Microeconomic Analyses of Old Indian Texts

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Für Corinna, Ben, Jasper, Samuel

Preface

What is this book about?

This is a course on both Old Indian literature and microeconomics. Old Indian literature covers a lot of ground–we focus on some of the material which is amenable to microeconomic analysis. The course has been introduced in 2013.

What about microeconomics ... ?

The course intends to be suitable for BA economics students as well as for students of indology. Therefore, no prior knowledge of microeconomics is assumed. The theoretical building blocks are provided whenever needed. However, one warning is in order. We need some mathematics: relations, functions, derivatives. A second however: these concepts will be explained carefully.

What about indology ... ?

The course is also self-contained with respect to the indological content. The reader is not supposed to have read, or even heard about, the *artha-śāstra* (the Indians' "Welfare of Nations"). The author is not an indologist himself but will do his very best to gather the Indological fine points whenever needed. Also, Sanskrit is not necessary to follow this course.

Exercises and solutions

The main text is interspersed with questions and problems wherever they arise. Solutions or hints are given at the end of each chapter.

Women, indologists and economists

While the lecturer is not ascetic, he likes to cite from Meiland (2009b, p. 237):

> The ascetic greeted the [indologists and economists] with friendly words of welcome, showing them the proper courtesies that delight guests. Taking advantage of the opportunity offered by their questions, and by way of a hospitality gift, he then preached a sermon, using discourses filled with examples and containing topics that [indologists and economists] find easy to grasp.

In fact, in the original, we read "women" instead.

Thank you!!

I am happy to thank many people who helped me with this book. Christian Alvermann, Katharina Lotzen, Kerstin Szwedek, ...

Leipzig, April 2013

Harald Wiese

CHAPTER I

Introduction: Old Indian literature and microeconomics

1. Four aims

Indian texts are full of classifications. A major one concerns the four aims of life and the four life stages. We borrow freely from Zimmer (1969), Olivelle (2004), Olivelle (2013), Boesche (2002) and , Oberlies (2012).

1.1. *artha* (material possessions). Artha is concerned with the achievement of worldly aims, in particular wealth and power. In this realm we deal with the following texts:

- An artha-śāstra is a treatise on economics and politics. Several are known to us today. The most famous one is ascribed to somebody called Kauṭilya. Well before the common era (or so common wisdom has it), this counselor to an important Indian king wrote a manual on "wise kingship", the *Artha-śāstra.* Among other topics, Kauṭilya deals with taxation, diplomacy, warfare, and the management of spies.
- The achievement of aims was also the content matter of the fable collections like the *Pañca-tantra* and the *Hitopadeśa.* In fact, the latter means Teachings for happiness. Among others, readers are told how to win friends or how sow mistrust between friends, on how to cheat others or how to prevent being cheated.

The *artha* realm is characterized by cold-blooded calculations.

1.2. *kāma* (pleasure, love). The best-known part of the literature on kāma deals with courting and love-making (see Vātsyāyana's Kāma-sūtra). Related are textbooks on poetics and acting (the Nāṭya-śāstras). This literature is not used in the present book.

1.3. *dharma* (religious and moral duties). Dharma is often translated as duty or moral obligation. A peculiarity of the Indian thought on dharma is the insistence on caste-related duties. There is a considerable overlap between artha and dharma. In this book, the following texts are important:

- Arguably, the main text on dharma is the dharma-śāstra due to Manu, also called Manu's code of law.

- Discussions on dharma are to be found in many texts. For example, the *Bhagavad Gītā* is concerned with the question of how to weigh the duties of warriors against the consequences of warfare.

1.4. *mokṣa* (liberation). Mokṣa lies at the center of Hindu religious thought. Roughly speaking, souls reside in humans (or animals or gods). The acts undertaken during a lifetime influence this human's (or animal's or god's) concrete form in the next life. The major aim (*paramārtha*) is to be released from the cycle of births.

For example, Garbe (1897, pp. 2-3) mentions these central ideas of Indian thought:

- "the doctrine of the transmigration of souls, and the theory intimately connected therewith of the subsequent effects of actions (*karman*)",
- the "belief that every individual unceasingly moves forward after death towards new existences in which it will enjoy the fruits of formerly won merits, and will suffer the consequences of formerly committed wrongs–whether in the bodies of men, animals, or plants, or in heavens and hells",

He adds that these ideas were "regarded as something self-evident, which, with the exceptions of the [*Cārvāka*s], or Materialists, no philosophical school or religious sect of India ever doubted." We will encounter these believes, including "heavens and hells", several times.

2. Four life stages

According to Hindu thought, life is to be divided into four stages (āśrama):

(1) *śiṣya* (student)
The student has to study śāstra in order to cope with the four aims described in the previous section.

(2) *gṛhastha* (householder)
The householder has founded a family and engages in the temperate pursuit of material welfare (*artha*) and sexual and esthetic pleasure (*kāma*). He needs to be attentive to his respective duties (dharma).

(3) *vanaprastha* (forest dweller)
After fulfilling his householder duties, a man retires to the forest for meditation.

(4) *bhikṣu* (wandering sage)
Finally, one is to walk around as a mendicant.

Thus, the first two stages are related to *grāma* (village), while the last two stages are connected with *vana* (forest). In the last two stages, the normal worldly preoccupations (with respect to *artha, kāma*, and *dharma*) are to be left aside. Meditation in the forest and wandering as a mendicant prepare for *mokṣa* (liberation).

3. Survey on Old Indian literature

Let us enumerate a few important works of literature and their (very partial!) treatment within this book:

3.1. Ṛg-veda. The oldest Old Indian literature known to us is the Ṛg-veda which is a collection of hymns as voluminous as the Bible. It dates from 1000 B.C. or sooner. We talk about the "social gods" Contract (*Mitra*), True-Speech (*Varuṇa*) and Hospitality (*Aryaman*) in chapter IX. Apart from the Ṛg-veda, three other Vedas exist. In the Old Indian literature, three or four of them are considered the most important ones.

3.2. Mahābhārata. While the Vedas (particularly the older ones) are written in Vedic (from which Sanskrit developed), the two large Indian epics are primarily written in Sanskrit. The largest one is the Mahabhárata which tells about a ruling family whose interior conflicts lead to a deadly battle. The Mahabharata consists of sixteen books. Within the sixth book, the *Bhagavad Gītā* is contained. For many Hindus, it is the most important religious document. We present a decision-theoretic analysis of the *Gītā* in chapter IV.

3.3. Artha-śāstra. The most famous manual on kingship or Artha-śāstra is ascribed to somebody called Kauṭilya. Apparently, several authors were involved at different time intervals (for a recent discussion, see Olivelle 2013, pp. 17). For our purposes, we might say that the text is about 2000 years old. In chapter V, we attempt to answer the question why the manuscript seems to have been lost for a span of about 1000 years.

The *Artha-śāstra*'s topic is how to run a kingdom. Successful monarchs need to study economics and foreign affairs (see the introduction by Olivelle 2013, pp. 40):

- Kauṭilya's economy was heavily regulated. In chapter V, we deal with the specific example of a complicated market tax.
- Kauṭilya's ideas on foreign policy rest on the maṇḍala theory which explains how a king should manage war and peace with direct and indirect neighbours. According to that theory, neighbours tend to be enemies and the enemies of enemies tend to be friends. Using game theory, we present a formal model of maṇḍala theory in chapter VIII.

3.4. Indian fables. Indian literature contains many beautiful stories with animal and human actors. Typically, these stories come with a specific conclusion or moral summarized in a subāṣita (nice saying, word of wisdom, proverb). Clearly, the subject matter dealt with in these stories belongs to the artha and the dharma literature. In our view, many stories contribute to applied microeconmics, to both decision theory and game theory. Microeconomic interpretations of selected subāṣita and stories will make this evident. Commonly, long stories contain shorter ones which may again contain further substories–this technique is called boxing.

Two collections of stories are especially important:

(1) The Pañca-tantra is probably the most influential story collection worldwide. Originally written in Sanskrit around 300 CE, it has been translated (and retold) into many Indian languages and more than 50 other languages (for more details, consult Olivelle 2006, pp. 17). Pañca-tantra can be translated as "Five discourses on worldly wisdom" and contains five books:
   - On causing dissension among allies
   - On securing allies
   - On war and peace
   - On losing what you have gained
   - On hasty actions

(2) The second famous collection clearly builds on the Pañca-tantra. It may be five hundred or six hundred years younger than its predecessor. It is the Hitopadeśa which can be translated as "teaching of happiness".

In chapters II, III, VII, and VI, some of the *subāṣita*s and some of the stories are dealt with.

3.5. Buddhist birth-stories. "Tradition holds that when the Buddha became enlightened he acquired the ability to see his own past lives as well as those of others. This belief in the possibility of knowing previous rebirths opened the door to a whole genre of literature called *jātaka* (literally "birth-story"), which was dedicated to depicting the past lives of the Buddha when he was still aspiring for enlightment." (Meiland (2009a, p. xv)) Āryaśūra's "Garland of the Buddha's past lives" is thought to have been composed in the 4th century AD. In his former lives, the Buddha exhibits extreme forms of

(1) giving (*dāna*) (giving away his life or family members)
(2) virtue (*śīla*) (accumulating merit and speaking truthfully)
(3) forbearance (*kṣānti*) (not becoming angry when tormented or even cut to pieces)

The giving aspect is looked at through the eyes of economic theories of altruism in chapter X. Virtue may be the issue in chapter III which is concerned with a tricky discussion between the *Bodhi-sattva* and a *Cārvāka* king (see below).

3.6. *Cārvāka* philosophy. In the form of the *Cārvāka* philosophy (see the monograph by Heera 2011), India has produced

- an atheistic (nāstika, i.e. (god) does not exist),
- non-Vedic (the authority of the Vedas is called into question),

- materialist (the existence of ātman ("soul") or *para-loka* ("afterworld") is denied), and
- hedonist

strand. Alas, the *Cārvāka* philosophy is indirectly (by writers of other traditions) attested only. Among other sources (see the list in Heera 2011, p. 54), *Cārvāka* philosophy is addressed in these books:

(1) In a the satirical play "Much Ado About Religion" (about 1100 years old, translated by Dezsö (2009)), a *Cārvāka* hedonist defends his ideas.
(2) "The Rise of the Wisdom moon" (nearly 1000 years old) is the first allegorical play in Sanskrit literature. In the translation by Kapstein (2009, p. lxvii), the persons comprise *Kāma* (Lord Lust), *Rati* (Lady Passion), *Mati* (Lady Intelligence), and *Cārvāka* (Hedonist), among others.
(3) A *Cārvāka* philosopher blames Yudhiṣṭhira, the eldest Pandava, after the Kurukṣetra battle (12. book of the Mahābhārata).
(4) In one of the Buddha's birth-stories, a *Cārvāka* king is diverted from his wrong views by the future Buddha (see the birth-story of Brahma in Meiland 2009b, pp. 267).

A few quotes from this literature are presented in the chapter X on altruism, as a counterpoint. We also have cause to dwell on the above mentioned birth-story in chapter III.

4. Microeconomics

4.1. Important parts of microeconomics. Microeconomics is concerned with the (optimal) decision of actors (households, firms, voters, ...) and how these decisions interact. Thus, microeconomics is about

- decision theory:
  - monopolistic firms decide on profit-maximizing prices, quantities, budgets for research and developement, ...
  - households buy the best bundle of goods among thosse bundles they can afford
  - governments set tax rates
- game theory:
  - several firms decide on prices where the price set by one firm influences the profit of another one
  - countries decide whether to attack each other
- general equilibrium theory:

  Can we find prices for all goods such that
  - all firms produce profit-maximizing quantities
  - households demand utility-maximizing bundles
  - all plans can be carried out

- Pareto optimality:

  Is it possible to improve the situation for one actor without getting a worse result for any other?

Rougly speaking, we consider decision theoretic models in part A, game theoretic ones in part B and consider very briefly general equilibrium theory and Pareto optimality in part C.

4.2. Microeconomic methods. Microeconomists build formal (mathematical models) that allow theoretical predictions. The microeconomic toolbox is filled with three instruments:

- rationality:
  It is assumed that actors know what they want and act accordingly.
- equilibrium:
  Microeconomists look for behaviors (actions, strategies, ...) such that no actor finds changing his behavior profitable.
- comparative statics:
  How do the parameters (input, model description) influence the variables (output, behavior in equilibrium)?

Part A

Decision theory

CHAPTER II

Preferences

1. Introduction

In this chapter, we learn about relations and preference relations and discuss these concepts in the light of some quotes from the Hitopadeśa and, to a lesser extent, from the Bhagavad Gita. In chapter X, we will present hedonist preferences and Buddhist altruism in some detail.

2. Relations

In this book, we need different sorts of relations for different purposes. Therefore, we introduce relations in general before turning to the specific case of preference relations. Consider these three examples:

EXAMPLE II.1. *For any two inhabitants from Leipzig, we ask whether*

- *one is the father of the other or*
- *they are of the same sex.*

EXAMPLE II.2. *For the set of integers $\mathbb{Z}$ (the numbers ..., -2, -1, 0, 1, 2, ...) , we consider the difference and examine whether this difference is an even number (i.e., from ..., $-2, 0, 2, 4$, ...).*

All three examples define relations, the first two on the set of the inhabitants from Leipzig, the last on the set of integers. Often, relations are expressed by the symbol $\sim$. To take up the last example on the set of integers, we have $5 \sim -3$ (the difference $5-(-3)=8$ is even) and $5 \nsim 0$ (the difference $5-0=5$ is not even). If x and y are related, we often write $x \sim y$.

Relations are defined on a set of "objects" or "elements". They have, or do not have, specific properties:

DEFINITION II.1 (properties of relations). *A relation $\sim$ on a set X is called*

- *reflexive if $x \sim x$ holds for all $x \in X$,*
- *transitive if $x \sim y$ and $y \sim z$ imply $x \sim z$ for all $x, y, z \in X$,*
- *symmetric if $x \sim y$ implies $y \sim x$ for all $x, y \in X$,*
- *antisymmetric if $x \sim y$ and $y \sim x$ imply $x = y$ for all $x, y \in X$,*
- *asymmetric if $x \sim y$ implies not $y \sim x$, and*
- *complete if $x \sim y$ or $y \sim x$ holds for all $x, y \in X$, $x \neq y$.*

Lemma II.1. *On the set of integers $\mathbb{Z}$, the relation $\sim$ defined by*

$$x \sim y :\Leftrightarrow x - y \text{ is an even number}$$

is reflexive, transitive, and symmetric, but neither antisymmetric nor complete.

":⇔" means that the expression left of the colon is defined by the expression right of the equivalence sign.

Proof. We have $x - x = 0$ for all $x \in \mathbb{Z}$ and hence $x \sim x$; therefore, $\sim$ is reflexive. For transitivity, consider any three integers x, y, z that obey $x \sim y$ and $y \sim z$. Since the sum of two even numbers is even, we find that

$$\begin{aligned} & (x-y)+(y-z) \\ = \; & x-z \end{aligned}$$

is also even. This proves $x \sim z$ and concludes the proof of transitivity. Symmetry follows from the fact that a number is even if and only if its negative is even.

