Microeconomic Analyses of Old Indian Texts

Preferences

Harald Wiese

University of Leipzig

Course overview

- Chapter I:
  Introduction: Old Indian literature and microeconomics

Part A. Decision theory

- **Chapter II:**
  Preferences
- Chapter III:
  Decisions
- Chapter IV:
  Decision theory for the Bhagavad Gita
- Chapter V:
  Monopoly theory and Kautilya's market tax

Chapter II: Preferences

1. Relations
2. Preference relations
3. Preferences in the Hitopadeśa
4. Functions and derivations
5. Preferences in two-dimensional space
6. Convex preferences and the Middle Way
7. Monotonic preferences and contentment
8. Equanimity

Relations

three examples

Example

For any two inhabitants from Leipzig, we ask whether

- one is the father of the other or
- they are of the same sex.

Example

For the set of integers $\mathbb{Z}$ (the numbers ..., -2, -1, 0, 1, 2, ...), we consider the difference and examine whether this difference is an even number (i.e., from ..., $-2, 0, 2, 4$, ...).

Relations

properties

Definition

A relation $\sim$ on a set X is called

- reflexive if $x \sim x$ holds for all $x \in X$,
- transitive if $x \sim y$ and $y \sim z$ imply $x \sim z$ for all $x, y, z \in X$,
- symmetric if $x \sim y$ implies $y \sim x$ for all $x, y \in X$,
- antisymmetric if $x \sim y$ and $y \sim x$ imply $x = y$ for all $x, y \in X$,
- asymmetric if $x \sim y$ implies not $y \sim x$, and
- complete if $x \sim y$ or $y \sim x$ holds for all $x, y \in X$, $x \neq y$.

Relations

difference is even number

Lemma

On the set of integers $\mathbb{Z}$, the relation $\sim$ defined by

$$x \sim y :\Leftrightarrow x - y \text{ is an even number}$$

is reflexive, transitive, and symmetric, but neither antisymmetric nor complete.

":$\Leftrightarrow$" means: the expression left of the colon is defined by the expression right of the equivalence sign.

Relations

difference is even number (proof)

- Reflexivity: $x - x = 0$ for all $x \in \mathbb{Z}$ and hence $x \sim x$
- Transitivity: any three integers x, y, z with $x \sim y$ and $y \sim z$. Then,

$$\begin{aligned} & (x - y) + (y - z) \\ = \quad & x - z \end{aligned}$$

  is also even.
- Symmetry: a number is even if and only if its negative is even.
- Not complete: neither $0 \nsim 1$ nor $1 \nsim 0$.
- Not antisymmetric: $0 \sim 2$, but not $0 = 2$

Relations

exercise on properties

Which properties do the relations "is the father of" and "is of the same sex as" have? Fill in "yes" or "no":

	relation	
property	is the father of	is of the same sex as
reflexive		
transitive		
symmetric		
antisymmetric		
asymmetric		
complete		

Definition (equivalence relation)

Let $\sim$ be a relation on a set X which obeys reflexivity, transitivity and symmetry. Then, any two elements $x, y \in X$ with $x \sim y$ are called equivalent and $\sim$ is called an equivalence relation.

$$[x] := \{y \in X : y \sim x\}.$$

is called an equivalence class.
Equivalence relation with two equivalence classes:

$$\begin{aligned} [0] &= \{y \in \mathbb{Z} : y \sim 0\} = \{..., -2, 0, 2, 4, ...\} \text{ and} \\ [1] &= \{y \in \mathbb{Z} : y \sim 1\} = \{..., -3, -1, 1, 3, ...\} \end{aligned}$$

Relations

exercise on equivalence classes

- Continuing the above example, find the equivalence classes $[17]$, $[-23]$, and $[100]$.
- Reconsider the relation "is of the same sex as". Can you describe its equivalence classes?

Preference relations

definition

Definition (preference relation)

A (weak) preference relation on X is denoted by $\succsim$ where $x \succsim y$ means "x is at least as good (as preferable, as virtuous) as y". Weak preference relations are always reflexive, transitive and complete. The indifference relation (derived from $\succsim$) is defined by

$$\begin{aligned} x &\sim y \text{ means} \\ x &\succsim y \text{ and } y \succsim x \end{aligned}$$

and the strict preference relation (derived from $\succsim$) is defined by

$$\begin{aligned} x &\succ y \text{ means} \\ x &\succsim y \text{ and not } y \succsim x. \end{aligned}$$

Preference relations

completeness

Every agent's preferences between any two objects x and y are

- either $x \prec y$: the agent strictly prefers y to x
- or $y \prec x$: the agent strictly prefers x is to y
- or $x \sim y$: the agent is indifferent between x and y.

Completeness means: the agent "knows what he wants"
Later chapter: Árjuna's decision problem as presented in the *Bhagavad Gītā*

Relations

exercise on preference relation

1. Assume

$$A \succsim B, C \sim E, C \succ A, D \sim A$$

Can you write the preferences for these bundles in one line where every bundle shows up once, only? How about E versus A?

