wagers are given by the strategy combination  $(w_d^*, w_a^*) = (0,0)$ .

**B. Using wagers for signalling purposes**

Let us now turn to a king who entertains the following belief: If one of the parties places a positive wager, while the other opts for a zero wager, the former is the honest party with probability 1. Thus, the king would simply disregard the evidence  $e$  and the quality  $q$  of the evidence in the case of differing wagers. If the wagers are both zero or both equal to  $w$ , the king cannot learn anything from the wagers and would be back to his informational status as in the previous section. We call this belief on the king's part the signalling belief.

If the king tries to use the wagers as a signal for truthtelling, he may succeed or he may fail. In the model below, equilibria are of the pooling type or of the separating type. Of course, separating equilibria are more interesting for our purposes because they show the wagers' potential to signal truthfulness. It is important to note that we do not just claim that wagers can serve as signals but show this function in a separating equilibrium.

The parties' payoff functions are as in the previous section. Based on the belief specified above, the king's payoff function is now given by

$$(3) \quad p_K(v, e, w_d, w_a) = \begin{cases} 0 \cdot J + w, & w_v = 0, w_{-v} = w \\ J + 0, & w_v = w, w_{-v} = 0 \\ qJ + w_{-v}, & v = e, w_d = w_a \\ (1 - q)J + w_{-v}, & v = -e, w_d = w_a \end{cases}$$

Some readers might find helpful the equivalent eq. 3<sup>App</sup> found in appendix II. In the first two lines of eq. (3), the parties have placed different wagers. In the first line, the king rules in favour of the party with a zero wager. Hence, the king's belief make him surmise that he has rules in favour of the undeserving party. Hence, his justice payoff is zero. However, since he has ruled against the party with a positive wager, he obtains this wager as a payoff. In the second line, the king believes that he rules in favour of the deserving party. Thus, his justice payoff is  $J$ . But now he does not obtain a wager payoff because he has ruled against the zero-wager party.

In the last two lines, the wagers are the same for both parties. Hence, the king cannot use the wagers as a signal for truthtelling. He resorts to the evidence which comes at a specific quality. The king's payoff is as in eq. (2) above.

On the basis of the king's signalling belief, we now look for equilibria which may turn out to be pooling or separating ones. Applying, again, backward induction, we start with the king's verdict. The king has observed the parties' wagers so that we distinguish between three cases:

1.  $w_d = w_a$

Comparing the last two lines in the king's payoff function (3), the king's wager payoff does not depend on whether he rules in line with the evidence or not. By  $q > 1/2$  the king's payoff is maximal if he follows the evidence.

$$2. w_d = w, w_a = 0$$

The king who rules against the accusant, obtains the justice payoff  $J$ , but no wager payoff. In contrast, if he rules against the defendant, he obtains the justice payoff zero, but the wager payoff  $w$ . Thus, the king follows the wager signal and rules in favour of the defendant if  $J > w$  holds. Such a king will be called a just king. For him, his justice payment is larger than the positive wager payment.

$$3. w_d = 0, w_a = w$$

Here, the wager placements are inverse to those in the previous case. Thus, the just king rules in favour of the accusant and the unjust king in favour of the defendant.

Let us summarize the king's verdict for all the four wager combinations:

**Proposition B1:**

Assume that the defendant is innocent. If the king entertains the signalling belief, his verdict is given by

$$v = \begin{cases} e, & w_d = w_a \\ d, & (w_d, w_a) = (w, 0) \text{ and } w < J \\ a, & (w_d, w_a) = (w, 0) \text{ and } w > J \\ a, & (w_d, w_a) = (0, w) \text{ and } w < J \\ d, & (w_d, w_a) = (0, w) \text{ and } w > J \end{cases}$$

After having shown how the king decides, we now turn to the parties' decisions. It is assumed they foresee the rational king's behaviour. Also, all the model's parameters are common knowledge. Thus, the parties take the king's decision as described above into account.