$\sim$ is not complete which can be seen from $0 \nsim 1$ and $1 \nsim 0$. Finally, $\sim$ is not antisymmetric. Just consider the numbers 0 and 2. □

Exercise II.1. *Which properties do the relations "is the father of" and "is of the same sex as" have? Fill in "yes" or "no":*

	relation	
property	*is the father of*	*is of the same sex as*
reflexive		
transitive		
symmetric		
antisymmetric		
asymmetric		
complete		

Definition II.2 (equivalence relation). *Let $\sim$ be a relation on a set X which obeys reflexivity, transitivity and symmetry. Then, any two elements $x, y \in X$ with $x \sim y$ are called equivalent and $\sim$ is called an equivalence relation.*

We can assemble all elements that are equivalent to some given element $x \in X$ in the set

$$[x] := \{y \in X : y \sim x\}.$$

$[x]$ is called an equivalence class. For example, our relation on the set of integers (even difference) is an equivalence relation. We have two equivalence classes:

$$\begin{aligned} [0] &= \{y \in \mathbb{Z} : y \sim 0\} = \{..., -2, 0, 2, 4, ...\} \text{ and} \\ [1] &= \{y \in \mathbb{Z} : y \sim 1\} = \{..., -3, -1, 1, 3, ...\} \end{aligned}$$

EXERCISE II.2. *Continuing the above example, find the equivalence classes* $[17]$, $[-23]$, *and* $[100]$. *Reconsider the relation "is of the same sex as". Can you describe its equivalence classes?*

3. Preference relations

We now turn to a specific relation.

DEFINITION II.3 (preference relation). *A (weak) preference relation on* X *is denoted by* $\precsim$ *where* $x \succsim y$ *means "*x *is at least as good (as preferable, as virtuous, as compatible with svadharma) as* y*". Weak preference relations are always reflexive, transitive and complete.*

The indifference relation (derived from $\succsim$*) is defined by*

$$\begin{aligned} x &\sim y \text{ means} \\ x &\succsim y \text{ and } y \succsim x \end{aligned}$$

and the strict preference relation (derived from $\succsim$*) is defined by*

$$\begin{aligned} x &\succ y \text{ means} \\ x &\succsim y \text{ and not } y \succsim x. \end{aligned}$$

Completeness of preferences means that the agent "knows what he wants". Of course, in real life, this is not always the case (see chapter IV, where we discuss Árjuna's decision problem presented in the *Bhagavad Gītā*).

Every agent's preferences between any two objects x and y are

- either $x \prec y$: the agent strictly prefers y to x
- or $y \prec x$: the agent strictly prefers x is to y
- or $x \sim y$: the agent is indifferent between x and y.

EXERCISE II.3. *Consider five bundles* A, B, C, D, *and* E.

(1) *Assume*

$$A \succsim B, C \sim E, C \succ A, D \sim A$$

Can you write the preferences for these bundles in one line where every bundle shows up once, only? How about E *versus* A*?*

(2) *How about*

$$A \succsim B, C \succ E, C \succ A, D \sim A$$

and

(3)

$$A \succsim B, B \sim E, C \succ A, E \succsim C$$

4. Preferences in the Hitopadeśa

In microeconomics, preferences are given and researches typically stay clear of criticizing preferences. The Hitopadeśa does not show any inhibition in this respect:

> Better to have a single virtuous son than a hundred fools! One moon destroys darkness, but not even a multitude of stars can do so.
>
> ...
>
> A large income, perpetual health, a wife who is dear and speaks pleasantly, an obedient son and money-making know-how–these are the six sources of happiness in this world, O king.

This quote from the Hitopadeśa has been taken from Torzsok (2007, p. 65), henceforth to be cited as "Hitopadeśa, p. 57" or sometimes just as "p. 57". Since preferences are relations, we need to perform the obvious translations: A large income is better than a small one etc. While these preferences seem obvious, the Hitopadeśa also warns against "beautiful wives" (p. 67). A reason is given much later:

> Falsehood, recklessness, cheating, jealousy, excessive greed, absence of virtues, and impurity–these are women's natural defects.

And, of course, beautiful women have more opportunities to be unfaithful, as exemplified by the story of the prince and the merchant's wife (pp. 191-201).

It will not come as a surprise that the four aims mentioned in the survey (chapter I) are valued highly:

> The birth of a person who does not succeed even in one of the four life-aims–to fulfill one's duties, obtain riches, satisfy one's desires or attain final release–is as useless as a nipple on a nanny-goat's neck. (p. 67)

While *mokṣa* ("final release from rebirth") is generally seen as a very important aim, it is also recognized as somewhat egoistic. For example, the elefant in a Buddha's birth-story is very noble:

> My endeavor is not for a good rebirth,
> nor for the glory of my royal parasol,
> nor for heaven with its fine undiluted pleasures,
> nor for Brahma's splendor, nor liberation's joy.
> (Meiland (2009b, p. 313))

Finally, and this is important for students and others:

> The wise spend their time diverting themselves with poetry and learned treatises [such as Wiese (2010a) or Wiese

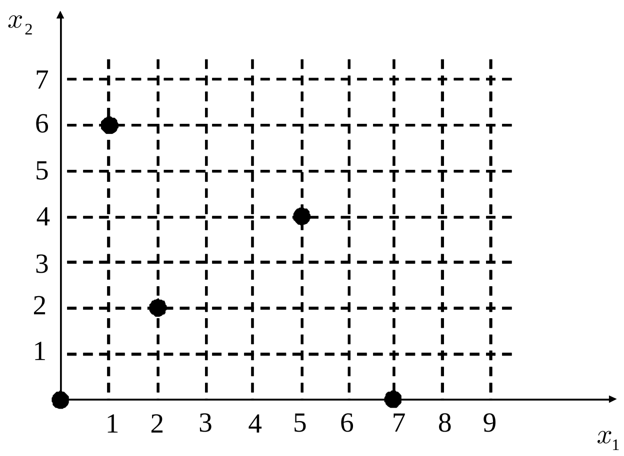

FIGURE 1

(2010b)], while fools succumb to vice, sleep or quarrels. (p. 79)

5. Functions and derivations

5.1. System of coordinates. Points in the two-dimensional space $\mathbb{R}^2$ are often denoted by (x_1, x_2) or by (x, y). Consider fig. 1. Where are the points $(7,0), (1,6), (4,5), (0,0)$?

5.2. Functions. In order to describe a function, we need

- arguments (the input)
  Examples are
  - cost functions with input: quantity of good to be produced
  - utility function with input: bundle of goods (3 apples and 2 bananas)
  - demand function with input: price
  - profit function with input: price
- values (the output)
  Examples are
  - cost functions with output: sum of money
  - utility function with output: utility of 5
  - demand function with output: quantity demanded
  - profit function with output: sum of money earned
- a formula or an algorithm that tells us which value to associate with a given argument
  Examples are
  - cost functions given by $c(y) = 2y^2$
  - utility function given by $U(x, y) = x + y$
  - demand function given by $X(p) = 100 - 2p$

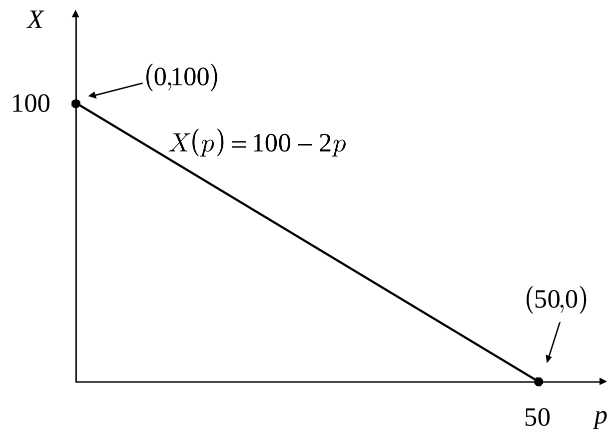

FIGURE 2

– profit function given by

$$\Pi(p) = \underbrace{R(p)}_{\text{revenue}} - \underbrace{C(p)}_{\text{cost}}$$

Let us consider the demand function given above. It is depicted in figure 2.

We distinguish:

- abscissa, here p-coordinate
- ordinate, here X-coordinate

EXERCISE II.4. *Consider the demand function given by*

$$X(p) = 200 - 4p.$$

Draw the function. Determine

- *the prohibitive price (i.e., the price at which the quantity demanded is zero: $X(p) = 0$)*
- *the satiation quantity (i.e., the quantity for price zero: $X(0)$)*

5.3. Slopes.

5.3.1. *The discrete case.* Consider the profit function depicted in fig. 3. The slope is

- positive whenever we have
  – The higher x, the higher Π, i. e., we have $\frac{\Delta\Pi}{\Delta x} > 0$.
  – The lower x, the lower Π, i. e., we have $\frac{\Delta\Pi}{\Delta x} > 0$.
- negative whenever we have
  – The higher x, the lower Π i.e., we have $\frac{\Delta\Pi}{\Delta x} < 0$.
  – The lower x, the higher Π, i.e., we have $\frac{\Delta\Pi}{\Delta x} < 0$.

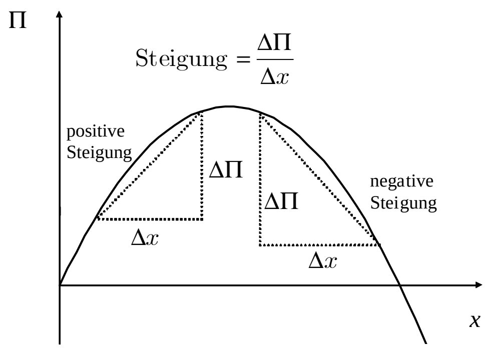

FIGURE 3

Maybe you find this procedure helpful. You say: on Δx, we have $\Delta\Pi$. For example, between the points

$$\begin{aligned}(x,\Pi) &= (0,5) \text{ and}\\ \left(\hat{x},\hat{\Pi}\right) &= (4,7)\end{aligned}$$

we have: On $\Delta x = 4-0$, we have $\Delta\Pi = 7-5$, i.e.,

$$\frac{\Delta\Pi}{\Delta x} = \frac{7-5}{4-0} = \frac{2}{4} = \frac{1}{2}.$$

You can also do it the other way around: On $\Delta x = 0-4$, we have $\Delta\Pi = 5-7$, i.e.,

$$\frac{\Delta\Pi}{\Delta x} = \frac{5-7}{0-4} = \frac{-2}{-4} = \frac{1}{2}.$$

Go back to fig. 1 for some exercise.

EXERCISE II.5. *Determine the slope between the points2*

$$\begin{aligned}(x,\Pi) &= (3,10) \textit{ and}\\ \left(\hat{x},\hat{\Pi}\right) &= (5,16)\end{aligned}$$

Determine the slope between the points

$$\begin{aligned}(x,\Pi) &= (3,10) \textit{ and}\\ \left(\hat{x},\hat{\Pi}\right) &= (5,2)\end{aligned}$$

5.3.2. *The continuous case.* Imagine that you let the interval Δx get smaller and smaller. Then, finally, you obtain the slope not between two points, but at one point. Consider fig. 4. Instead of $\frac{\Delta\Pi}{\Delta x}$ in the discrete case, we use the notation

$$\frac{d\Pi}{dx} \text{ or } \Pi'(x)$$

in the continuous case.

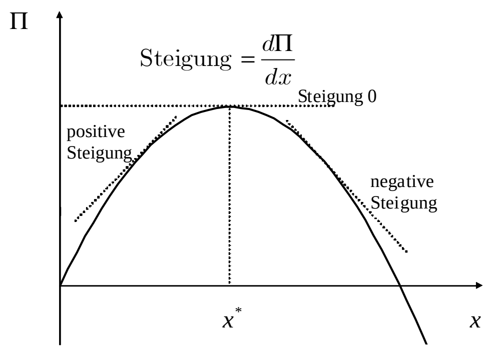

FIGURE 4

One advantage of the continuous case is that it is often easy to calculate the slope. Indeed, the slope is calculated by way of the first derivative. Let us see a few examples:

function	first derivative
$f(x) = 4x$	$f'(x) = 4$
$f(x) = 6$	$f'(x) = 0$
$f(x) = 7x^2$	$f'(x) = 2 \cdot 7x$
$f(x) = 7x^2 + 4x + 6$	$f'(x) = 2 \cdot 7x + 4$

Derivation is simple. Just follow these rules:

- $f(x) = 4x$ is a straight line through the origin $(0,0)$. Therefore, the slope is constant and the same as in the discrete case. Thus, the slope of $4x$ is the slope between the points

$$\begin{aligned} (0, f(0)) &= (0,0) \text{ and} \\ (1, f(1)) &= (1,4) \end{aligned}$$

  and hence, by "on $\Delta x = 1 - 0$, we have $\Delta f = 4 - 0$", $\frac{4}{1} = 4$.
- $f(x) = 6$ is a horizontal line (for every input x, you get the output 6). Therefore the slope is zero. This you can also see from "on Δx, we have $\Delta f = 0$".
- Deriving functions like $f(x) = 7x^2$ goes as follows:
  - Consider the exponent of x which happens to be 2.
  - Prefix this exponent as a factor.
  - Reduce the exponent by 1.
  - Done.

Thus, we have

$$f'(x) = \underbrace{2}_{\substack{\text{the old exponent}\\ \text{prefixed}}} \cdot 7x^{\overbrace{2-1}^{\substack{\text{the old exponent}\\ \text{reduced by 1}}}} = 14x^1 = 14x$$

(Actually, the derivation of $4x$ and of 6 follow the same rule!)

- When deriving a sum, just derive the summands.

EXERCISE II.6. *Determine the slope at $x = 2$ and $x = 3$ for the functions given by*

- $f(x) = 7 - x^2$,
- $g(x) = 18$
- $\Pi(x) = 4x^6 - 2x^2$

5.3.3. *Slopes in economics.* Sometimes, economists have a funny way of speaking. Instead of slope or first derivative, they often use the word "marginal". They speak of "marginal utility", "marginal cost", or "marginal profit". For example: marginal cost is $\frac{dc}{dy}$ or $\frac{\Delta c}{\Delta y}$ and means: If I produce one extra unit, by how much does the cost increase?

Sometimes, a function may depend on two inputs. For example, utility depends on the number of apples and bananas consumed: $U(a, b)$. Then, one may ask the question of by how much utility increases if one extra apple is consumed. Then, in order to indicate that we are interested in the slope with respect to apples but not with respect to bananas, instead of

$$\frac{dU}{da} \text{ (not quite correct)}$$

we write

$$\frac{\partial U}{\partial a} \text{ (correct).}$$

6. Preferences in two-dimensional space

6.1. Bundles of goods. For many economic applications, it is necessary to describe "bundles of goods". Depending on the problem at hand, these goods can stand for

- apples,
- material goods,
- leisure and material goods, or
- time spent for meditation.

Thus, the concept of a good is broad and may well encompass aspects of spiritual life.

To simplify the analysis, we deal with two goods at a time, only. Thus, we deal with bundles of goods, such as

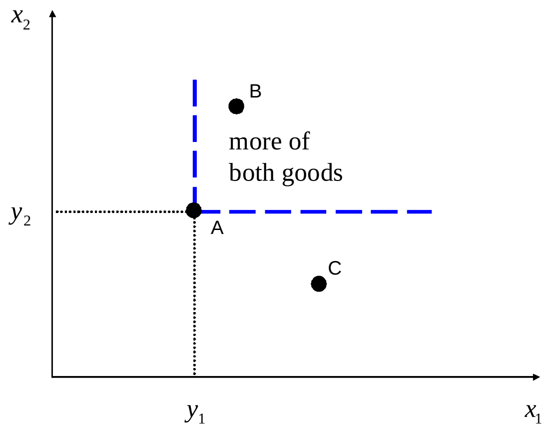

FIGURE 5. Bundles of goods in two-dimensional space

- 4 apples and 2 pears,
- 10 hours leisure and monetary income of 60 Euros for consumption purposes, or
- 2 hours meditation and 10 hours consumption of material goods.