2. How about

$$A \succsim B, C \succ E, C \succ A, D \sim A$$

and

3.

$$A \succsim B, B \sim E, C \succ A, E \succsim C$$

Preferences in the Hitopadeśa I

In microeconomics, preferences are given and researches typically stay clear of criticizing preferences. The Hitopadeśa does not show any inhibition in this respect:

> *Better to have a single virtuous son than a hundred fools! One moon destroys darkness, but not even a multitude of stars can do so.*
>
> *...*
>
> *A large income, perpetual health, a wife who is dear and speaks pleasantly, an obedient son and money-making know-how–these are the six sources of happiness in this world, O king.*

Microeconomics: A large income is better than a small one etc.

Preferences in the Hitopadeśa II

Four aims:

> *The birth of a person who does not succeed even in one of the four life-aims–to fulfill one's duties, obtain riches, satisfy one's desires or attain final release–is as useless as a nipple on a nanny-goat's neck.*

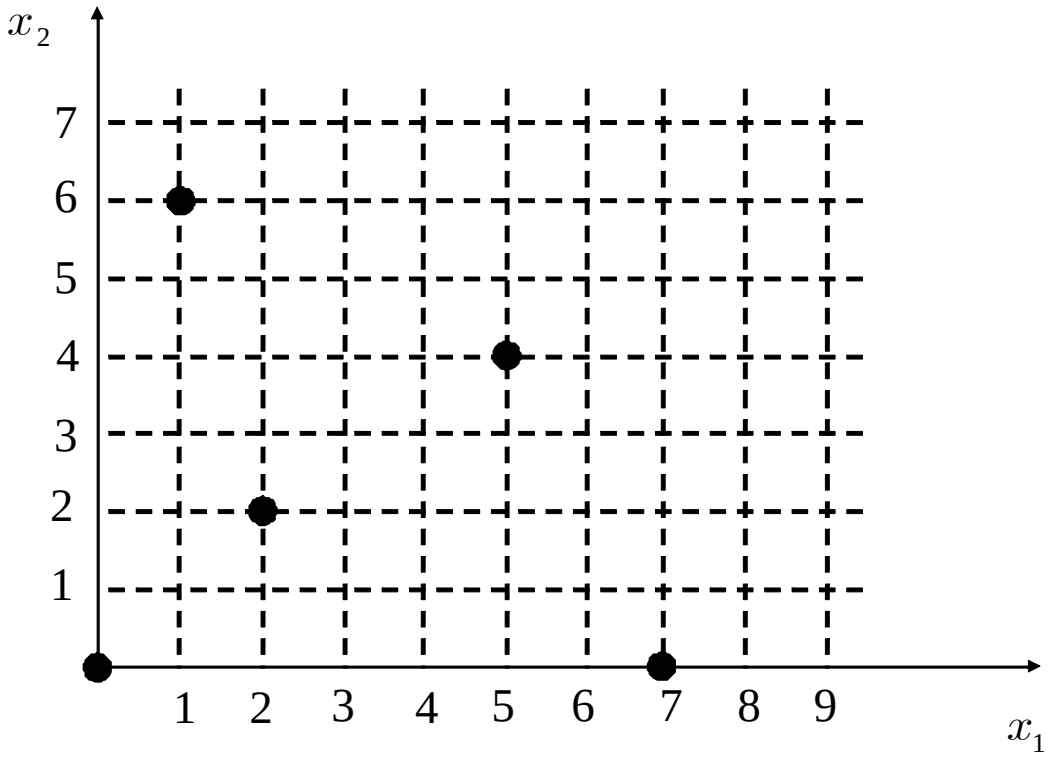
But: *mokṣa* is also recognized as somewhat egoistic.
The Buddha as elefant:

> *My endeavor is not for a good rebirth,*
> *nor for the glory of my royal parasol,*
> *nor for heaven with its fine undiluted pleasures,*
> *nor for Brahma's splendor, nor liberation's joy.*

Functions and derivations

System of coordinates

Points in the two-dimensional space $\mathbb{R}^2$ denoted by (x_1, x_2) or by (x, y).
Where are the points $(7, 0)$, $(1, 6)$, $(4, 5)$, $(0, 0)$?