Assume that the defendant is innocent, a fact known to the two parties, but not to the king. However, the king might infer the truth-telling party from differing wagers (in line with his belief) or might rely on the evidence which is correct with probability  $q$ . In case of an unjust king, we obtain the following payoff matrix for the two parties:

	$w_a = 0$	$w_a = w$
$w_d = 0$	$[(1 - q)(-x), (1 - q)x]$	$[0, -w]$
$w_d = w$	$[-x - w, x]$	$[(1 - q)(-x - w), q(-w) + (1 - q)x]$

Now compare the defendant's payoffs in case of  $w_a = 0$ . Apparently,  $w_d = 0$  is the best action for the defendant. And  $w_d = 0$  is also the best action in case of

$w_a = w$ . Thus, the defendant has a dominant strategy, namely to choose the zero wager. By a similar argument, the accuser also has the zero wager as a dominant action. Summarizing, we obtain:

**Proposition B2:**

Assume that the defendant is innocent. Assume an unjust king (for whom  $w > J$  holds). On the basis of Proposition B1, the action combination  $(w_a^*, w_d^*) = (0,0)$  is a pooling equilibrium (in dominant actions). The unjust king then obtains the payoff  $qJ$ .

Let us now turn to the more interesting case of a just king ( $J > w$ ). Since he decides differently from an unjust king, we obtain another payoff matrix for the two parties:

	$w_a = 0$	$w_a = w$
$w_d = 0$	$[(1 - q)(-x), (1 - q)x]$	$[-x, x]$
$w_d = w$	$[0,0]$	$[(1 - q)(-x - w), q(-w) + (1 - q)x]$

If one party chooses the zero wager, the other party can safely choose the positive wager, because the just king will then rule in favour of the latter party. From this observation,  $(0,0)$  cannot be a Nash equilibrium. Consult appendix III for a proof of the following proposition:

**Proposition B3:**

Assume that the defendant is innocent. Assume a just king (for whom  $w < J$  holds). We obtain three parameter constellations with the following equilibrium outcomes:

P  $\frac{x}{w} \leq \frac{1-q}{q}$  (small stake  $x$ , large wager  $w$ , and/or low quality of evidence  $q$ )

From theoretical grounds, we predict the separating equilibrium  $(w, 0)$ .

The king's payoff in that equilibrium is just  $J$ .

Q  $\frac{1-q}{q} < \frac{x}{w} < \frac{q}{1-q}$  (medium stake, wager, and/or quality of evidence)

In this case, there exists exactly one equilibrium, namely the separating equilibrium  $(w, 0)$ . Again, the king's payoff is  $J$ .

R  $\frac{x}{w} > \frac{q}{1-q}$  (large stake, small wager, and/or low quality of evidence)

In this case, there exists exactly one equilibrium, namely the pooling equilibrium  $(w, w)$ . The king's payoff is  $qJ + w$ .

The reader is invited to consult the figure below. If the king is unjust ( $J < w$ ), we obtain the pooling equilibrium of Proposition B2. A just king is able to achieve separating equilibria, namely constellations P and Q where the stake is sufficiently small, the wager sufficiently large, and/or the evidence of sufficiently high quality. In a separating equilibrium the king rules in favour of the

party that has placed a positive wager and against the party with the zero wager. Then the king can be sure to obtain the justice payoff, but does not receive any wager payoff.