Formally, bundles of goods are elements of the two-dimensional space $\mathbb{R}_+^2$ (where "+" means that we have non-negative amounts of both goods). Figure 5 shows three bundles of goods. x_1 stands for the amount of good 1 (apples, for example) while x_2 represents the amount of good 2 (pears). At point A (i.e., point (y_1, y_2)) the agent consumes y_1 units of good 1 and y_2 units of good 2. If one moves from A to B (or to any point north-east of A) the units of good 1 and good 2 increase. In contrast, moving from A to C means that consumption of good 1 increases while consumption of good 2 decreases.

Oftentimes, agents have monotonic preferences – they prefer to have more rather than less. In that case, the agent would strictly prefer bundle B over bundle A in figure 5.

EXERCISE II.7. *Consider the three bundles* $A = (3, 2)$, $B = (4, 7)$ *and* $C = (5, 5)$. *If the agent has monotonic preferences, what does that imply about his preferences concerning these bundles?*

6.2. Utility functions and indifference curves. Economists use two convenient methods to describe preferences, utility functions and indifference curves. Utility functions attach numbers to bundles such that a better bundle has a higher utility number ($U(x) > U(y)$ in case of $x \succ y$) and equally valued bundles the same ($U(x) = U(y)$ in case of $x \sim y$).

For example, $U(x_1, x_2) = x_1 + 2x_2$ is a utility function which expresses the preferences $(1, 2) \sim (5, 0) \sim (3, 1)$ or $(2, 1) \prec (1, 2)$. The same preferences are expressed by the utility functions $V(x_1, x_2) = 2U(x_1, x_2)$ or

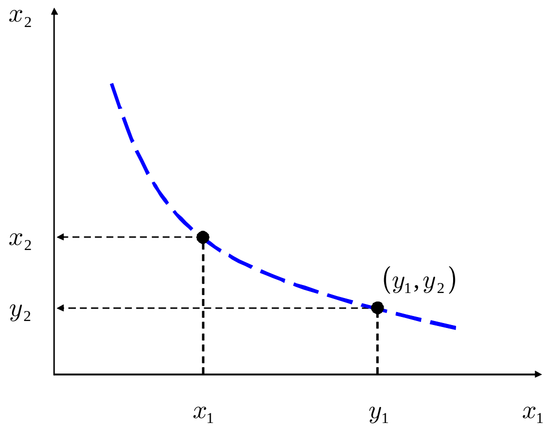

FIGURE 6. Points x and y lie on one indifference curve

$W(x_1, x_2) = \sqrt{U(x_1, x_2)}$. We say that the utility functions U, V and W are equivalent (in expressing the same preferences). Indeed, the absolute numbers are of no relevance since modern microeconomics is wedded to ordinal preference theory. The only task of utility functions is to describe preferences in a handy manner.

EXERCISE II.8. *Assume the utility function* $U(x_1, x_2) = x_1 + 2x_2$ *and consider the three bundles* $A = (3, 6)$, $B = (4, 7)$ *and* $C = (5, 5)$. *Infer the agent's preferences.*

A famous utility function is given by $U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, $0 \le \alpha \le 1$, the so-called Cobb-Douglas utility function. We can safely assume that the sum of the exponents is 1, because otherwise, we can find an equivalent utility function where the sum of the exponents is just 1.

The second way to describe preferences uses the x_1-x_2 diagram and links indifferent bundles (i.e., bundles with the same utility). Every bundle lies on one indifference curve. Given such a bundle, one can ask the question which other bundles are seen as indifferent from the agent's point of view. An indifference curve links all these bundles.

Consider point $x = (x_1, x_2)$ in figure 6. Let us increase the consumption of good 1 by $y_1 - x_1$ units. If the household is to stay indifferent, he needs to give up $x_2 - y_2$ units of good 2. He then ends up in point y and we have indifference between points x and y. In the same fashion, we can derive other points on that indifference curve.

However, we need the additional information about which indifference curve is preferred to another one. This information is provided by numbers attached to indifference curves. Consider figure 7. On the left-hand side, we have an example of monotonic preferences (more is better) while the right-hand diagram shows non-monotonic preferences for noise and dirt.

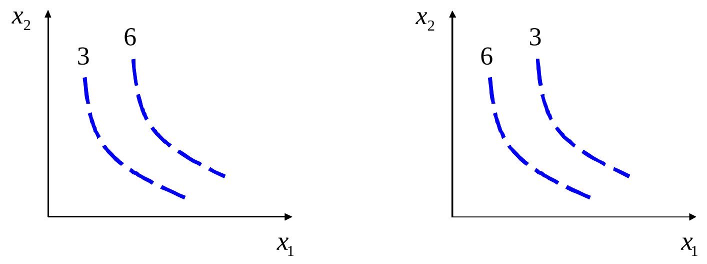

FIGURE 7. Indifference curves with numbers

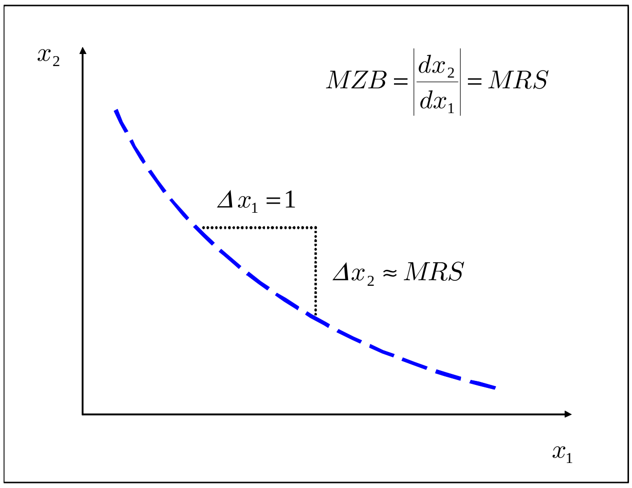

FIGURE 8. The marginal rate of substitution

EXERCISE II.9. *Sketch indifference curves for a goods space with just 2 goods and, alternatively,*

- *good* 2 *is a bad (the consumer would like to have less of that good),*
- *good* 1 *represents red matches and good* 2 *blue matches,*
- *good* 1 *stands for right shoes and good* 2 *for left shoes.*

6.3. The marginal rate of substitution. Finally, we need to discuss how much of good 2 the household is prepared to give up for additional consumption of good 1. A discrete version of this "rate of substitution" is given in figure 8. In that figure, we increase consumption of good 1 by one unit. MRS then tells us the number of units of good 2 that the household can give up while still staying on the same indifference curve. We could also say: MRS measures the willingness to pay for one additional unit of good 1 in terms of good 2.

If we want to use calculus, we need to focus on a "very small" unit of good 1. we arrive at the "marginal rate of substitution" which we abbreviate by MRS. Graphically, it is the absolute value of the slope of an indifference curve at a bundle (x_1, x_2).

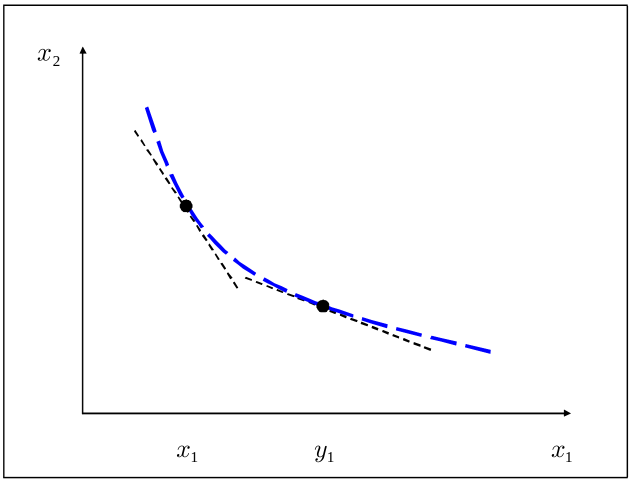

FIGURE 9. Concave indifference curve, increasing MRS

As an example, consider the utility function given by $U(x_1, x_2) = ax_1 + bx_2$, $a > 0$ and $b > 0$, i.e., the case of perfect substitutes (or red and blue matches). Along an indifference curve, the utility is constant at some level k. We look at all bundles (x_1, x_2) fulfilling $ax_1 + bx_2 = k$. We find the slope of that indifference curve by

- solving for x_2 and obtaining $x_2(x_1) = \frac{k}{b} - \frac{a}{b}x_1$ and
- forming the derivative with respect to x_1 which yields $\frac{dx_2}{dx_1} = -\frac{a}{b}$.

Therefore, the marginal rate of substitution for perfect substitutes is $\frac{a}{b}$.

So far, we did not make use of a utility function (possibly) representing the preferences. If such a function is available, calculating the marginal rate of substitution is an easy exercise:

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}.$$

Here, we make us of the partial derivatives of the utility function, $\frac{\partial U}{\partial x_1}$ and $\frac{\partial U}{\partial x_2}$, for goods 1 and 2, respectively. They are called ... yes (!), marginal utility. Let us return to the case of perfect substitutes considered above. The marginal rate of substitution is found easily:

$$MRS(x_1) = \frac{\frac{\partial(ax_1+bx_2)}{\partial x_1}}{\frac{\partial(ax_1+bx_2)}{\partial x_2}} = \frac{a}{b}$$

7. Convex preferences and the Middle Way

Consider fig. 9. Here we have $x_1 < y_1$ and $MRS(x_1) > MRS(y_1)$. If the consumption of good 1 (wine) increases while the consumption of good 2 (cheese) decreases, the MRS often decreases. Indeed, the extra wine is not worth a lot of cheese if I consume a lot of wine already.

For example, the MRS of Cobb-Douglas utility functions (given by $U(x_1, x_2) = x_1^a x_2^{1-a}$, $0 < a < 1$) is

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{a x_1^{a-1} x_2^{1-a}}{(1-a)\, x_1^a x_2^{-a}} = \frac{a}{1-a}\frac{x_2}{x_1}.$$

If we increase x_1, we need to decrease $x_2 > 0$ along any indifference curve (Cobb-Douglas preferences are monotonic) – $\frac{x_2}{x_1}$ is therefore a decreasing function of x_1. These preferences are called strictly convex by economists.

An alternative definition is this:

- Take two bundles of goods between which you are indifferent.
- Then, draw a line between these two goods.
- Now,
  - if you weakly prefer every bundle on the line to the two extreme bundles, your preferences are convex;
  - if you strictly prefer every bundle strictly in between the two extreme bundles, your preferences are strictly convex.

At this point, we need to learn how to add bundles and how to multiply bundles with a real number. Just solve the next exercise:

EXERCISE II.10. *Consider the vectors* $x = (x_1, x_2) = (2, 4)$ *and* $y = (y_1, y_2) = (8, 12)$. *Find* $x + y$, $2x$ *and* $\frac{1}{4}x + \frac{3}{4}y$*!*

$\frac{1}{4}x + \frac{3}{4}y$ is called a linear combination of vectors x and y because the coefficients are non-negative and they sum up to 1. It is to be found on the line between x and y. $\frac{1}{4}x + \frac{3}{4}y$ is closer to y because y's coefficient is the highest. An extreme case is $0x + 1y$.

Now we can say: preferences are convex if the linear combination of two indifferent bundles is preferred to each of these bundles.

EXERCISE II.11.

Apparently, the Buddha had a liking for convex preferences–that is at least one possibility of interpreting his "Middle Way" (for an interesting discussion of the Middle Way, see Bahm 1993).

EXERCISE II.12. *If you are a Buddhist and happen to be indifferent between*

- *two glasses of wine and 4 meditations on the one hand, and*
- *four glasses of wine and 2 meditations on the other hand,*

do you then prefer three glasses of wine and three meditations to two glasses of wine and 4 meditations?

8. Monotonic preferences and contentment

8.1. Tanha versus chanda. Payutto (1994) develops Buddhist household theory around the concepts of tanha versus chanda. In chapter 2 (pp.

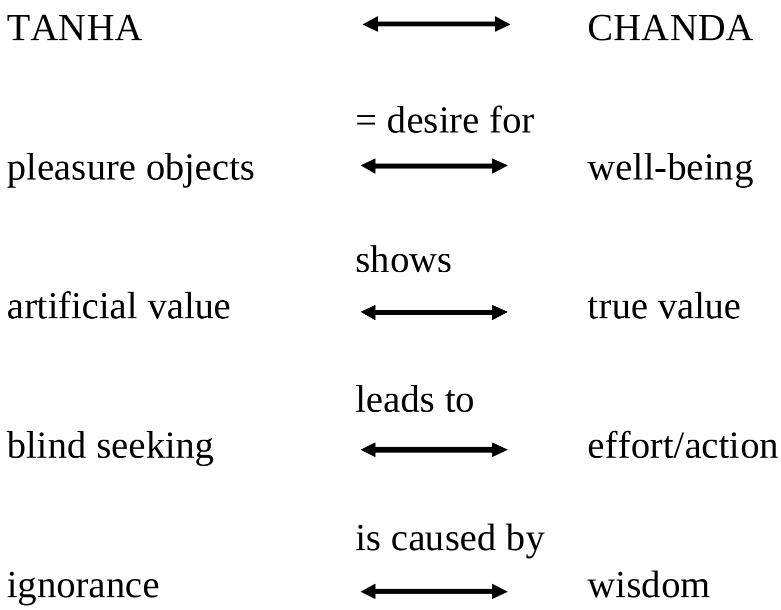

FIGURE 10. Tanha versus chanda

29), he explains these two words (this subsection) while chapter 3 is devoted to particular aspects of preference and household theory (section **??**).

Tanha is "blind craving", "wanting to have", or "seeking of objects which pander to self interests and is supported and nourished by ignorance". Marketing departments all over the world try to address the "five sense pleasures" sought by tanha: sights, sounds, smells, tastes, and bodily feelings. In short, tanha is the "desire for pleasure objects".

In contrast, chanda is "directed toward benefit, it leads to effort and action, and is founded on intelligent reflection." The short translation is "desire for well-being". Figure 10 juxtaposes these two important preference concepts. For example, tanha means desire for pleasure objects and shows artificial value whereas chanda is desire for well-being and shows true value.

8.2. Monotonic preferences in the Hitopadeṣa. Monotonic preferences are well-known to the writers of the Hitopadeṣa:.

> Fire never has enough wood, nor is the ocean fully satisfied with the rivers, nor Death with all creatures, nor a beautiful-eyed woman with all men. (p. 287)

However, warnings against monotonic preferences are also common place:

> Greed makes one lose one's mind, greed breeds desire; and if a man is tormented by desire, he will suffer in this world and the next. (p. 161)

Or:

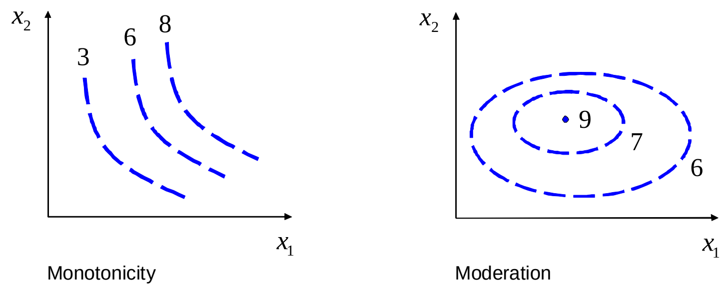

FIGURE 11. Monotonicity (tanha) versus moderation (chanda)

> Happy are those peaceful-minded folk who are satisfied with the ambrosia of contentment. But how can that same happiness be shared by moneygrubbers who keep racing here and there? (p. 163)

8.3. Translations. What might be an appropriate translation of "the ambrosia of contentment" into preference theory? The difference between monotonic preferences (more is better) and non-monotonic preferences (after a certain point) is depicted in fig. 11. Of course, non-monotonic preferences are not foreign to the archetypal homo oeconomicus who knows that there may be "too much of a good thing" such as cheese or wine. Also, dynamic consumption models have been presented to show how consumption in the past influences consumption and well-being in the future (see, for example Becker & Murphy 1988).