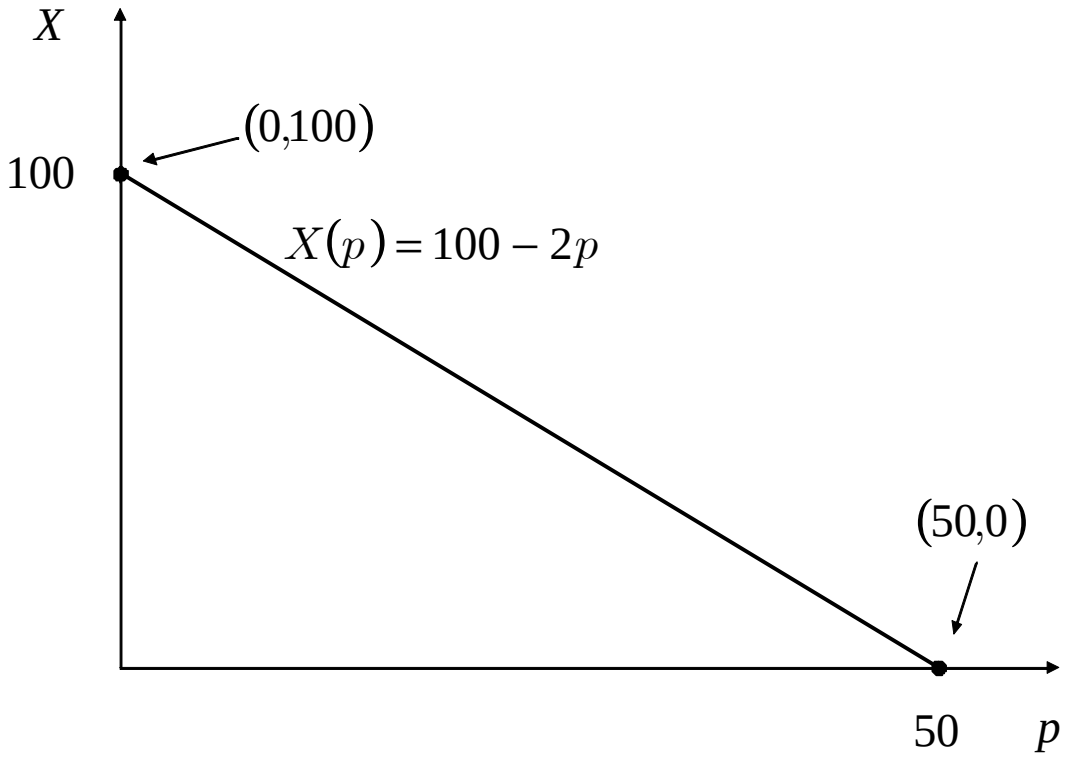
Functions and derivations

Functions I

- arguments (the input)
  Examples are
  - cost functions with input: quantity of good to be produced
  - utility function with input: bundle of goods (3 apples and 2 bananas)
- values (the output)
  Examples are
  - cost functions with output: sum of money
  - utility function with output: utility of 5
- a formula
  Examples are
  - cost functions given by $c(y) = 2y^2$
  - utility function given by $U(x, y) = x + y$

Functions and derivations

Functions II



- abscissa, here p-coordinate
- ordinate, here X-coordinate

Functions and derivations

Exercise on demand function

Consider the demand function given by

$$X(p) = 200 - 4p.$$

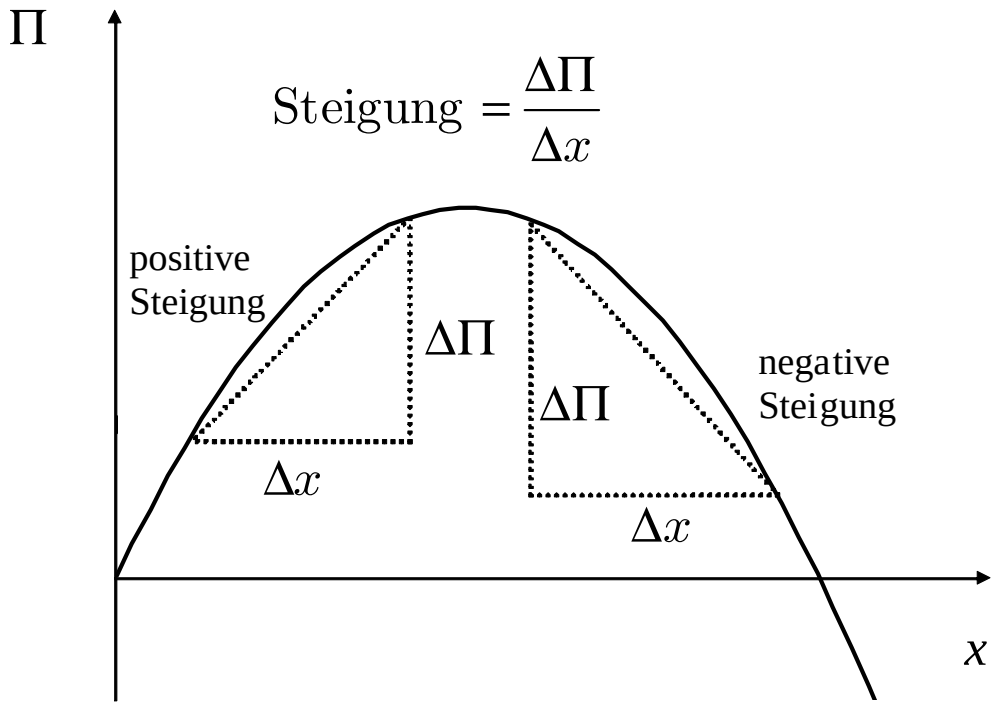
Draw the function. Determine

- the prohibitive price (i.e., the price at which the quantity demanded is zero: $X(p) = 0$)
- the satiation quantity (i.e., the quantity for price zero: $X(0)$)