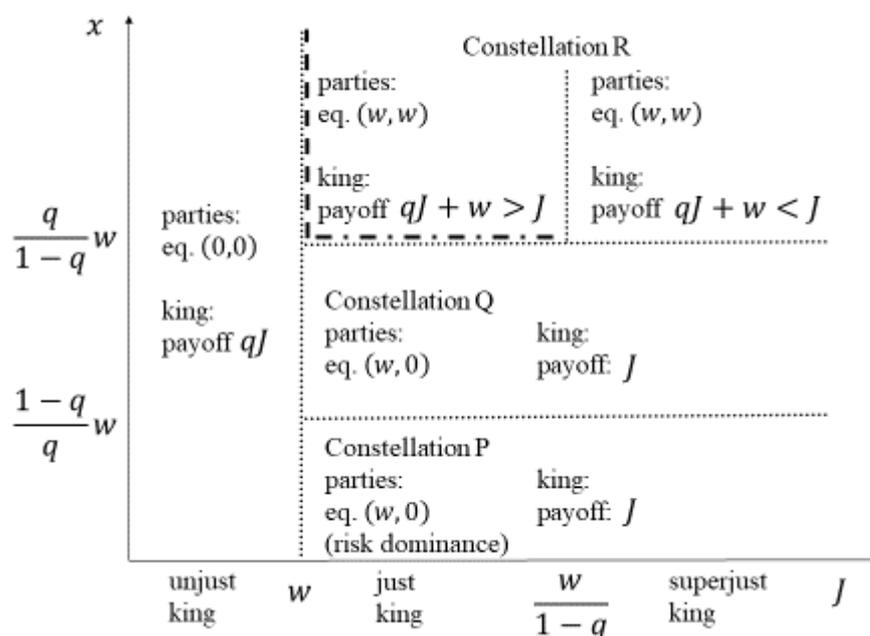


Figure: Pooling and separating equilibria

In constellation R the king is just. However, the wager is so small (or the stake so large) that both parties choose the positive wager. In that case, the just king does not achieve separation. His justice payoff is uncertain and depends on the quality of evidence. However, this just king has the advantage of obtain the wager, either from the defendant or from the accuser.

An interesting comparison concerns the payoff  $qJ + w$  in constellation R in relation to the payoff  $J$  in the constellations P or Q. One finds that the non-pooling payoff  $qJ + w$  is larger than the separating payoff  $J$  if  $J < \frac{w}{1-q}$  holds. Note that the interval  $w < J < \frac{w}{1-q}$  is nonempty. Thus, the payoff  $qJ + w$  is obtained by a king who is just (so that the pooling equilibrium  $(w, w)$  comes about), but not overly just (so that the king prefers to cash in on a wager rather than pronouncing a correct verdict with probability 1). In contrast, we call the king superjust if  $J > \frac{w}{1-q}$  holds. This king would prefer a separating equilibrium in constellations P or Q over the pooling equilibrium in constellation R.

### C. The optimal wager amount from the king's point of view

So far, we have considered two-stage models with the two stages of simultaneous wager decisions by the parties and verdict by the king. Let us now assume that the king himself fixes the amount of the wager  $w$ , before the parties decide for a zero wager or for the wager  $w$ . That is, we turn to a three stage model. By backward induction, the last two stages are solved as above. Thus, we can turn to the king who determines  $w$ .

By choosing a sufficiently large  $w$  (higher than  $J$ ), the king can make sure of being considered a just king. In fact, the king can even make sure that constellations P or Q hold. Again, he just needs to choose a sufficiently large wager amount  $w$ . In this manner, the king can guarantee a separating equilibrium and successfully employ wagers as signals of truth-telling. Since the (just) king's payoff in constellations P and Q is higher than the (unjust) king's payoff, it is surely in the king's interest to strive for P or Q rather than the left (unjust) region in the figure. The following proposition summarizes the main result, but turn to appendix IV for specifics.

**Proposition C:**

Assume that the defendant is innocent. The king will never choose a wager amount that would make him unjust. In case of  $qJ < x$ , the payoff maximizing king chooses the wager  $w^* = \min\left(J, \frac{1-q}{q}x\right)$  leading to the pooling equilibrium  $(w, w)$ . If, on the contrary,  $qJ > x$  holds, the king chooses a sufficiently high wager so as to guarantee a separating equilibrium.

Thus, it turns out that a king with reliable evidence (large  $q$ ), with a lot to gain by correct verdicts in terms of the justice payoff  $J$  and when facing a low-stake trial, can achieve and likes to achieve a separating equilibrium.

### III. Conclusion

In this conclusion, I like to comment on related judicial institutions and come back to the two puzzles mentioned in the introduction.

If “objective” evidence is not used by a judge, ordeals or wagers may have been used in pre-modern India. Related to both ordeals and wagers is the nearly 1000 years old English institution of “trial by battle” used to settle unclear land disputes. Here, representatives of the opponents fought against each other with clubs, and the winning party obtained (or kept) the contested land. An economic analysis is provided by Leeson (2011). The opponents hire champions to fight for them and the outcome is mainly dependent on the money spent to hire a champion (or even several, in order to dry out the champions market for the opponent). There are important differences between a trial by battle and a trial with

a wager. The important similarity consists in the fact that the opponents need to risk money. In the Indian case, the *pana* is wagered and has to be paid only if the king's ruling is adverse. In the English trials by battle, the money spent for champions is lost for both good or bad outcomes. Significantly, this English institution did not survive for long.