Another aspect of contentment can be explained by way of figure 12. The left-hand side shows the original set of indifference curves for (materialistic?) goods 1 and 2. After becoming a more content person (e.g., a Buddhist), the person in question has a higher level of contentment with less consumption of material goods. That is, you would rather be a content person with bundle A (right-hand diagram) than a discontent person with the same bundle (left-hand diagram). Or, you are indifferent between consuming bundle A with content preferences (right-hand diagram) and consuming bundle B with greedy preferences (left-hand diagram).

As a practical matter, being content may be influenceable by means of rational arguments, meditation, or prayer (similar to Christians who thank God for providing the means of daily life).

In some places, the Hitopadeśa is even taking the more extreme view that wealth is bad:

> It generates suffering to earn it, anxiety in hard times and it deludes people into prosperity–how can wealth lead to happiness? (p. 181)

Or:

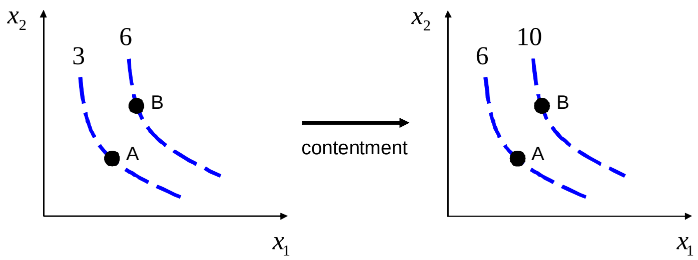

FIGURE 12. A content person is happy with less (material goods)

> Abandon desire, and who is a pauper, who a lord? But if you give desire some leeway, you'll be a slave first and foremost. (p. 183)

9. Equanimity

At some stage, the Hitopadeśa advises equanimity with respect to good and bad events:

> Whether happiness or misery befalls you, you should accept it. Happy and unhappy events take turns, revolving like a wheel. (p. 177)

We also like to quote from the Bhagavad Gita where Krishna also recomments equanimity:

> "He whose mind is unperturbed in times of sorrow, who has lost the craving for pleasures, and who is rid of passion, fear and anger, is called a sage of steadied thought. His wisdom is secure who is free of any affections and neither rejoices nor recoils on obtaining anything good or bad."
> (Cherniak (2008, p. 191))

To us, Krishna seems to advocate a preference relation $\succsim$ with

$$\text{pleasure} \sim \text{sorrow}.$$

In the philosophical literature, there is a discussion about whether one can decide to have specific preferences or desires (see the discussion by Millgram 1998). Indeed, the economist Frank (1987) asks the question: "If homo economicus could choose his own utility function, would he want one with a conscience?" Here, we might ask the related question: "If man could choose his own utility function, would he want one governed by equanimity?" We do not pursue this discussion here.

Equanimity seems to be closely related to contentment. Indeed, consider a Cobb-Douglas utility function with a prefixed $ch > 0$ so as to obtain $U(x_1, x_2) = ch \cdot x_1^{\alpha} x_2^{1-\alpha}$, $0 \leq \alpha \leq 1$, where $ch > 0$ is high for a person trained to be content and low for a malcontent person. However, how could

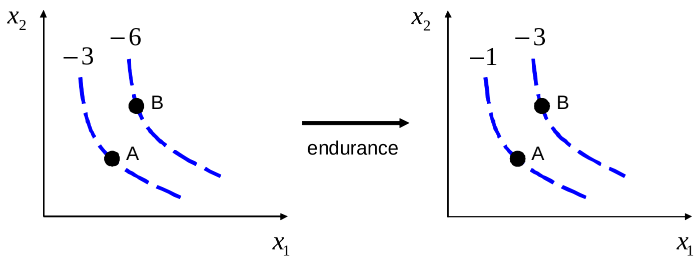

FIGURE 13. Endurance helps to cope with disagreeable things more easily

we incorporate sorrow or misery? In terms of our microeconomic model we could consider two bads (dirt and disagreeable noise) as in figure 13. Normal people get very upset about the state of affairs at A in the left-hand part of the figure, while the equanimous agent keeps a peaceful mind. Indeed, after equanimity training, he is as unhappy about the very bad state of affairs in B (right panel) as he would be in A.

The corresponding utility function is $U(x_1, x_2) = -(1 - ch) \cdot x_1^\alpha x_2^{1-\alpha}$, $0 \leq \alpha \leq 1$, where $0 < ch < 1$ is high for a chanda person.

Of course, it is not quite clear psychologically whether this utility function is possible for negative aspects while the above contentment utility function can be brought to bear on positive things. Indeed, the idea of equanimity is to accept both positive aspects and negative aspects of life. A similar attitude is hidden behind the verse from the old testament (book Job): "The Lord gave, and the Lord has taken away; blessed be the name of the Lord."

10. Solutions

Exercise II.1

Did you also obtain

	relation	
property	is the father of	is of the same sex as
reflexive	no	yes
transitive	no	yes
symmetric	no	yes
antisymmetric	no	no
asymmetric	yes	no
complete	no	no

Exercise II.2

We have $[17] = [-23] = [1]$ and $[100] = [0]$. The relation "is of the same sex as" is an equivalence relation (see exercise II.1). The equivalent classes are "the set of all males" and "the set of all females".

Exercise II.3

(1) The preferences

$$A \succsim B, C \sim E, C \succ A, D \sim A$$

can be rewritten as

$$C \sim E \succ D \sim A \succsim B.$$

By transitivity, we have $E \succ A$.

(2) The second preferences

$$A \succsim B, C \succ E, C \succ A, D \sim A$$

amount to

$$E \prec C \succ D \sim A \succsim B.$$

Here, we do not know whether E is preferred to A or not.

(3) Finally,

$$A \succsim B, B \sim E, C \succ A, E \succsim C$$

implies

$$A \succsim B \sim E \succsim C \succ A.$$

Transitivity then leads to the contradiction $A \succ A$. In this situation, the preference information is not useful.

Exercise II.4

The prohibitive price is defined by $X(p) = 200 - 4p = 0$. It is 50.

The satiation quantity is $X(0) = 200 - 4 \cdot 0 = 200$.

The demand function is a straight line between the points $(0, 200)$ and $(50, 0)$.

Exercise II.5

The slope between the points

$$\begin{aligned} (x, \Pi) &= (3, 10) \text{ and} \\ \left(\hat{x}, \hat{\Pi}\right) &= (5, 16) \end{aligned}$$

is

$$\frac{\Delta\Pi}{\Delta x} = \frac{16 - 10}{5 - 3} = \frac{6}{2} = 3$$

and hence positive. Between the points

$$\begin{aligned} (x, \Pi) &= (3, 10) \text{ and} \\ \left(\hat{x}, \hat{\Pi}\right) &= (5, 2) \end{aligned}$$

the slope is negative:

$$\frac{\Delta\Pi}{\Delta x} = \frac{2 - 10}{5 - 3} = \frac{-8}{2} = -4$$

Exercise II.6

Did you obtain

- $f'(x) = -2x$,

- $g'(x) = 0$
- $\Pi'(x) = 24x^5 - 4x$

Exercise II.7

If the agent has monotonic preferences, then

$$B \succ A \text{ and } C \succ A,$$

but it is unclear whether we have $B \prec C$, or $C \prec B$ or $B \sim C$.

Exercise II.8

We have

$$\begin{aligned} U(A) &= U(3,6) = 15, \\ U(B) &= U(4,7) = 18 \text{ and} \\ U(C) &= U(5,5) = 15 \end{aligned}$$

and hence

$$B \succ A \sim C.$$

Exercise II.9

Your four pictures should look like this:

- If good 2 is a bad, the indifference curve is upward sloping.
- Red and blue matches are perfect substitutes. They are depicted by linear indifference curves with slope -1.
- Left and right shoes are perfect complements and the indifference curves are L-shaped.

Exercise II.10

Adding two vectors reduces to adding the components:

$$\begin{aligned} x + y &= (2,4) + (8,12) \\ &= (2+8, 4+12) = (10,16) \end{aligned}$$

Multiplying a vector with a real number is also defined component-by-component:

$$2x = 2(2,4) = (2 \cdot 2, 2 \cdot 4) = (4,8)$$

Using both operations, we find

$$\begin{aligned} \frac{1}{4}x + \frac{3}{4}y &= \frac{1}{4}(2,4) + \frac{3}{4}(8,12) \\ &= \left(\frac{1}{4} \cdot 2, \frac{1}{4} \cdot 4\right) + \left(\frac{3}{4} \cdot 8, \frac{3}{4} \cdot 12\right) \\ &= \left(\frac{1}{2}, 1\right) + (6,9) \\ &= \left(6\frac{1}{2}, 10\right) \end{aligned}$$

Exercise II.11

The preferences indicated in (a) are strictly convex, while those in (b) and (c) are convex but not strictly convex. The preferences depicted in (d) are not convex.

Exercise II.12

Yes, by

$$\frac{1}{2}(2,4)+\frac{1}{2}(4,2)=(3,3)$$

you should indeed.

CHAPTER III

Decisions

1. Introduction

We now turn to decision theory where actions, states of the world, and preferences are the central building blocks. After explaining the basic model, we present a few simple examples from the Hitopadesa. In the following chapter, we penetrate deeper into decision theory in order to analyze the Bhagavad Gita in some detail.

2. Decision theory: the simple models

Preference relations, actions, consequences, states of the world, choice functions etc. form the ingredients of decision theory. We provide the necessary building blocks by borrowing freely from Kreps (1988), Rubinstein (2006), or Simon (1955, p. 102). Preference relations have already been introduced in chapter II.

Consider a simple example. A firm can producte umbrellas or sunshades. Umbrellas lead to a profit of 100, sunshades yield 64.

action	production of umbrellas	100
	production of sunshades	64

FIGURE 1. Payoffs depend on actions

In this most basic microeconomic decision model we have

- a set of actions A (production of umbrellas, production of sunshades)
- a set of consequences C (profits)
- a consequence function $f : A \to C$ that attributes a consequence $c \in C$ to every action $a \in A$ (the production of umbrellas leads to the profit of 100) and
- a preference relation $\succsim$ on C (presumably, 100 is better than 64)

In the standard decision model, an agent chooses an action $a \in A$, earns the consequence $f(a)$ which may be better or worse than consequences obtained from other actions. The theoretical prediction is an action a^* that obeys

$$\underbrace{f(a^*)}_{\in C} \succsim \underbrace{f(a)}_{\in C} \text{ for all } a \in A.$$

Differently put, the decision maker chooses an action a^* with consequence $f(a^*)$ such that no other action a exists that leads to a consequence $f(a)$ which is better than $f(a^*)$. In our example, the firm will produce umbrellas.

In more involved models, a set of states of the world is also added. Reconsider the firm that produces umbrellas or sunshades. The firm's profits now depend on the weather, also. There are two states of the world, good or bad weather. The following payoff matrix indicates the profit as a function of the firm's decision (strategy) and of the state of the world.

		state of the world	
		bad weather	good weather
action	production of umbrellas	100	81
	production of sunshades	64	121

FIGURE 2. Payoff matrix

The highest profit is obtained if the firm produces sunshades and the weather is good. However, the production of sunshades carries the risk of a very low profit, in case of rain. The payoff matrix examplifies important concepts in our basic decision model: actions, states of the world, payoffs and payoff functions.

- The firm has two strategies, producing umbrellas or producing sunshades.
- There are two states of the world, bad and good weather.
- The payoffs are 64, 81, 100 or 121.
- The payoff function determines the payoffs resulting from strategies and states of the world. For example, the firm obtains a profit of 121 if it produces sunshades and it is sunny.

Let us translate this example into a somewhat more formal model. By W, we denote the set of states of the world. We always assume that A and W are set up so that the decision maker can choose one and only one action from A and that one and only one state of the world from W can actually happen.

Since the outcomes (the payoffs or profits in the umbrella-sunshade example) depend on both actions and states of the world, we need to consider tuples (a, w) with $a \in A$ and $w \in W$. The set of these tuples is denoted by $A \times W$. Instead of a consequence function f, we then deal with an uncertain-consequence function $g : A \times W \to C$, i.e., a consequence $c \in C$ is determined by both an action $a \in A$ and a state of the world $w \in W$.

For the analysis of this decision situation, it is helpful to ask the following question: Given a specific state of the world, which action is best? We also say: Which action is a best response to a state of the world. In our example, if the weather is bad, the production of umbrellas yields a higher profit than the production of sunshades. This is indicated by $\boxed{\text{R}}$ in the following matrix:

		state of the world	
		bad weather	good weather
action	production of umbrellas	100 $\boxed{\text{R}}$	81
	production of sunshades	64	121 $\boxed{\text{R}}$

FIGURE 3. Payoff matrix

Sometimes, one action is better than another one for all states of the world. We then say that a dominates b. Let us change the payoffs a bit:

		state of the world	
		bad weather	good weather
action	production of umbrellas	100 $\boxed{\text{R}}$	181 $\boxed{\text{R}}$
	production of sunshades	64	121

FIGURE 4. Payoff matrix

Then, the production of umbrellas dominates the production of sunshades because we have

$$\begin{aligned} g\,(\text{umbrella, bad weather}) &\succ g\,(\text{sunshade, bad weather}) \text{ and} \\ g\,(\text{umbrella, good weather}) &\succ g\,(\text{sunshade, good weather}) \end{aligned}$$

In terms of best responses, umbrella production dominates sunshade production because we have the $\boxed{\text{R}}$ everywhere in the umbrella row.

Dominance makes deciding easy. If there is no dominant action, the decision maker may have information about the probabilities for the states which help him to come to a conclusion. We turn to probabilities later on.

Exercise III.1. *In the payoff matrix below, does any action dominate any other one?*

		weather		
		bad	medium	good
action	production of green icecream	10	20	30
	production of blue icecream	25	25	20
	production of red icecream	8	15	25

3. Simple models in the Hitopadeśa

3.1. Investment and duty in short and long lifes. The Hitopadeśa starts with an invocation (to Lord Shiva). Also, the investments (in financial and human capital) and fulfillment of religious duties are considered for people who live short or long lives:

> A wise man should think about knowledge and money as if he were immune to old age and death; but he should perform his duties as if Death had already seized him by the hair. (p.57)

Here, p. 57 is to be understood as "Hitopadeśa, p. 57" and refers to Torzsok (2007, p. 57). Apparently, this subashita deals with two different decision situations. One is concerned with investment in "knowledge and money", the other with performing duties. At first sight, the advice seems to be contradictory. Why work with different assumption when dealing with these different problems?

Let us try decision-theoretic analyses. With respect to the first decision, we propose the actions

$$\begin{aligned} \text{invest} &= \text{save money/increase knowledge,} \\ \text{do not invest} &= \text{spend money/do not labor for education} \end{aligned}$$

and the states of the world

$$\text{short life,}$$
$$\text{long life.}$$

The consequences can be seen in the following decision matrix:

		state of the world	
		short life	long life
action	invest	no use for capital/knowledge	long use of capital/knowledge [R]
	do not invest	enjoyment of money/leisure [R]	material poverty/ spiritual poverty **VB**

FIGURE 5. The investment payoff matrix

In case of a short life, investments do not pay and the decision maker would rather like to enjoy his money and leisure as long as he lives (see [R]**).** In contrast, if the agent lives for a long time, investments pay off (see [R]). Indeed, the very bad outcome **VB** of poverty (material and spiritual one) occurs if the agent neglects investments and lives for a long time.

The Hitopadeśa's advice of imagining a very long life amounts to the advice of investing. It may also be seen as a means to avoid the very bad outcome. A maxmin strategy would also lead to this recommendation. The maxmin strategy works as follows: For every action, consider the worst outcome. Then, choose the action with the best (of these worst) outcomes. Thus,

action invest — > no use for capital/knowledge,

action do not invest — > material or spiritual poverty

and, since "poverty" (the very bad outcome) is worse than "no use", the decider should choose to invest. Alternatively, one may try to translate this risk-averse approach by a "better safe than sorry" attitude.