Functions and derivations

Slopes: discrete

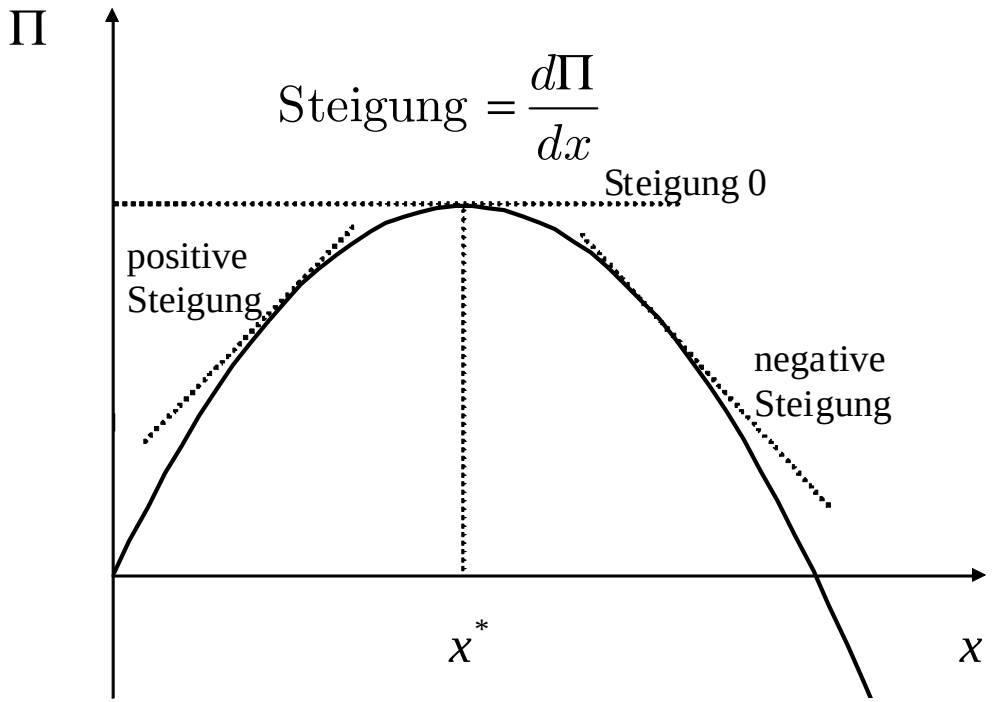
- Slope is positive for $\frac{\Delta \Pi}{\Delta x} > 0$
- Slope is negative for $\frac{\Delta \Pi}{\Delta x} < 0$



Functions and derivations

Slopes: continuous I

Let the interval Δx get smaller and smaller ...



The slope can be calculated by the first derivative of a function.

Functions and derivations

Slopes: continuous II

- $f(x) = 4x \rightarrow f'(x) = 4$
- $f(x) = 4 \rightarrow f'(x) = 0$
- $f(x) = 7x^2 \rightarrow f'(x) = 2 \cdot 7x$
- $f(x) = 7x^2 + 4x + 6 \rightarrow f'(x) = 2 \cdot 7x + 4$

Derivation is simpler than sandhi rules in sanskrit:

$$f(x) = 7x^2$$

$$f'(x) = \underbrace{2}_{\text{the old exponent prefixed}} \cdot 7x \overbrace{2-1}^{\text{the old exponent reduced by 1}} = 14x^1 = 14x$$

Functions and derivations

Exercise on derivatives

Determine the slope at $x = 2$ and $x = 3$ for the functions given by

- $f(x) = 7 - x^2$,
- $g(x) = 18$
- $\Pi(x) = 4x^6 - 2x^2$

Functions and derivations

Slopes in economics

Instead of slope or first derivative, economist use the word "marginal":

- marginal utility
- marginal cost
- marginal profit

For example, utility depends on the number of apples and bananas consumed: $U(a, b)$.
By how much does utility increase if one extra apple is consumed?

$$\frac{dU}{da} \text{ (not quite correct)}$$

$$\frac{\partial U}{\partial a} \text{ (correct).}$$

Preferences in two-dimensional space

Bundles of goods

Goods can be

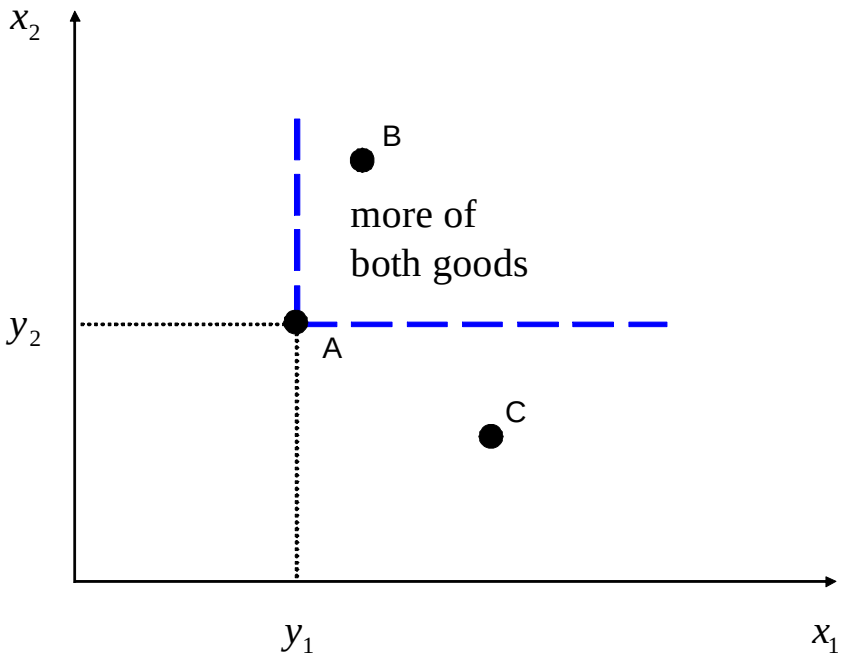
- apples,
- material goods,
- leisure and material goods, or
- time spent for meditation.