While this paper deals with judicial wagers in India, Matthiass (1888, pp. 5–18, 1912, pp. 341–347) argues that they were also present in other Indo-European judicial traditions. The author understands wagers as central to the transition from “self-help, that is, physical combat” (Matthiass (1912, p. 342)) to increasingly formalised third-party involvement: “If the contending parties appealed to a trusted person they merely gave up the personal encounter, in the place of which there now appeared the assertion by each party that he was right; in other words, contending opinions took the place of personal conflict. Each of the parties had to show evidence of the earnestness of his opinion and of his firm belief in his contention, else there would have been no inducement why he should surrender his right to self-help. Substitution of personal opinion was appropriately followed by a deposit made with the trusted party; this deposit, which the trusted party was to surrender to the victor, constituted the penalty that the defeated party incurred. Thus, we see that the parties made a wager and that the oldest form of arbitral court was the wager-court” (Matthiass (1912, p. 342)). Later on, argues Matthiass (1912, p. 343), wagers became increasingly important as fees for these third parties. Matthiass links wagers to deposits and court fees. Both connections are plausible steps in the evolution of law. Note, however, that it would be unusual to let the conflicting parties individually decide on these fees. In any case, the motives for voluntarily placing positive wagers are still left unexplained. And this is where the present paper tries to make a contribution.

Addressing the incentive puzzle, wagers can be rationalised in the following manner: The honest party to a conflict is more willing to risk a wager than the dishonest party. Indeed, if both parties have placed a positive wager, the innocent one can hope to win if the quality of evidence is sufficiently large. Having the possibility of differing signals in mind, the king may be happy to choose relative high wagers that make the honest party risk the wager and make the dishonest one choose the zero wager. This holds if the stakes are small in relation to the expected justice payment. Inversely, however, the king may choose to be just, but not superjust and to determine a wager amount that is relatively large, but sufficiently small so that both parties choose the positive wager.

It is clear from our model, that judicial wagers have serious drawbacks. First, a cash-stripped party may just not be able to place the wager amount required by the king. Then, separation is not driven by the honesty or truthfulness of the parties, but by their more or less deep pockets. This fact will surely make a king's subjects suspicious of that institution. Additionally, the subjects will sometimes

observe that the king obtains the wager amount. That, also, will not contribute to the king's reputation. The parties may suspect that the king has financial reasons when using the wagers as a basis for his judgement. Doing so and/or the suspicion that he might do so, will certainly undermine any confidence in the justice system.<sup>10</sup> Indeed, our model has shown how the king may then be torn between two motives. On the one hand, he takes the positive wager as an indication for truthful behavior and tends to rule in favor of the only party risking the wager. On the other hand, ruling against the party with the positive wager is financially profitable for the king. For these mixed motives, one may conjecture that a third party like the Brahmins, rather than the king himself, was the recipient. However, the *nibandhakāra* evidence collected by Lariviere (1981b) does not provide any support in this direction.

Leeson (2012) and Wiese (2016a) show why ordeals might have been quite sensible institutions. The theoretical ideas put forward in these papers go together well with the fact that ordeals were quite successful and in use for many centuries. In contrast, judicial wagers seem to have gone out of fashion many centuries before ordeals did (compare Lariviere (1981b, p. 144) and Lariviere (1981a)). From the point of view of the current paper, the problematic nature of judicial wagers just sketched may underlie their factual failure, somewhat similar to the failure of trial by battle. Of course, *dharmasāstra* authors may not find good reason to write extensively about an institution long gone extinct. This is probably the solution to the scarce-evidence puzzle.

## Appendix I

Eq. (2) can be rewritten as follows:

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<sup>10</sup> In this context, note the “*Varuṇa* rule”. It stipulates that the king is to throw confiscated property into water. This apparent waste of resources calls out for an explanation (attempted by Wiese 2016b).

$$(2^{App}) \quad p_K(v, e, w_d, w_a) = \begin{cases} qJ, & v = d, e = d, w_d = 0, w_a = 0 \\ (1 - q)J, & v = d, e = a, w_d = 0, w_a = 0 \\ (1 - q)J, & v = a, e = d, w_d = 0, w_a = 0 \\ qJ, & v = a, e = a, w_d = 0, w_a = 0 \\ qJ + w, & v = d, e = d, w_d = w, w_a = w \\ (1 - q)J + w, & v = d, e = a, w_d = w, w_a = w \\ (1 - q)J + w, & v = a, e = d, w_d = w, w_a = w \\ qJ + w, & v = a, e = a, w_d = w, w_a = w \\ qJ, & v = d, e = d, w_d = w, w_a = 0 \\ (1 - q)J, & v = d, e = a, w_d = w, w_a = 0 \\ (1 - q)J + w, & v = a, e = d, w_d = w, w_a = 0 \\ qJ + w, & v = a, e = a, w_d = w, w_a = 0 \\ qJ + w, & v = d, e = d, w_d = 0, w_a = w \\ (1 - q)J + w, & v = d, e = a, w_d = 0, w_a = w \\ (1 - q)J, & v = a, e = d, w_d = 0, w_a = w \\ qJ, & v = a, e = a, w_d = 0, w_a = w \end{cases}$$