Let us now model the second decision problem with the actions

fulfill dharma now

fulfill dharma later

and the states of the world as above. We then obtain the decision matrix:

The best situation is g (fulfill dharma later, long life) and described by

- long life
- enjoyment of life in youth
- fulfill dharma later and
- heaven or good karma.

However, postponing the focus on dharma is risky. If his life is short, the agent suffers eternal damnation in hell or bad karma for his future lifes.

		state of the world	
		short life	long life
action	fulfill dharma now	good karma [R]	good karma, little enjoyment in youth
	fulfill dharma later	bad karma **VB**	good karma, some enjoyment in youth [R]

FIGURE 6. The duty-now-or-duty-later matrix

In order to prevent this very bad outcome (**VB**), it is best to choose "fulfill dharma now" which is also the best action for "short life". This is the recommendation given in the Hitopadeśa!

It turns out that both decision problems can be analyzed along the same lines so that the advice given is not contradictory, after all.

Proverbs and their recommendations often come in pairs, one telling the opposite of the other. We have just seen that the Hitopadeśa advocates the long-term perspective with respect to investments in financial and human capital. In other parts of that same collection of stories and advice, the reader is warned against hoarding. For example,

> The wealth of a rich man lie in what he gives away or enjoys; once he dies, others will play with his wife as well as his wealth.
>
> Furthermore,
>
> I think that your wealth is what you give away to distinguished people or what you consume day by day; the rest is what you keep for somebody else. (pp. 173)

(An interpretation with a life-cycle model may be possible.)

As a final remark, the authors of the Hitopadeśa also warn against wealth in general, even for the purpose of giving it away:

> It generates suffering to earn it, anxiety in hard times and it deludes people in prosperity–how can wealth lead to happiness.
>
> And,
>
> If you want money to spend for religious purposes it's better not to desire anything at all. It's better not to touch mud and avoid it from a distance than to wash it off. (p. 173)

3.2. Fate and human effort. In the Hitopadeśa's prologue, king Sudarśana muses about the relationship of fate and human effort. Criticizing lazy people who just rely on fate, he makes his point of view clear:

> One should not give up one's efforts, even when acknowledging the role of fate; without effort, one cannot obtain oil from sesame seeds.
>
> And there is another verse on this:
>
> Fortune gravitates towards eminant men who work hard;
> only cowards say it depends on fate.
> Forget about fate and be a man–use your strength!
> Then, if you don't succeed inspite of your efforts, what is there to blame?
> (p. 69)

Thus, according to king Sudarśana, effort and fate co-determine the outcomes. We offer this interpretation: First, the decision maker may be lazy or busy, i.e., we have

$$A = \{\text{lazy, busy}\}.$$

Second, fate may be favorable or unfavorable:

$$W = \{\text{favorable, unfavorable}\}$$

The outcomes for all pairs from $A \times W$ should obey these preferences

$$\begin{aligned} g(\text{busy, favorable}) &\succ g(\text{lazy, favorable}) \succ g(\text{lazy, unfavorable}), \\ g(\text{busy, favorable}) &\succ g(\text{busy, unfavorable}) \succ g(\text{lazy, unfavorable}) \end{aligned}$$

while it is not clear how to rank

$$g(\text{lazy, favorable}) \text{ and } g(\text{busy, unfavorable}).$$

For example, the following numerical payoffs, reflect the above preferences:

		state of the world (fate)	
		favorable	unfavorable
action	lazy	10	2
	busy	50	10

FIGURE 7. A payoff matrix

Thus, a payoff of 10 may result from lazyness and luck or else from high effort and ill fate. As king Sudarśana says, the payoff of 10 is no reason for reproach if it accrues to a person who has used his strength.

The matrix interpretation seems well in line with Sudarśana's thinking. Indeed, he adds to the above verse this line (Hitopadeśa p. 71): "Just as a cart cannot move on one wheel, so fate itself cannot be fulfilled without human effort."

It may seem "busy" dominates "lazy" ($50 > 10$ and $10 > 2$). However, effort does not come without costs. If we include the costs of effort c, we obtain the matrix

		state of the world (fate)	
		favorable	unfavorable
action	lazy	10	2
	busy	$50 - c$	$10 - c$

EXERCISE III.2. *In the payoff matrix above, can you find out for which cost intervals "busy" is dominant, lazy" is dominant, or neither is dominant?*

4. Decision theory: lotteries

Let us revisit the producer of umbrellas and sunshades whose payoff matrix is given below, but this time we add "belief" (probability assessment) that bad weather occurs with probability $\frac{1}{4}$ (and hence good with with probability $\frac{3}{4}$).

		state of the world	
		bad weather, $\frac{1}{4}$	good weather, $\frac{3}{4}$
action	production of umbrellas	100	81
	production of sunshades	64	121

FIGURE 8. Umbrellas or sunshades?

Then, the action "produce umbrellas" yields the payoff 100 with probability $\frac{1}{4}$ and 81 with probability $\frac{3}{4}$. Thus, the probability distribution on the set of states of the world leads to a probability distribution for payoffs, in this example denoted by

$$L_{\text{umbrella}} = \left[100, 81; \frac{1}{4}, \frac{3}{4}\right].$$

Here, L stands for lottery. For lotteries, one is often interested in the expected value. In order to calculate the expected value, we multiply the probability for each state of the world with the corresponding payoff and sum over all states. In our example, we have

$$\begin{aligned} E\left(L_{\text{umbrella}}\right) &= \frac{1}{4}\cdot 100+\frac{3}{4}\cdot 81 \\ &= 85.75 \end{aligned}$$

EXERCISE III.3. *Find the expected value for the lottery that results form the production of sunshades.*

5. Lotteries in the Hitopadeśa and the Pañcatantra

5.1. Fate and human effort revisited. One might try to apply lotteries to the Hitopadeśa. However, there is, as far as we know, no indication that the writers of the Hitopadeśa used probabilities. With this caveat in mind, let us assume probabilities for favorable and unfavorable states of the world in the example of subsection 3.2. We then get this payoff matrix:

		state of the world (fate)	
		favorable, p_{fav}	unfavorable, p_{unfav}
action	lazy	10	2
	busy	$50-c$	$10-c$

FIGURE 9. A payoff matrix

The two actions lead to the lotteries

$$\begin{aligned} L_{\text{lazy}}^{c} &= \left[10,2;p_{\text{fav}},p_{\text{unfav}}\right] \text{ and} \\ L_{\text{busy}}^{c} &= \left[50-c,10-c;p_{\text{fav}},p_{\text{unfav}}\right] \end{aligned}$$

with expected payoffs

$$\begin{aligned} E\left(L_{\text{lazy}}^{c}\right) &= p_{\text{fav}}\cdot 10+\left(1-p_{\text{fav}}\right)\cdot 2 \\ &= 2+8p_{\text{fav}} \text{ and} \\ E\left(L_{\text{busy}}^{c}\right) &= p_{\text{fav}}\cdot\left(50-c\right)+\left(1-p_{\text{fav}}\right)\cdot\left(10-c\right) \\ &= 10+40p_{\text{fav}}-c \end{aligned}$$

respectively. Now, a comparison of the two expected payoffs yields

$$E\left(L_{\text{busy}}^{c}\right)>E\left(L_{\text{lazy}}^{c}\right) \text{ or } c<8+32p_{\text{fav}}$$

Thus, whenever the cost of being busy is sufficiently low, the decision maker prefers to make an effort.

5.2. Investment and duty in short and long lifes revisited. Let us also apply the expected-payoff maximiziation procedure to the two other problems considered above. The investment-problem in a short or long life is given by this payoff matrix with probabilities:

		state of the world	
		short life, p_{short}	long life, p_{long}
action	invest	no use for capital/knowledge	long use of capital/knowledge R
	do not invest	enjoyment of money/leisure R	material poverty/ spiritual poverty **VB**

FIGURE 10. The investment payoff matrix

Here, we do not have any numerical values for the outcomes. Let us assume that these numerical values exist. They can be denoted by $u\,(\text{no use})\,, u\,(\text{long use})\,,\ u\,(\text{enjoyment})$ and $u\,(\text{poverty})$, respectively. Then, we have the two lotteries

$$\begin{aligned} L_{\text{invest}} &= [u\,(\text{no use})\,, u\,(\text{long use})\,; p_{\text{short}}, p_{\text{long}}] \text{ and} \\ L_{\text{not invest}} &= [u\,(\text{enjoyment})\,, u\,(\text{poverty})\,; p_{\text{short}}, p_{\text{long}}]\,. \end{aligned}$$

It is preferable to choose invest (the recommendation given in the Hitopadeśa) if

$$\begin{aligned} E\,(L_{\text{invest}}) &> E\,(L_{\text{not invest}})\,, \\ p_{\text{short}} &< \frac{u\,(\text{long use}) - u\,(\text{poverty})}{[u\,(\text{enjoyment}) - u\,(\text{no use})] + [u\,(\text{long use}) - u\,(\text{poverty})]} \text{ or} \\ p_{\text{short}} &< \frac{1}{\frac{[u(\text{enjoyment}) - u(\text{no use})]}{[u(\text{long use}) - u(\text{poverty})]} + 1} \end{aligned}$$

holds. Thus, investment is best, if the following conditions hold:

- p_{short} is small or p_{long} is large,
- $u\,(\text{enjoyment})$ or $u\,(\text{poverty})$ are small,
- $u\,(\text{no use})$ and $u\,(\text{long use})$ are large.

Differently put, it is best to invest if the payments associated with that decision are large. Also, investment is a good idea if the probability for the very bad outcome is large.

EXERCISE III.4. *Can you do a similar analysis for "dharma now" versus "dharma later" (see p. 36)?*

5.3. Dissension among allies.

5.3.1. *The lion and the bull.* The first book of the Pañca-tantra is entitled “On causing dissension among allies”. The frame story is about a jackal called Damanka who is counselor to the king, a lion, with the name Piṅgalaka. The lion has made friends with a bull, named Saṃjīvaka. His counselors and all the other animals at the court are not happy about the exclusive nature of this friendship:

> From that time onwards, every day Piṅgalaka and Saṃjīvaka spent their time together in mutual affection ... As time went by, the lion made fewer kills and food became scarce. As a result the same two, Karataka and Damanka [jackal counselors, HW], became very hungry ...
> (Pañcatantra, p. 107)

where Pañcatantra stands for Olivelle (2006). The jackal Damanka decides to destroy this friendship. Basically, he tells both sides that the other party is about to kill his erstwhile friend. Both the lion and the bull find the jackal’s assertion unlikely, but they become suspicious in time. Finally, when the two former friends meet, while still hoping that the highly valued friendship can be preserved, the lion attacks the bull who finally dies.

Should we classify the bull’s killing by the lion as “a rational preemptive action or a (possibly counterproductive) act of panic” (Elster (1999, p. 233))?

5.3.2. *The model.* We use the letter L for the lion and the leter B for the bull. Both consider attacking or not attacking, i.e., they have the action set

$$A = \{\text{attack, not attack}\}$$

The outcomes are framed in terms of

- F (payoff for friendship)
- V (payoff for victory over friend)
- NF (payoff for loss of friendship and death of one animal or both animals, resulting from fighting)
- D (payoff for death)

where these four payoffs are indexed by L or B, respectively. We assume

$$F > V > NF > D$$

so that both animals value friendship highly. Even if friendship cannot be sustained, both prefer not to kill the former friend ($NF > V$). However, the worst outcome for each is his own death. We make the following assumptions:

- If one animal attacks while the other waits, the latter will be killed.
- If both animals attack, friendship will be destroyed and one or both animals will die.
- If no animal attacks, the two animals can restore their friendship.

Thus, we obtain this payoff matrix:

		Bull: attack (A), p_B	Bull: not attack (NA), $1-p_B$
Lion	attack (A), p_L	NF_L, NF_B	V_L, D_B
	not attack (NA), $1-p_L$	D_L, V_B	F_L, F_B

5.3.3. *Uncertainty about actions (two decision problems).* In chapter VI, we will attempt a game-theoretic analysis of the highly suspenseful moment where the two animals approach each other. Here, we content ourselves with analyzing the situation as two separate decision problems.

The lion assumes that the bull will attack with a probability of p_B. Given this probability it is best for the lion to attack whenever

$$p_B NF_L + (1-p_B)\, V_L > p_B D_L + (1-p_B)\, F_L$$

or

$$p_B > \frac{1}{1+\frac{NF_L - D_L}{F_L - V_L}}.$$

Thus, the lion will attack whenever a suitable combination of the following holds:

- The lion assumes that the bull will attack with a high probability.
- NF_L and V_L are large, i.e., the lion does not fear the loss of friendship and enjoys victory over the bull.
- D_L and F_L are small, i.e., the lion is afraid of death and does not value friendship very highly.

Similarly, the bull will attack if he assumes a probability p_L for an attack by the lion and if

$$p_L > \frac{1}{1+\frac{NF_B - D_B}{F_B - V_B}}$$

holds. In the story, it is the lion who attacks first. Maybe, the lion imputed a higher attack probability to the bull than the other way around. Also, the bull is not too worried about dying in battle:

> To die in a war is for men the most glorious death of all.
> (Olivelle (2006, p. 191))

6. Lotteries against and in favor of God's existence

6.1. The loan lottery. One of the Buddha's birth-stories, the birth-story of Brahma, is about a king who does not believe in the afterworld (*para-loka*) and holds other *Cārvāka* views, also (see chapter X, p. 146). The former Buddha was a Brahma deity (a god) who tried to benefit the king by converting him to virtuous attitudes and behavior:

> "If convinced that good and bad deeds have
> happy and unhappy results in the next life,
> one avoids evil and strives for purity.
> But non-believers follow their whims.

The king's pernicious false view was an affliction that spelled ruin, bringing calamity on the world. As a result, the Great One, that divine seer, felt compassion for the king. So, one day, while the monarch was staying in a beautiful and secluded grove [Hain, HW], caught up in sense pleasures, the Great One descended from the Brahma Realm and blazed [aufflammen, lodern, HW] in front of him." (Meiland (2009b, p. 271))

The god and the king engage in an argument on whether the "next world" exists. The king is not convinced and comes up with a clever proposal:

"Well. great seer!

> If the next world is not a bogey man [Schreckgespenst, HW]
> for children,
> and if you think I should believe in it,
> then give me five hundred nishkas
> and I'll return you a thousand in another life!"
>
> (Meiland (2009b, p. 279))

Thus, the king proposes a lottery. Let p_{asti} be the probability the the next world exists and $p_{n\bar{a}sti} = 1 - p_{asti}$ the probability for non-existence. Then, the lottery is given by

$$\begin{aligned} L_{\text{loan}} &= \left[\underbrace{1000-500}_{\substack{\text{loan is repaid} \\ \text{in other world} \\ \text{that exists}}}, \underbrace{-500}_{\substack{\text{loan is not repaid} \\ \text{in other world} \\ \text{as other world does not exist}}} ; p_{asti}, p_{n\bar{a}sti} \right] \\ &= [500, -500; p_{asti}, 1 - p_{asti}] \end{aligned}$$

If the god does not accept the lottery, he obtains the payoff of 0. Now, so the king seems to argue, the lottery is worth accepting whenever its expected value

$$\begin{aligned} E\left(L_{\text{loan}}\right) &= p_{asti} \cdot 500 + (1 - p_{asti}) \cdot (-500) \\ &= -\underbrace{500}_{\substack{\text{loan given} \\ \text{in both cases}}} + p_{asti} \cdot \underbrace{1000}_{\substack{\text{repayment} \\ \text{if other world exists}}} \end{aligned}$$

is larger than zero, i.e., if

$$p_{asti} > \frac{500}{1000} = \frac{1}{2}$$

This seems a good test of whether the god himself believes in the other world. If he assumes a probability larger than $\frac{1}{2}$, he should–so argues the king–accept the lottery.