Bundles of goods:

- 4 apples and 2 pears,
- 10 hours leisure and monetary income of 60 Euros for consumption purposes, or
- 2 hours meditation and 10 hours consumption of material goods.

Preferences in two-dimensional space

Monotonicity



Problem

Three bundles $A = (3, 2)$, $B = (4, 7)$ *and* $C = (5, 5)$. *Preference order!*

Preferences in two-dimensional space

Utility functions

Two convenient methods to describe preferences:

1. utility functions
2. indifference curves

Utility functions attach numbers to bundles such that

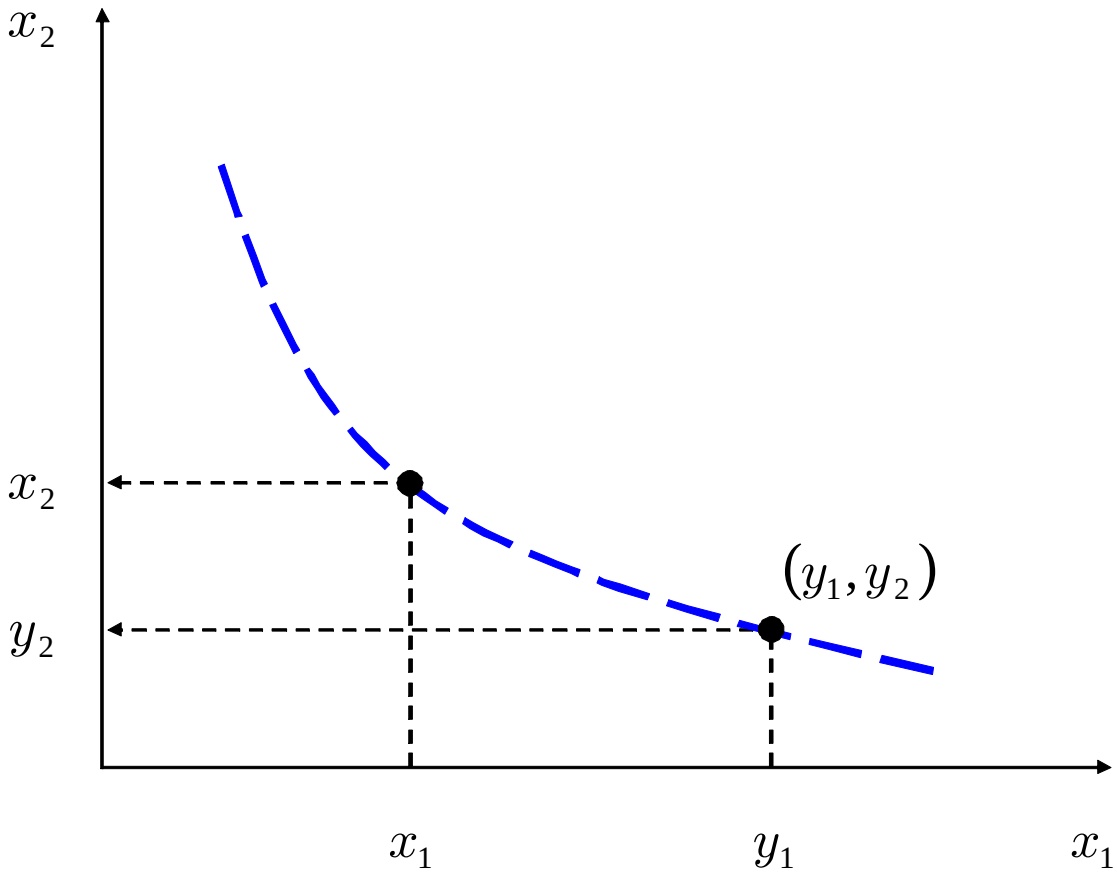
- a better bundle has a higher utility number
  $U(x) > U(y) \Leftrightarrow x \succ y$
- equally valued bundles have the same
  $U(x) = U(y) \Leftrightarrow x \sim y$

Problem

Assume the utility function $U(x_1, x_2) = x_1 + 2x_2$ *and consider the three bundles* $A = (3, 6)$, $B = (4, 7)$ *and* $C = (5, 5)$. *Infer the agent's preferences.*