## Appendix II

Eq. (3) can be rewritten as follows:

$$(3^{App}) \quad p_K(v, e, w_d, w_a) = \begin{cases} qJ, & v = d, e = d, w_d = 0, w_a = 0 \\ (1 - q)J, & v = d, e = a, w_d = 0, w_a = 0 \\ (1 - q)J, & v = a, e = d, w_d = 0, w_a = 0 \\ qJ, & v = a, e = a, w_d = 0, w_a = 0 \\ qJ + w, & v = d, e = d, w_d = w, w_a = w \\ (1 - q)J + w, & v = d, e = a, w_d = w, w_a = w \\ (1 - q)J + w, & v = a, e = d, w_d = w, w_a = w \\ qJ + w, & v = a, e = a, w_d = w, w_a = w \\ J, & v = d, w_d = w, w_a = 0 \\ w, & v = a, w_d = w, w_a = 0 \\ w, & v = d, w_d = 0, w_a = w \\ J & v = a, w_d = 0, w_a = w \end{cases}$$

## Appendix III

Note the following inequalities

$$(1 - q)(-x) < 0 \text{ (D1) and}$$

$$-x \geq (1 - q)(-x - w) \text{ (D2)}$$

for the defendant and similarly

$(1 - q)x < x$  (A1) and  
 $q(-w) + (1 - q)x \leq 0$  (A2)  
 for the accuser. D1 and A1 always hold.

We need to consider three parameter constellations P, Q and R:

$$P \quad \frac{x}{w} \leq \frac{1-q}{q}$$

The inequality of constellation P is a rewriting of inequality D2. Here,  $(0, w)$  is an equilibrium by D2 and A1. Observe that  $\frac{1-q}{q} < \frac{q}{1-q}$ . Hence, we also have  $\frac{x}{w} \leq \frac{q}{1-q}$  or, equivalently, A2. Thus,  $(w, 0)$  is a second equilibrium, namely by A2 and D1. Of course, two equilibria amounts to two theoretical prediction in constellation P. The criterion of risk-dominance allows to favour the second equilibrium over the first one. In applying that criterion (see Harsanyi and Selten (1988)), one looks at each equilibrium and calculates the products of deviations. Consider the second equilibrium with payoffs 0 for both parties. If the defendant deviates, his payoff is reduced from 0 to  $(1 - q)(-x)$ , with difference  $0 - (1 - q)(-x)$ . If the accuser deviates, his payoff is reduced from 0 to  $q(-w) + (1 - q)x$ , with difference  $0 - (q(-w) + (1 - q)x)$ . In the present case, the second equilibrium risk-dominates the first one if  $[0 - (1 - q)(-x)][0 - (q(-w) + (1 - q)x)] > [-x - (1 - q)(-x - w)][x - (1 - q)x]$  holds. This inequality is equivalent to  $q > \frac{1}{2}$ . Therefore, our theoretical prediction in case P is the equilibrium  $(w, 0)$ .

$$Q \quad \frac{1-q}{q} < \frac{x}{w} < \frac{q}{1-q}$$

As shown under constellation P,  $(w, 0)$  is an equilibrium. Since D2 is violated,  $(0, w)$  is not an equilibrium in this case. By these two observations, the other two possible action combinations are also ruled out as equilibria.

$$R \quad \frac{x}{w} > \frac{q}{1-q} \text{ (large stake, small wager, and/or low quality of evidence)}$$

In constellation R,  $(w, 0)$  is also ruled out as an equilibrium. By violating both D2 and A2,  $(w, 0)$  is an equilibrium and, indeed, the only one.