If the king's lottery is cleverly constructed, the god's answer is surely ingenious. The god has no doubt about the other world, but does not think it realistic that he will get repaid:

> "Even in this world, wealth seekers
> do not offer money to the wicked,
> nor to the greedy, fools or indolents [träge, HW].
> For whatever goes there comes to ruin.
>
> But if they see someone who is modest,
> naturally calm and skilled in business,
> they will give him a loan, even without witnesses.
> For money entrusted to such a man brings reward.
>
> The same procedure for giving a loan
> should be used for the next world, king.
> But it would be improper to entrust money to you;
> for your conduct is corrupted by wicked views.
>
> Who would harrass you for a thousand nishkas
> when you lie in hell, senseless, sick with pain,
> brought there by your own actions
> caused by the evil of your false views?
>
> ...
>
> In the next world, where nihilists [*nāstika* in the original, HW] live
> a thick darkness and icy wind tortures
> people by tearing through their very bones.
> What prudent man would go there to get money?
>
> ...
>
> When blazing iron nails fasten your body
> to the ground red with smokeless flames

and you wail pitifully as your body burns,
who would ask you for your debt then?"

(Meiland (2009b, pp. 279-281, 287))

Understandably, the god (if a "wealth seeker" at all) does not find this lottery attractive:

$$L_{\text{loan, not paid back}} = \left[\underbrace{-500}_{\substack{\text{loan is not (!) repaid}\\ \text{in other world}\\ \text{that exists}}}, \underbrace{-500}_{\substack{\text{loan is not repaid}\\ \text{in other world}\\ \text{that does not exist}}}; p_{asti}, p_{n\bar{a}sti} \right] = [-500; 1]$$

6.2. The hell lottery. As seen in the previous subsection, the god predicts the king a dire future. The horrors of hell are not specific to the birth-stories. In the "Life of Buddha by Ashvaghosah", we are treated to similarly uncomfortable descriptions (see Olivelle 2009, pp. 407, 409). Returning to the birth-story of Brahma, let us quote some additional horrors:

"Some are wrapped in blazing iron turbans.
Others are boiled to a broth [Fleischbrühe, HW] in iron pots.
Others are cut by showers of sharp weapons,
their skin and flesh ripped by hordes of beasts.

Tired, others enter Váitarani's acrid [ätzend, HW] waters,
which scroches [brennen, HW] them on contact like flames.
Their flesh wastes away but not their lives,
for they are sustained by their evil deeds."

(Meiland (2009b, p. 285))

Understandably, the king is convinced:

"My mind almost runs wild with fear
at learning of the punishments in hell.
It practically burns with blazing thoughts
regarding my plight on meeting that fate.

Shortsightedly I trod the wrong path,
my mind destroyed by evil views.
Be then my path, recourse of the good!
By my resort and refuge, sage!

As you dispelled the darkness of my views
like the rising sun dispels night,

so tell me, seer, the path I should follow
to avoid a bad rebirth after this life."

(Meiland (2009b, p. 285))

The god is prepared to give this advice:

Conquer vice, so difficult to vanquish!
Pass beyond greed, so difficult to overcome!
You will thus reach the gleaming gold-gated city
of the king of heavens, ablaze with fine gems [Juwel, HW].

May your mind, which once praised evil views,
firmly cherish the creeds valued by good men.
Abandon immoral beliefs proclaimed
by those eager to pleasure fools.

...

With glory as its banner,
pity as its retinue [Gefolge, HW]
and tranquility as
its lofty flag, king,
if you travel in this chariot [the virtue chariot, HW]
glittering with wisdom
to benefit others and yourself,
you will certainly not enter hell.

(Meiland (2009b, pp. 291, 295))

An interpretation in terms of lotteries is plausible. After all, the king has shown to understand this concept in the previous subsection. The two lotteries are

- $L_{C\bar{a}rv\bar{a}ka}$, standing for his usual *nāstika* views and conduct, versus
- L_{virtue} standing for his new life of pity, tranquility, and wisdom.

Thus, we have

$$L_{C\bar{a}rv\bar{a}ka} = \left[\underbrace{\text{pleasures in this life, but hell with endless horrors}}_{\text{other world exists}}, \underbrace{\text{pleasures in this life}}_{\text{other world does not exist}}; p_{asti}, p_{n\bar{a}sti} \right]$$

or in utility numbers

$$L_{C\bar{a}rv\bar{a}ka} = [-100\ 000,\ 10; p_{asti}, p_{n\bar{a}sti}]$$

with expected payoff

$$\begin{aligned} E\left(L_{C\bar{a}rv\bar{a}ka}\right) &= p_{asti} \cdot (-100\ 000) + (1 - p_{asti}) \cdot 10 \\ &= 10 - 100\,010 p_{asti} \end{aligned}$$

and

$$L_{\text{virtue}} = [\text{life of pity, tranquility, and wisdom, no hell; } 1]$$

with (expected) payoff

$$E\left(L_{\text{virtue}}\right) = 2.$$

Which choice maximizes expected payoff? The virtuous life is the better choice if

$$E\left(L_{\text{virtue}}\right) > E\left(L_{C\bar{a}rv\bar{a}ka}\right),$$

i.e., if the probability of the existence of the other world is sufficiently high. For our numbers, the above inequality is equivalent to

$$p_{asti} > \frac{8}{100010} \approx \frac{8}{10000} = 0.0008.$$

Thus, the king may not (really) believe in the other world, but prefers to play it safe.

Incidentally, Blaise Pascal (paragraph 680 in édition Sellier, paragraph 418 in édition Lafuma, paragraph 233 in édition Brunschvicg) presented a similar argument for believing in God (see, for example Sellier 1991, p. 469).

7. No analyses as yet of these recommendations

7.1. Avoid greed. According to the Hitopadeśa (p. 93), greed is to be avoided:

> Greed produces anger and desire; greed brings delusion and destruction: greed is the cause of evil.

7.2. Don't be inactive. Do not indulge in non-activity because of fear something might go wrong.

> If disaster were near, we would listen to the words of our elders. But if we were to think so carefully in all cases, we wouldn't even get round to eating.
> (Hitopadeśa, p. 91)

Similarly:

> Not to embark on a project for fear of failure is the sign of a coward. My brother, who would refuse to eat food for fear of indigestion?
> (Hitopadeśa, p. 243)

7.3. Avoid two times six faults. Avoid these six states or attitudes:

> A man who wants to prosper in this world should avoid six faults: drowsiness, exhaustion, fear, anger, laziness and procrastination.
> (Hitopadeśa, p. 97)

Also: don't be like this:

> He who is envious, he who criticizes everything, he who is never satisfied, the angry man and the ever fearful, and he who lives off someone else's fortune–these six have a miserable fate.
> (Hitopadeśa, p. 93)

7.4. Be secretive. Things not to post on facebook:

> A wise man should not tell others about his financial losses, his anxiety, his troubles at home, or that he's been hookwinked [hoodwinked = getäuscht, HW] or humiliated.
> (Hitopadeśa, p. 155)

Also, do not tell anybody about:

> Health, wealth, family secrets, magic charms, one's sex life, medicines, penances, donations, humiliations–these are the nine things one should keep strictly private.
> (Hitopadeśa, p. 155)

8. Solutions

Exercise III.1

The first action (green icecream) dominates the third action (red icecream) by

$$10 > 8, 20 > 15, 30 > 25.$$

Furthermore,

- the first action does not dominate the second (see the state of medium weather with $20 < 25$),
- the second action does not dominate the first (see the state of good weather with $20 < 30$),
- the second does not dominate the third (see the state of good weather with $20 < 25$), and
- the third does not dominate the second (see the state of bad weather with $8 < 25$).

Exercise III.2

"Busy" is dominant for $50 - c > 10$ and $10 - c > 2$, i.e., for $c < 8$. "Lazy" is dominant for $50 - c < 10$ and $10 - c < 2$, i.e., for $c > 40$. Otherwise ($8 < c < 40$), neither is dominant (do not worry about $c = 8$ or $c = 40$).

Exercise III.3

The production of sunshades yields the lottery

$$L_{\text{sunshades}} = \left[64, 121; \frac{1}{4}, \frac{3}{4}\right]$$

which has the expected payoff

$$\begin{aligned} E\left(L_{\text{sunshades}}\right) &= \frac{1}{4} \cdot 64 + \frac{3}{4} \cdot 121 \\ &= 106.75. \end{aligned}$$

Exercise III.4

Adding the probabilities, we obtain this payoff matrix:

<table>
<tr><td></td><td></td><td colspan="2">state of the world</td></tr>
<tr><td></td><td></td><td>short life, p_{short}</td><td>long life, p_{long}</td></tr>
<tr><td rowspan="2">action</td><td>fulfill dharma now</td><td>good karma [R]</td><td>good karma, little enjoyment in youth</td></tr>
<tr><td>fulfill dharma later</td><td>bad karma VB</td><td>good karma, some enjoyment in youth [R]</td></tr>
</table>

FIGURE 11. The duty-now-or-duty-later matrix

Again, we assume numerical values for the outcomes which we denote by u (good karma), u (good karma, no fun), u (bad karma) and u (good karma, fun), respectively. We obtain the two lotteries

$$\begin{aligned} L_{\text{fulfill dharma now}} &= \left[u\left(\text{good karma}\right), u\left(\text{good karma, no fun}\right); p_{\text{short}}, p_{\text{long}}\right] \text{ and} \\ L_{\text{fulfill dharma later}} &= \left[u\left(\text{bad karma}\right), u\left(\text{good karma, fun}\right); p_{\text{short}}, p_{\text{long}}\right]. \end{aligned}$$

The Hitopadeśa's recommendation of fulfilling dharma now is best if

$$E\left(L_{\text{fulfill dharma now}}\right) > E\left(L_{\text{fulfill dharma later}}\right)$$

holds or if we have

$$\begin{aligned} p_{\text{short}} &> \frac{u\left(\text{good karma, fun}\right) - u\left(\text{good karma, no fun}\right)}{\left[u\left(\text{good karma}\right) - u\left(\text{bad karma}\right)\right] + \left[u\left(\text{good karma, fun}\right) - u\left(\text{good karma, no fun}\right)\right]} \text{ or} \\ p_{\text{short}} &> \frac{1}{\frac{\left[u(\text{good karma}) - u(\text{bad karma})\right]}{u(\text{good karma, fun}) - u(\text{good karma, no fun})} + 1} \end{aligned}$$

Therefore, opting for a virtuous life when young is best, if the following conditions hold:

- p_{short} is large or p_{long} is small,
- u (good karma) or u (good karma, no fun) are large,
- u (bad karma) and u (good karma, fun) are small.

In other words, fulfilling *dharma* now is good if the payments for that decision (first line in the above matrix) are relatively large. Also, one will avoid the "fulfill dharma later" if this action leads to bad karma (the very bad outcome) with a high probability p_{short}.

CHAPTER IV

Decision theory for the *Bhagavad Gītā*

Don't let the actions's fruit be your motivation
(Bhagavad Gita, translated by Cherniak 2008, pp. 188, verse 26.47)

An act may [...] be identified with its possible consequences (The Foundations of Statistics (a major foundational work in decision theory), written by Savage 1972, p. 14)

1. Introduction

[1]The basic model of decision theory consists of acts, consequences and preferences. A decision maker chooses an act which leads to a consequence. Some consequences are preferred to others. In this setting, the theoretical prediction is obvious: The decision maker chooses the action with the consequence which he prefers to any other consequence obtainable by some alternative action. In the standard models, consequences are more important than acts. As Savage (1972, p. 14) remarks: "If two different acts had the same consequences [...], there would [...] be no point in considering them two different acts at all. "

Thus, microeconomic decision theory is unabashedly consequentialist. Therefore, it may seem impossible to analyze the central writing of Hinduism, the *Bhagavad Gītā*, from a decision-theoretic viewpoint. After all, one of Lord Krishna's famous dictums stipulates: "Don't let the action's fruits be your motivation" where the Sanskrit term for fruit is *phala* which may alternatively be translated as consequence/utility/profit – terms used again and again in microeconomic texts.

The *Bhagavad Gītā* (*Gītā* for short) is part of book six (out of 18 books) of the great Indian epic Mahabhárata. The setting is this: The great warrior Árjuna is about to engage in a fight where the Pandavas (five brothers, among them Árjuna) and their allies are found on one side while the other side consists of the Pandavas' cousins and their respective allies. Árjuna's charioteer is his friend Krishna who reveals himself as God Krishna later on. Árjuna realizes that many of his relatives and teachers can be found on the other side. Imagining the consequences of a deadly fight, he decides

[1]Valuable hints have been given by participants of the research seminar of the Leipzig institute for indology and by Andre Casajus, Frank Hüttner, Katharina Lotzen, Karin Szwedek, and Ulla Wessels.

against fighting and tells Krishna about his decision. Krishna then uses many different arguments and manners to convince Árjuna that, after all, he should fight. Finally, Árjuna is convinced and the battle can begin.

In this chapter, we try to analyze the discussion between the two protagonists, and Krishna's preaching to Árjuna, in decision-theoretic terms. It seems to us that we might be the first to do so–the history of research given by Malinar (2007, pp. 17) does not mention any work done in this direction.

Among others, we obtain the following findings:

- A decision-theoretic reconstruction of some parts of the *Bhagavad Gītā* is possible. In particular, we can express the above quotation and others in a formal manner.
- It may seem that Árjuna, initially, argues in a purely consequentialist manner while Krishna argues in terms of svadharma (the duty of the respective persons). However, a closer reading reveals that Krishna does not shy away from consequentialists arguments.
- Krishna puts a new twist on the standard decision-theory model by pointing out that actions are not only relevant because of their consequences.
- We show how Krishna's *svadharmic* point of view can be seen as an example of the *Rational Shortlist Method* (to be introduced below).
- While Krishna's insistence on *svadharma* (duty in line with one's social standing) seems radical, less extreme versions are in use in almost all societies. We argue that Krishna's insistence on svadharma can be made fruitful for a new decision model that we like to call "svadharmic decision theory".

As befitting a *tīka*, we present our arguments in several small steps. The next section presents the basics of decision theory. We then turn to Árjuna's arguments against fighting in section 4 while Krishna's counter arguments are analyzed in section 5. In section 6, we develop svadharmic decision theory. Section 7 concludes the chapter.

2. A little bit of set theory

In order to prepare the following definition of a choice function, we need some set theory:

DEFINITION IV.1 (set and elements). *Set – any collection of "elements" that can be distinguished from each other. Set can be empty:* $\varnothing$.

DEFINITION IV.2 (set and subset). *Let M be a nonempty set. A set N is called a subset of M (denoted by $N \subseteq M$) if and only if every element from N is contained in M. We use curly brackets $\{\}$ to indicate sets.*

EXERCISE IV.1. *True?*

- $\{1,2\} = \{2,1\}$
- $\{1,2,3\} \subseteq \{1,2\}$

Definition IV.3 (cardinality). *Let M be a nonempty set. The cardinality of M is the number of elements in M and is denoted by $|M|$.*

For example, the cardinality of $\{1,2,3\}$ is $|\{1,2,3\}| = 3$.

Definition IV.4. *Let M be a nonempty set. A tuple on M is an* ***ordered*** *list of elements from M. Elements can appear several times. A tuple consisting of n entries is called n-tuple. () are used to denote tuples.*
$(a_1, ..., a_n) = (b_1, ..., b_m)$ if $n = m$ and $a_i = b_i$ for all $i = 1, ..., n$.