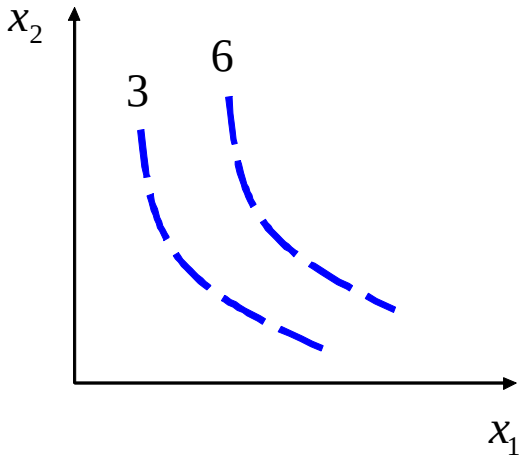
Preferences in two-dimensional space

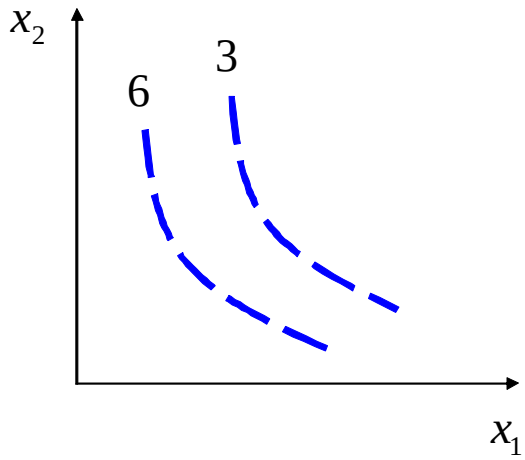
Indifference curves I



Preferences in two-dimensional space

Indifference curves II





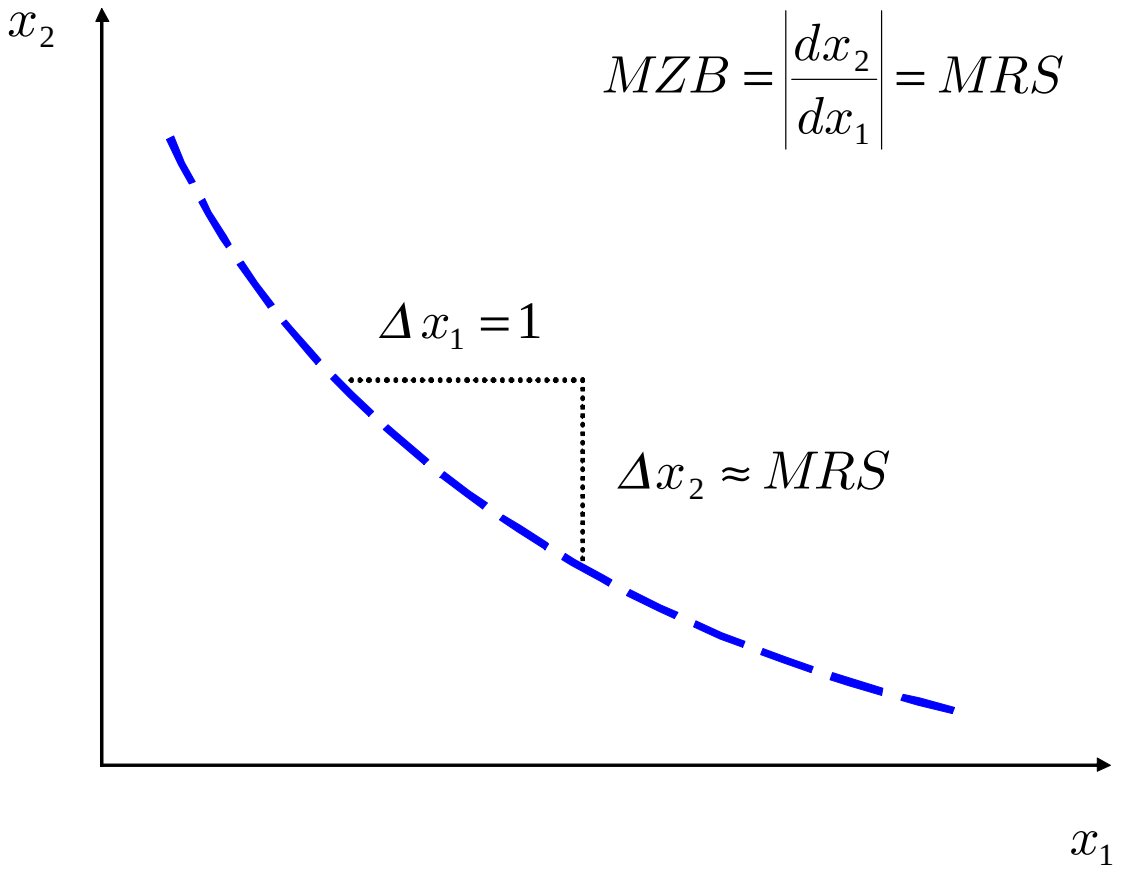
Problem

Sketch indifference curves:

- *good* 2 *is a bad (the consumer would like to have less of it),*
- *good* 1 *represents red matches and good* 2 *blue matches,*
- *good* 1 *stands for right shoes and good* 2 *for left shoes.*

Preferences in two-dimensional space

Marginal rate of substitution



Relations

exercise on properties

Fill in "yes" or "no":

property	relation: weak preference	relation: strong preference	relation: indifference
reflexive			
transitive			
symmetric			
complete			

Hence, indifference is an equivalence relation. Its equivalence classes are ...