## Appendix IV

Let us first look for an optimal  $w$  within region R where the king's payoff is  $qJ + w$ . The payoff maximizing king chooses a maximal wager amount  $w$ , but has to make sure that he is considered just ( $w < J$ ) and that both parties choose

the positive wager ( $w < \frac{1-q}{q}x$ ). Within constellation R, the king therefore chooses the payoff maximizing wager  $w^* = \min\left(J, \frac{1-q}{q}x\right)$ .

Since the king can achieve a separating equilibrium in constellations P or Q, the relevant comparison is between  $qJ + \min\left(J, \frac{1-q}{q}x\right)$  and  $J$ . We obtain two cases.

1. case  $J < \frac{1-q}{q}x$  (or, equivalently,  $x > \frac{q}{1-q}J$ ) with  $w^* = J$

Here, the king chooses to be barely just. The wager is so small that both parties may choose this positive wager in equilibrium. However, they both might also choose the zero wager in equilibrium. The king prefers this pooling equilibrium over the separating one by  $qJ + J > J$ . In terms of the figure, the vertical dashed line in bold type is addressed.

2. case  $J > \frac{1-q}{q}x$  (or, equivalently,  $x < \frac{q}{1-q}J$ ) with  $w^* = \frac{1-q}{q}x$

Here, the king is just with  $J > w$ . Both parties can choose the positive wager in equilibrium. The king prefers this pooling equilibrium over the separating one if  $qJ + \frac{1-q}{q}x > J$  or, equivalently, if  $x > qJ$  holds. Note that the interval given by  $qJ < x < \frac{q}{1-q}J$  is nonempty. In terms of the figure, the horizontal irregularly dashed line in bold type is addressed. Inversely, in case of  $x < qJ$ , the king would choose sufficiently large wagers that lead to constellations P or Q.

## ***G Publication bibliography***

Brick, David (2015): *Brahmanical Theories of the Gift*. Cambridge (MA): Harvard University Press.

Gibbons, Robert (1992): *A primer in game theory*. New York, NY, Harlow: Harvester Wheatsheaf; Prentice Hall Financial Times.

Harsanyi, John C.; Selten, Reinhard (1988): *A general theory of equilibrium selection in games*. Cambridge: MIT Press.

Kangle, R. P. (1969): *The Kauṭilya Arthaśāstra. Part I: Sanskrit Text with a Glossary*. 2<sup>nd</sup> ed. Bombay: Bombay University.

- Lariviere, Richard W. (1981a): *The Divyatattva of Raghunandana Bhaṭṭācārya. Ordeals in classical Hindu law.* New Delhi: Manohar.
- Lariviere, Richard W. (1981b): *The Judicial Wager in Hindu Law.* In *Annals of the Bhandarkar Oriental Research Institute* 62, pp. 135–145.
- Leeson, Peter T. (2011): *Trials by Battle.* In *Journal of Legal Analysis* 3, pp. 341–375.
- Leeson, Peter T. (2012): *Ordeals.* In *The Journal of Law and Economics* 55 (3), pp. 691–714. DOI: 10.1086/664010.
- Matthiass, Bernhard (1888): *Die Entwicklung des römischen Schiedsgerichts.* Rostock: Commissions-Verlag der Stiller'schen Hof- u. Universitätsbuchhandlung.
- Matthiass, Bernhard (1912): *Partial extract of Matthiass (1888) in English.* In *The American Journal of International Law* 6, pp. 341–347.
- Olivelle, Patrick (2005): *Manu's code of law. A critical edition and translation of the Mānava-Dharmaśāstra (South Asia research).*
- Olivelle, Patrick (2013): *King, governance, and law in ancient India. Kauṭilya's Arthaśāstra.* New York, NY: Oxford Univ. Press.
- Olivelle, Patrick (2019): *Yajnavalkya: a Treatise on Dharma.* Cambridge (MA): Cambridge University Press.
- Rasmusen, Eric (2009): *Games and information. An introduction to game theory.* 4<sup>th</sup> ed. Malden: Blackwell.
- Wiese, Harald (2016a): *An economic vindication of Ancient Indian 'Nonsense'.* In *Bulletin of the School of Oriental and African Studies* 79, pp. 513–534.

Wiese, Harald (2016b): Indian principal-agent theory, or, how Varuṇa helps the king to be just. In *Asiatische Studien – Études Asiatiques* 70, pp. 571–589.