Exercise IV.2. *True?*

- $(1,2,3) = (2,1,3)$
- $(1,2,2) = (1,2)$

Definition IV.5 (power set). *Let M be any set. The set of all subsets of M is called the power set of M and is denoted by 2^M.*

For example, $M := \{1,2,3\}$ has the power set

$$2^M = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

Thus, the power set is a set whose elements are sets. Note that the empty set $\emptyset$ also belongs to the power set of M, indeed to the power set of any set. $M := \{1,2,3\}$ has eight elements which is equal to $2^3 = 2^{|\{1,2,3\}|}$. This is a general rule: For any set M, we have $\left|2^M\right| = 2^{|M|}$. 2 plays a special role in the definition of a power set. The reason is this–every element belongs to or does not belong to a given subset.

3. Decision theory

3.1. Relations and preference relations. For the sake of this chapter, we redefine preference relations. As in chapter II, they are denoted by $\precsim$ where $x \precsim y$ means "x is at least as good (as preferable, as virtuous, as compatible with svadharma) as y". However, we do not, in general, assume that preferences are complete.

Completeness of preferences means that the agent "knows what he wants". Of course, in real life, this is not always the case. In this chapter, we will discuss complete and incomplete preference relations. Strict preference relations are said to be complete if $x \succ y$ or $y \succ x$ holds for all $x, y \in X$, $x \neq y$. The strict preference relation $\succ$ may be complete (depending on the set X), but need not. It is not complete if there are two objects x and y in X which the agent finds equally attractive ($x \sim y$). Strict preferences obey asymmetry. This means: $x \succ y$ implies "not $y \succ x$".

It is important to note that preferences do not necessarily refer to egotistic motives. Indeed, in this chapter, we are mainly concerned with moral arguments weighed by Árjuna and Krishna. Árjuna's selfish motives never play a role in his own arguments, and only sometimes those by Krishna.

3.2. Actions, consequences, and states of the world. As in chapter III, we distinguish between

- a set of actions A,
- a set of consequences C,
- a consequence function $f : A \to C$ that attributes a consequence $c \in C$ to every action $a \in A$, and
- a preference relation $\succsim$ on C.

In this basic decision model, it does not really matter whether preferences are defined on C or on A. However, for the analysis of the *Bhagavad Gītā*, we need to distinguish between these preference definitions carefully (see subsection 3.5 below).

Sometimes, we want to consider a subset A' of the whole action set A. Let $\succ$ be an asymmetric relation on A (e.g., a strict preference relation). By

$$\max\left(A'; \succ\right) \subseteq A'$$

we then denote "best" actions from A', i.e., those actions a from A' for which no other action $b \in A'$ with $b \succ a$ exists. In particular, a^* from above is a best action from A. Abusing notation, if $\max(A'; \succ)$ contains only one element, we will sometimes consider $\max(A'; \succ)$ an element of A', rather than a subset of A'.

As in chapter III, we also use a set of states of the world W and define dominant actions in the same manner.

3.3. Choice functions and axioms. Following Manzini & Mariotti (2007, p. 1826), we introduce the concept of a (point-valued) choice function (a set-valued definition is used by Kreps 1988, p. 12). The idea of a choice function is this: Consider a set of actions A and a nonempty subset $A' \subseteq A$. Now, given A', choose exactly one element from A'.

Formally, we have

Definition IV.6 (choice function). *Let A be a set of actions with $|A| > 2$. A choice function γ on A is given by*

$$\begin{aligned} \gamma &: 2^A \to A, \text{ with} \\ \gamma\left(A'\right) &\in A' \text{ for every } A' \in 2^A. \end{aligned}$$

For example, if the strict preference relation $\succ$ on A is complete, $\gamma(A') = \max(A'; \succ)$ defines a choice function. Consider, however, a subset $A' = \{a, b\}$ with neither $a \succ b$ nor $b \succ a$. Then, we have $\max(A'; \succ) = A'$ and, hence, $\gamma(A') := \max(A'; \succ)$ does not define a choice function.

Exercise IV.3. *Consider the set*

$$A = \{(1,3), (2,2), (3,2), (3,5), (4,2)\}$$

where the first entry refers to the number of bananas and the second to the number of grapes. Let the agent pick no bundle, one bundle or several bundles according to the following rules

(1) *The agent has monotonic preferences with respect to bananas and grapes. He prefers bananas to grapes. Whenever there are more bananas than grapes in one bundle, he strictly prefers that bundle. If a bundle contains the same number of bananas in two bundles, he lets the number of grapes decide.*
(2) *The agent has monotonic preferences and likes bananas and grapes equally. In fact, he is interested in the sum of fruit, only.*

Which (if any) of these rules define a choice function γ *by* $\gamma(A') := \max(A'; \succ)$*?*

Choice functions γ may, or may not, obey the weak axiom of revealed preference (**WARP**): If action a is chosen in a situation where b is also feasible, then b cannot be chosen in a situation where both a and b are feasible. The idea is this: The fact that a (and not b) was chosen in the first situation tells us that a is preferred over b.

WARP is considered an axiom of rationality. Consider the following violation of the axiom. When confronted with the action set $A' = \{a, b, c\}$, you choose a, but from $A'' = \{a, b\}$ you pick b. That seems an odd choice indeed. It is as if you were in a pizzeria intent on ordering a cheese pizza. Then the waiter comes and apologizes: quatro formaggio is out today. Then you alter your choice: a pizza funghi, please.

In the appendix, we provide the formal definition of this axiom and two others that will become relevant later on.

3.4. The rational short-list method. Manzini & Mariotti (2007) present and axiomatize the *Rational Shortlist Method*. According to this decision procedure, agents use two (or more) rationales in a prespecified order. Let $\succ_1$ and $\succ_2$ be asymmetric relations on A. Let A_1 be the set of actions surviving application of $\succ$, i.e., $A_1 = \max(A; \succ_1)$. Then, we apply $\succ_2$ to A_1 to obtain $A_2 = \max(A_1; \succ_2)$. For example, in order to choose a car, the decision maker first rejects all cars that cost more than € 10.000. Then, among the remaining cars, he chooses the one (let us assume there is only one) with the smallest milage.

Definition IV.7 (rational shortlist method). *A choice function* γ *is a rational shortlist method (RSM), if a pair of asymmetric relations* $(\succ_1, \succ_2)$ *exists such that*

$$\gamma(A') = \max(\max(A'; \succ_1); \succ_2)$$

holds for all $A' \in 2^A$.

This definition implies that, for each subset of A, the sequential application of the two rationales leads to exactly one choice.

Manzini & Mariotti (2007) show that RSM does not, in general, obey the weak axiom of revealed preference (**WARP**). Therefore, it is somewhat lacking in rationality. However, the authors can show that RSM fufills two weaker axioms, and that every choice function that fulfills these two axioms can be expressed as an RSM. The interested reader can find these two axioms in the appendix.

3.5. Distinguishing between four kinds of preference relations. Broadly speaking, Árjuna's arguments refer to consequences and Krishna's, to actions. Therefore, we propose to distinguish between four kinds of preferences:

(1) a preference relation $\succsim_C$ on C,
(2) a preference relation $\succsim_A$ on A, and
(3) a preference relation $\succsim_{A\times C}$ on $A \times C$, where elements from $A \times C$ are action-consequence tuples $[a, c]$.

Of course, actions and consequences cannot be mixed arbitrarily. Let us assume a model without states of the world (certain consequences). Then we can derive

4.: another preference relation, $\succsim$ on A, by defining

$$a \succsim b :\Leftrightarrow [a, f(a)] \succsim_{A\times C} [b, f(b)] .$$

With respect to $\succsim_C$ (on C), Árjuna argues against fighting by pointing to the fierce killing involved. Krishna uses the same preference relation to convince Árjuna that shying away from fighting is bad for the latter's reputation as a fearless warrior. The preference relation $\succsim_A$ on A is used to bring home Krishna's insistance on svadharma. Irrespective of the consequences, doing one's duty is better than not doing it.

In general, preferences $\succsim_{A\times C}$ for both actions and consequences may be relevant. Finally, an action has to be chosen. One main topic of the *Gītā* is how to find and argue for preferences $\succsim$ on A.

4. Despondent Árjuna

4.1. The *Gītā*. Arguing for a human decision theory that deviates from the simplistic decision models, Selten (1978, pp. 147) suggests three levels of decision making: the levels of (i) routine, (ii) imagination, and (iii) reasoning. Let us associate these three states with (i) the very early Árjuna, (ii) the early Árjuna, and (iii) the late Árjuna.

(i): It may be argued that the very early Árjuna, willing to fight, is on the routine level. After all, fighting is a warrior's duty (*kṣatradharma*).

(ii): Then, after inspecting the opposing side, the early Árjuna is horrified: "Krishna, at the sight of my own kin standing here ready to fight, my limbs feel tired and my mouth has gone dry, my body is trembling and my hair is standing on end." (25.28 - 29 in Cherniak

(2008, p. 177), henceforth cited as *Gītā* 25.28 - 29). The warrior imagines the consequences of fighting. He expounds a cascade of consequences leading, in several steps, from (a) the destruction of the family clan (*kula*) over (b) the loss of *dharma* and (c) the cessation of offerings to ancestors to (d) eternal hell (*Gītā* 25.40 - 44). Here, Árjuna invokes *kuladharma*. The warrior is also aware that he may suffer from a bad conscience: "Better in this world to live on alms without killing the mighty elders; for were I to kill the elders, eager though they are for worldly gain, in this very world I would taste pleasures smeared with blood." (*Gītā* 26.5)

The implication drawn by the early Árjuna is clear: "It would be better for me if Dhrita·rashtra's sons [Árjuna's cousins, HW], armed with weapons, were to kill me in battle unresisting and unarmed!" (*Gītā* 25.46)

4.2. Decision-theoretic analysis. The very early Árjuna routinely considers fighting, only. One interpretation is this: Árjuna is a warrior whose action set is not

$$A = \{\text{fight, not fight}\}$$

but the smaller action set

$$A_{sv} = \{\text{fight}\}.$$

Here, *sv* refers to *svadharmic* where *dharma* is translated as duty/religion etc. and *sva* means "own". For the warrior Árjuna *svadharma* is *kṣatradharma* (warrior duty). In normal decision-theoretic parlance, actions from A are called feasible. Here, A is not feasible but A_{sv}, only. The restricted action set may be the result of dharma feasibility, rather than technical feasibility or financial feasibility in economic models.

The early Árjuna becomes aware of his full action set $A = \{\text{fight, not fight}\}$. He also contemplates on the possibility that fighting might result in victory or defeat. We can use action set A and the set of states of the world

$$W = \{\text{good luck, bad luck}\}$$

to formalize the interplay of actions and states of the world. It seems that Árjuna does not entertain the hope that renouncing fighting might lead to his cousins' seeking an amiable solution. Therefore, the action "not fight" is automatically (with probability 1) associated with defeat, i.e., we have

$$\begin{aligned} g(\text{fight, good luck}) &= \text{victory and family destruction,} \\ g(\text{fight, bad luck}) &= \text{defeat and family destruction,} \\ g(\text{not fight}, \cdot) &= \text{defeat without family destruction.} \end{aligned}$$

or the matrix of fig. 1.

		state of the world	
		good luck	bad luck
Arjuna	fight	victory and family destruction	defeat and family destruction
	not fight	defeat without family destruction	defeat without family destruction

FIGURE 1. The early Árjuna's payoff matrix

We argue that the early Árjuna's assessment of the consequences is given by

$$\begin{aligned} & g\,(\text{fight, victory and family destruction}) \\ \prec_C\; & g\,(\text{not fight, defeat without family destruction}) \\ \succ_C\; & g\,(\text{fight, defeat and family destruction})\,. \end{aligned}$$

Árjuna says: "And we don't even know which is preferable: to vanquish or to be vanquished." (*Gītā* 26.6). "not knowing" might be translated by indifference:

$$\begin{aligned} & g\,(\text{fight, victory and family destruction}) \\ \sim_C\; & g\,(\text{fight, defeat and family destruction}) \end{aligned}$$

Incomplete preferences (Árjuna is not able to rank these two consequences one way or another) provide an alternative plausible interpretation.

Be that as it may, for the early Árjuna, "not fight" is a dominant action. Therefore, we do not need to speculate about any probabilities attached to good or bad luck, or to victory or defeat (in case of fighting). (Formally, the expected utility of "not fight" would be higher than the expected utility of fight for every probability distribution on W (see, for example, Kreps 1988, pp. 31).)

Since the routine level (*kṣatradharma*) and the imagination level (*kuladharma*) militate for contradictory recommendations, Árjuna is despondent and does not know what to do. He turns to Krishna for help: "... my mind confused over my duty [translation of *dharma*, not *svadharma*, HW], I ask you to tell me for sure what would be best." (*Gītā* 26.7) Here, "what would be best" is clearly to be understood in terms of $\succsim$ on A.

Thus, the despondent Árjuna is torn between the routine-level decision of fighting (very early Árjuna) and the imagination-level decision of not fighting (early Árjuna). One might say that Árjuna is confronted with a hard choice (see the title of a book by Levi 1986). In hard-choice situations, a decision maker sees no obvious way to come to a conclusion. Could Árjuna not just consult his preferences $\succsim$ on A (see subsection 3.5)? No, if his preferences were complete (note our definition of preferences above), the deliberation process would be finished. However, this process is what the *Bhagavad Gītā* is about (from the decision-theoretic standpoint). Indeed, Kliemt (2009, pp. 48) and other philosophers of decision theory argue that complete "preferences are not reasons to act". In line with that sentiment, decision theorists set up, and put to experimental tests, models of reason-based choice (a survey is presented by Shafir, Simonson & Tversky 2008): "... decision makers often seek and construct reasons in order to resolve the conflict and justify their choice, to themselves and to others." Obviously, this is what happens in the *Bhagavad Gītā*. Krishna offers many reasons to Árjuna why the latter should indeed put up the fight. It is these arguments and therefore Selten's reasoning level that we now turn to.

5. Krishna's counter-arguments

5.1. The body-as-garment argument. Krishna tries to persuade Árjuna by means of many different arguments that we will look at one by one. While Krishna does not deny that many people might be killed (as they indeed will in great numbers), he tries to influence Árjuna's preferences with respect to family destruction. Krishna argues that the body is of minor importance, it is the soul that counts. He uses the word *deh-in* for (embodied) soul, i.e., the (soul who is the) possessor of the body. In Krishna's words: "Whoever thinks this soul can kill or be killed, doesn't understand. It neither kills, nor is it killed. It isn't born; it never dies ... . Just as a man casts off his worn-out clothes and puts on other new ones, so the embodied soul casts off its worn-out bodies and takes other new ones." (*Gītā* 26.19 - 22)

Even if Árjuna were not to accept his body-as-garment argument, Krishna has a second line of attack on Árjuna's preferences: "... death is certain for those who are born, and birth is certain for those who die; and so, this being inevitable, you shouldn't grieve." (*Gītā* 26.27)

Thus, Krishna's first arguments take exception not to Árjuna's function g, but to Árjuna's preferences $\succsim_C$ on C. To our mind, these arguments have a consequentialist flavor. Krishna is telling Árjuna that the consequences of many people dying are not as serious as he makes them out to be.