Preferences in two-dimensional space

Exercise on marginal rate of substitution

Problem

Find the MRS for the utility function $U(x_1, x_2) = ax_1 + bx_2$, $a > 0$ *and* $b > 0$ *(perfect substitutes)*

Along an indifference curve, the utility is constant at some level k.

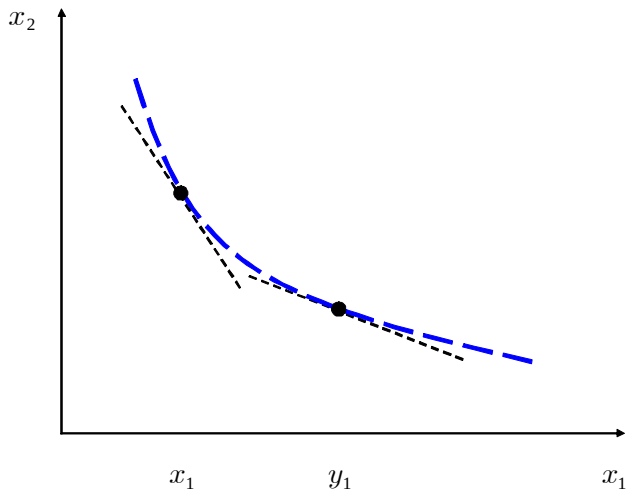
- Solve for x_2 and obtain $x_2(x_1) = \frac{k}{b} - \frac{a}{b}x_1$
- Form the derivative with respect to x_1 which yields $\frac{dx_2}{dx_1} = -\frac{a}{b}$.

Therefore, the marginal rate of substitution for perfect substitutes is $\frac{a}{b}$.
Sometimes easy:

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}.$$

Convex preferences

Definition



1. As x_1 increases, *MRS* decreases.
2. Bundles in between (convex combinations) are better.

Problem

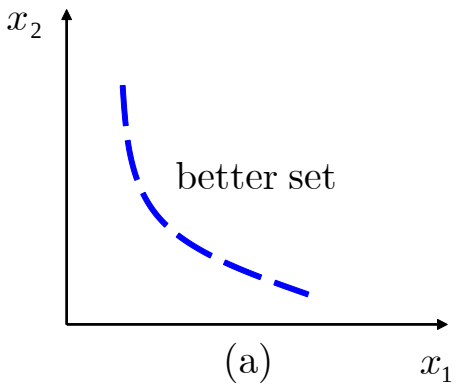
Consider the vectors $x = (x_1, x_2) = (2, 4)$ *and* $y = (y_1, y_2) = (8, 12)$. *Find* $x + y$, $2x$ *and* $\frac{1}{4}x + \frac{3}{4}y$!

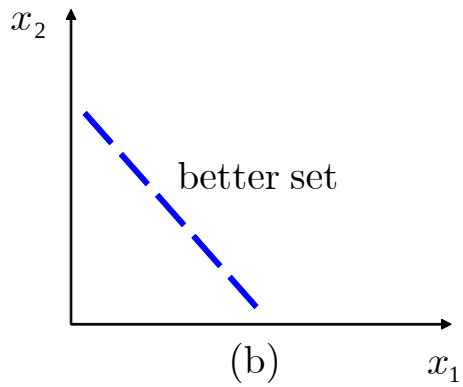
Convex preferences

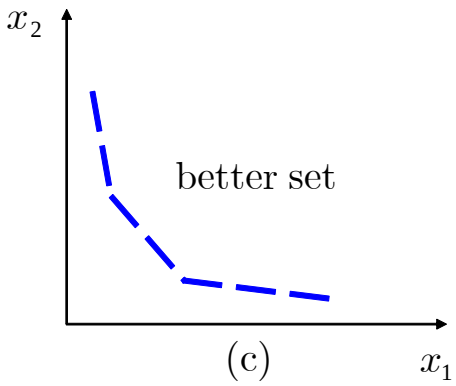
Exercise

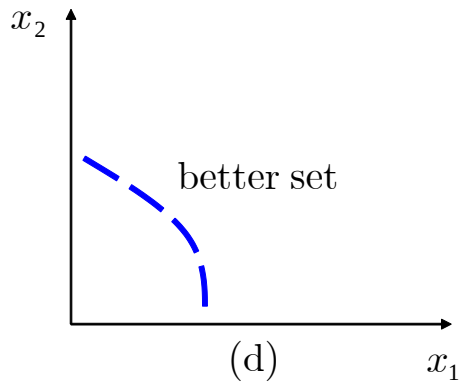
Problem

Convex preferences?









Convex preferences

Exercise on the Middle Way

If you are a Buddhist and happen to be indifferent between

- two glasses of wine and 4 meditations on the one hand, and
- four glasses of wine and 2 meditations on the other hand,

do you then prefer three glasses of wine and three meditations to two glasses of wine and 4 meditations?

Monotonic preferences and contentment

Tanha versus chanda

In Buddhist thought, tanha is bad, chanda is good:

TANHA	⟷	CHANDA
	= desire for	
pleasure objects	⟷	well-being
	shows	
artificial value	⟷	true value
	leads to	
blind seeking	⟷	effort/action
	is caused by	
ignorance	⟷	wisdom

Monotonic preferences and contentment

Hitopadeśa I

Monotonic preferences are well-known to the writers of the Hitopadeśa:.

> *Fire never has enough wood, nor is the ocean fully satisfied with the rivers, nor Death with all creatures, nor a beautiful-eyed woman with all men.*

However, warnings against monotonic preferences are also common place:

> *Greed makes one lose one's mind, greed breeds desire; and if a man is tormented by desire, he will suffer in this world and the next.*

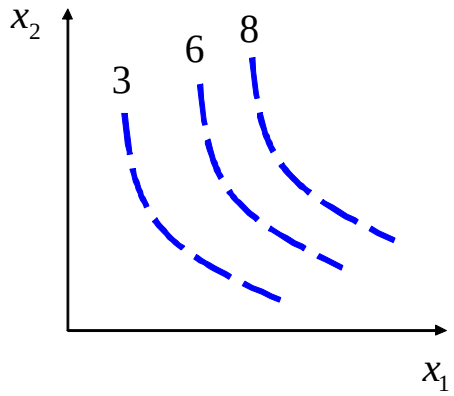
Monotonic preferences and contentment

homo oeconomicus

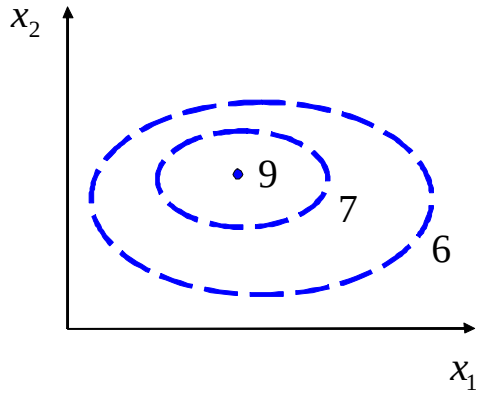
- non-monotonic preferences are not foreign to the archetypal homo oeconomicus who knows that there may be “too much of a good thing” such as cheese or wine.
- dynamic consumption models have been presented to show how consumption in the past influences consumption and well-being in the future (by Gerry Becker).

Monotonic preferences and contentment

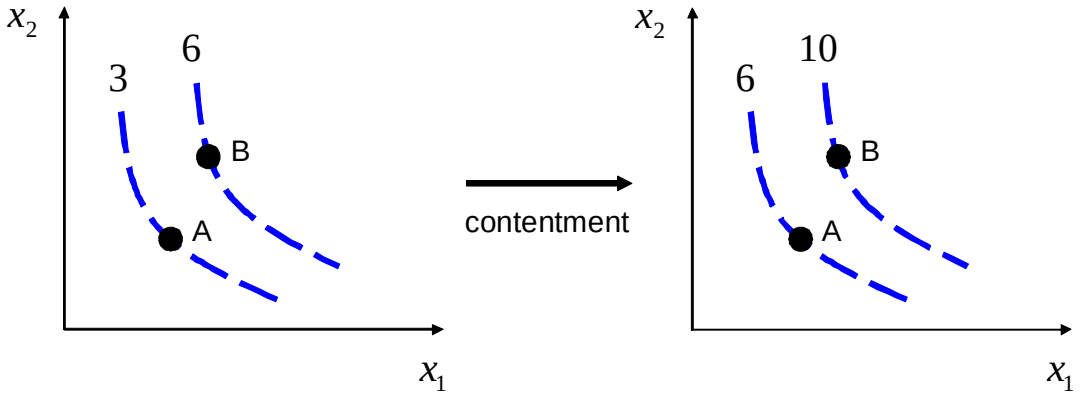
Monotonicity (tanha) versus moderation (chanda)



Monotonicity



Moderation



Monotonic preferences and contentment

Hitopadeśa II

In some places, the Hitopadeśa is even taking the more extreme view that wealth is bad:

> *It generates suffering to earn it, anxiety in hard times and it deludes people into prosperity–how can wealth lead to happiness? (p. 181)*

Or:

> *Abandon desire, and who is a pauper, who a lord? But if you give desire some leeway, you'll be a slave first and foremost. (p. 183)*

Equanimity

Hitopadeśa and Gītā

At some stage, the Hitopadeśa advises equanimity with respect to good and bad events:

> *Whether happiness or misery befalls you, you should accept it. Happy and unhappy events take turns, revolving like a wheel. (p. 177)*

We also like to quote from the Gītā where Krishna also recomments equanimity:

> *"He whose mind is unperturbed in times of sorrow, who has lost the craving for pleasures, and who is rid of passion, fear and anger, is called a sage of steadied thought. His wisdom is secure who is free of any affections and neither rejoices nor recoils on obtaining anything good or bad."*

Equanimity

Translation

Krishna seems to advocate a preference relation $\succsim$ with

$$\text{pleasure} \sim \text{sorrow}.$$

But: Is it possible to choose one's preferences?