5.2. A dominance argument. If Krishna's arguments from the first subsection are consequentialist, this is a fortiori true for the ones he then offers. Krishna points out to Árjuna the two-fold negative personal consequences of withdrawing from battle. First of all, Árjuna would miss the chance to attain heaven: "You should attend to your own duty [*svadharma*] and stand firm, for there is nothing better for a warrior than a legitimate battle. Happy the warriors who find such a battle, Partha [son of Pritha, i.e., Kunti, HW]–an open door to heaven ..." (*Gītā* 26.31-32)

Second, Árjuna is warned against serious reputational damage: "The great warriors will think you withdrew from the battle out of fear, and though highly regarded by them before, you will be slighted. Your enemies too will say many unseemly things, disparaging your ability; and what could be more painful than that? Get up, son of Kunti [Kunti is Árjuna's mother, HW], and resolve to fight! For you will either be killed and attain heaven, or you will prevail and enjoy the earth." (*Gītā* 26.35-37)

With these two arguments, Krishna draws attention to consequences of Árjuna's unwillingness to fight, that might have gone unnoticed by Árjuna himself. In this manner, Krishna corrects Árjuna's view of the consequence function g. We can safely assume that the protagonists share the same preference assessment of these consequences. If we concentrate on them (as Krishna wants Árjuna to), "fight" becomes a dominant action (see the matrix of fig. 2).

		state of the world	
		good luck	bad luck
Arjuna	fight	prevail and enjoy the earth	be killed and attain heaven
	not fight	shameful loss of reputation	shameful loss of reputation

FIGURE 2. Árjuna's payoff matrix, as argued by Krishna

5.3. Exculpation. Krishna also presents an argument against Árjuna's bad conscience ("pleasures smeared with blood"). He exculpates the hesitating warrior from the consequences of fighting by claiming: "I am Time, the world destroyer, ripened, and here I am busy crushing the worlds. Even without you, all the warriors drawn up in the opposing ranks will cease to exist. ... I have myself long since doomed them to perish; you just be the instrument ... ". (*Gītā* 35.32-33)

Thus, Krishna tells Árjuna that he is wrong about the consequences ensuing from "not fight" (see fig. 1). Árjuna cannot prevent family destruction.

5.4. Equanimity. Krishna recommends equanimity to Árjuna by saying "Don't let the actions's fruit be your motivation" (*Gītā* 26.47) and also "He whose mind is unperturbed in times of sorrow, who has lost the craving for pleasures, and who is rid of passion, fear and anger, is called a sage of steadied thought. His wisdom is secure who is free of any affections and neither rejoices nor recoils on obtaining anything good or bad." (*Gītā* 26.56-57)

We invite the reader to go back to chapter II where we offer a few comments on equanimity from a preference perspective.

5.5. Svadharma and paradharma. While Krishna suggests equanimity with respect to $\succsim_C$, he does not do so with respect to $\succsim_A$ on A. "Actions, not fruits" seems to be his anti-consequentialist motto: "You have a right to the action alone, never to its fruits. Don't let the action's fruit be your motivation, and don't be attached to inactivity. ... the wise ones of disciplined understanding renounce the fruit produced by action and ... attain the perfect state." (*Gītā* 26.47-51). A little later, Krishna then specifies the action Árjuna is to perform: "One's own duty [*svadharma*, HW], even if done imperfectly, is better than another's [*paradharma*, HW], even if done well. The duty of others is fraught with danger; better to die while fulfilling one's own." (*Gītā* 27.35)

We propose the following decision-theoretic interpretation. Let us turn to the preference relation on $A \times C$ where $[a, c] \succsim_{A\times C} [a', c']$ means that the action-consequence tuple $[a, c]$ is weakly prefered to $[a', c']$. Thus, we now admit the possibility that both actions and consequences may be relevant.

Definition IV.8. *Assume preferences* $\succsim_{A\times C}$ *on* $A\times C$. *They are purely consequentialist, if there is a preference relation* $\succsim_C$ *on* C *such that*

$$\begin{aligned} & [a,c] \succsim_{A\times C} \left[a',c'\right] \\ \Leftrightarrow \quad & c \succsim_C c'. \end{aligned}$$

$\succsim_{A\times C}$ *are purely action-oriented if a preference relation* $\succsim_A$ *on* A *exists with*

$$\begin{aligned} & [a,c] \succsim_{A\times C} \left[a',c'\right] \\ \Leftrightarrow \quad & a \succsim_A a'. \end{aligned}$$

We remind the reader of Savage's (1972, p. 14) remark: "If two different acts had the same consequences [...], there would [...] be no point in considering them two different acts at all." In a sense, the standard decision-theoretic attitude is purely consequentialist (a more balanced comment is offered in the conclusions). In contrast, Krishna's insistence on *svadharma* can be expressed by

$$\begin{aligned} A &= A_{sv} \cup A_{pa}, \\ A_{sv} \cap A_{pa} &= \emptyset \end{aligned}$$

where *pa* refers to *paradharma* or laws for others and

$$[a_{sv}, c] \succ_{A\times C} \left[a_{pa}, c'\right]$$

whenever $a_{sv} \in A_{sv}$ and $a_{pa} \in A_{pa}$, for any c and $c' \in C$. Thus, Krishna's *svadharmic* point of view is a special instance of pure action orientation. It also implies $[a_{sv}, f(a_{sv})] \succ_{A\times C} [a_{pa}, f(a_{pa})]$ and hence, by the fourth definition given in subsection 3.5, $a_{sv} \succ a_{pa}$.

5.6. Sequentially rationalizable choice and svadharmic RSM. We now show that Krishna's *svadharmic* point of view can be seen as an example of the *Rational Shortlist Method* (RSM) due to Manzini & Mariotti (2007) (see subsection 3.4 above). In the context of the *Bhagavad Gītā*, the *svadharma* check comes first, and the consequence check second. Thus, we consider the relation $\succ_1 = \succ_{sv}$ on A given by

$$a \succ_{sv} b :\Leftrightarrow a \in A_{sv} \wedge b \in A_{pa}$$

which is asymmetric because A_{sv} and A_{pa} are disjoint. For a subset $A' \subseteq A$, $\max(A'; \succ_{sv})$ contains *svadharmic* actions, only, or *paradharmic* actions, only. Consider these three cases:

(1) A' contains *svadharmic* and *paradharmic* actions. Then, we have $\max(A'; \succ_{sv}) = A' \backslash A_{pa}$.
(2) A' contains *svadharmic* actions, only. Then, we have $\max(A'; \succ_{sv}) = A'$.
(3) A' contains *paradharmic* actions, only. Then, we have $\max(A'; \succ_{sv}) = A'$.

Thus, the *svadharma* check is effective if and only if both *svadharmic* and *paradharmic* actions are available. This is, of course, the situation the despondent Árjuna finds himself in.

Let us assume an asymmetric relation $\succ_C$ on $f(A)$ that is complete. Consider the function $\gamma_{sv} : 2^A \to A$ defined by

$$\gamma_{sv}(A') = \max\left(f\left(\max(A'; \succ_{sv})\right); \succ_C\right), A' \subseteq A.$$

It is a (well-defined) choice function by completeness of $\succ_C$. It is also an RSM because we can define $a \succ_2 b$ by $f(a) \succ_C f(b)$ and then rewrite γ_{sv} as

$$\gamma_{sv}(A') = \max\left(\max(A'; \succ_{sv}); \succ_2\right), A' \subseteq A.$$

As we have noted in subsection 3.4, RSMs do not, in general, obey **WARP**. However, we obtain the following theorem:

Theorem IV.1. *The choice function γ_{sv} fulfills **WARP**.*

The proof is relegated to the appendix.

6. Svadharmic decision theory

While Krishna's svadharmic point of view seems extreme, a less extreme version may be interesting for decision theory in general. After all, it is held by many people that specific behaviors are, or are not, "befitting somebody's station". However, in contrast to Krishna, we like to recognize the fact that appropriate behavior is a matter of degree rather than one of just two categories, *svadharma* versus *paradharma*. Therefore, we introduce a *svadharmic* distance function d on A where $d(a)$ means the distance of action a to $A_{sv} \subseteq A$. In particular, $d(a) = 0$ for every $a \in A_{sv}$. Also, very inappropriate actions a are characterized by high distance $d(a)$.

Definition IV.9. *Consider an action set A, a* svadharmic *subset $A_{sv} \subseteq A$ and a* svadharmic *distance function $d : A \to \mathbb{R}$ with $d(a) = 0$ for all $a \in A_{sv}$. Let $f : A \to C$ be a certain-consequence function and let $\succsim_C$ be a preference relation on $f(A) \subseteq C$. A relation $\succsim_{wsv}$ on A is called a weak* svadharmic *relation if it obeys the following properties*

- *(importance of* svadharma*): For every two actions a, b with $f(a) \sim_C f(b)$ and $d(a) < d(b)$, we have*

$$a \succ_{wsv} b$$

- *(importance of consequences): For every two actions a, b with $f(a) \succ_C f(b)$ and $d(a) = d(b)$, we have*

$$a \succ_{wsv} b$$

An agent encounters a hard svadharmic choice between actions a and b if

- *$d(a) > d(b)$ (b more in line with* svadharma *than a),*
- *$f(a) \succ_C f(b)$ (consequence of a preferred to consequence of b), and*
- *neither $a \succsim_{wsv} b$ nor $b \succsim_{wsv} a$*

hold.

Note that preference relations are defined as complete. Hard svadharmic choices can occur for non-complete relations, only.

In a very influential paper, Simon (1955) argues for a satisficing decision model. Actors search for better alternatives until they happen upon an action whose consequence is deemed satisfactory. Consider the following *svadharma* version of satisficing:

Definition IV.10. Svadharmic *satisficing is defined by the following procedure:*

Assume a minimum consequence $\hat{c}$. Keep searching until an action from the set

$$A_{sv} \cap \{a \in A : f(a) \succsim_C \hat{c}\}$$

is found. If such an action does not exist, choose any action from

$$A_{pa} \cap \{a \in A : f(a) \succsim_C \hat{c}\}$$

7. Conclusions

The topics raised by the *Bhagavad Gītā* have been attacked from quite diverging points of view: theological and philosophical (see the recent monograph by Malinar 2007), or psychological (see Rank 1914, Goldman 1978). This chapter explores a decision-theoretical approach. Broadly speaking, one may classify the early Árjuna's point of view as consequentialist and Krishna's standpoint as action-oriented. We develop a *svadharmic* decision theory that builds on Krishna's arguments.

We would now like to put the arguments presented so far into perspective:

- The different aspects of *dharma*
  Olivelle (2009, pp. xlv-xlix) differentiates between six meanings of *dharma*. It seems that the *Gītā*'s discussion is concerned with Dh_4: "dharma .. belonging to or within the domain of a particular category of people or a particular goal toward which it is directed" (p. xlvii).
- *Svadharmic* decision theory and consequences
  While we think that *svadharmic* decision theory is well-suited for decisions of agents in the context of status, rank, social classes and the like, we need to point out that standard decision theory is also capable of taking these aspects into account, albeit in a different manner. Whenever an action is considered as especially fitting or unfitting to a particular person, this fact (known to the agent and/or known to others) may be counted among the consequences of that action. Indeed, it is Krishna who alerts the fight-averse warrior Árjuna to the bad reputation that would result from a refusal to fight (see subsection 5.2).

  The reader may also note that we did not discuss the reasons why specific acts are judged as *svadharmic*. One could argue that beneficial consequences (grosso modo or on average) provide these reasons. Then, the contrast between consequentialism and action orientation becomes less stark. When we argue for rules or *svadharma*, consequences are important. However, when an individual decision maker has to act, he should be guided by these rules without worrying about consequences. From this viewpoint, *svadharmic* decision theory and rule consequentialism (see the collection of articles in Hooker, Mason & Miller 2000) are close cousins.
- *Svadharma* and identity
  Svadharmic decision theory is closely related to research on identity undertaken by psychologists, sociologists, and even economists. Akerlof & Kranton (2000) belongs to the third group but is clearly inspired by the two other strands.
- Simplification and bounded rationality:
  We feel that our decision-theoretic interpretation of the *Gītā* focuses on some central points. However, our analysis is not complete. For example, Krishna also argues for *svadharma* (i.e., for choosing actions from A_{sv}, only) by pointing to the simplification involved: "There is one resolute understanding here, delight of the Kurus, but the understanding of the irresolute are multifarious without limit." (*Gītā* 26.41) The interested reader may consult Rubinstein (1998, pp. 14) on simplification in the context of bounded rationality.

- Complete preferences as reasons to act:
  In subsection 4.2, we mention that complete preferences are usually not considered "reasons to act" by philosophers of decision theory. In the context of the present chapter, we note that this observation holds for preferences $\succsim$ on A (or, in particular, for $\succsim_{wsv}$ on A) but not for (sub) preferences used to educe them. Preferences $\succ_{sv}$ (see subsection 5.6) or $\succsim_C$ (in subsection 6) enter the deliberation process and can be considered "reasons to act".
- Alternative citations within the *Gītā*:
  It goes without saying that alternative citations from the *Gītā* could have been chosen. In particular, Krishna's teachings on *sattva, rajas*, and *tamas* (see Cherniak 2008, pp. 273) provide suitable examples. Broadly speaking, Krishna views the attitudes preferred by him as an instance of *sattva*, while he warns Árjuna against the *rajas* mode.
- Mahabhárata's fifth book
  In this article, we focus on the *Bhagavad Gītā* which belongs to the sixth book of the Mahabhárata. However, Malinar (2007, pp. 35) rightly stresses that the discussion between Árjuna and Krishna is foreshadowed by somewhat similar arguments in the fifth book (see, for example Garbutt 2008). Yudhishthira's doubts and arguments focus on *kuladharma* and resemble those of the early Árjuna while Krishna himself, Kuntī (Yudhishthira's and Árjuna's mother), Vidulā (who is a woman from the *kṣatriya varna*/caste and written "Vidurā" by Malinar) and even Duryodhana (the eldest of the Pandavas' cousins) advocate the *kṣatradharma* and *svadharma* point of view.
- Mahabhárata's twelfth book
  After the war, Yudhiṣṭhira condemns the war and its consequences. Interestingly, *Cārvāka* (*Cārvāka* philosophy is often characterized as atheistic, non-Vedic, materialist, and hedonist) makes his appearance (see Heera 2011, pp.19). He does not talk about pleasure, but seems to side with the early Árjuna and the current Yudhiṣṭhira. *Cārvāka* blames Yudhiṣṭhira for the Kurukṣetra battle: "What have you gained by destroying your own people and murdering your own elders?" Finally, *Cārvāka* is considered a demon in disguise and burned to ashes.
- Sen's advocacy of the early Árjuna's position
  Economists have not contributed a lot to any discussion about the *Gītā*. A noteworthy exception is the Indian Nobel price winner of 1998, Amartya Sen, who published a paper in the Journal of Philosophy on "Consequential Evaluation and Practical Reason". In that paper, Sen (2000, p. 482) takes Árjuna's side and argues that

> "one must take responsibility for the consequences of one's actions and choices, and that this responsibility cannot be obliterated by any pointer to a consequence-independent duty or obligation."

8. Appendix: axioms WARP, WEAK WARP, and EXPANSION

Choice functions γ may, or may not, obey the following axioms (see Manzini & Mariotti 2007, p. 1828).

WARP (weak axiom of revealed preference)**:** If action $a \in A$ is chosen for A' $(a = \gamma(A'))$ where $a, b \in A'$ holds, then $a \in A''$ implies $b \neq \gamma(A'')$.

WEAK WARP: If action $a \in A$ is chosen for A' $(a = \gamma(A'))$ where $a, b \in A'$ holds and also $b_1, ..., b_l, ..., b_k \in A'$, then $a \in A''$ and $b_1, ..., b_l \in A''$ imply $b \neq \gamma(A'')$.

WEAK WARP is weaker than **WARP** because we need the additional existence of actions b_1 through b_k, some of which are still present in A'', to conclude $b \neq \gamma(A'')$.

EXPANSION is the third axiom mentioned by the authors. It says that an action chosen for both action sets A' and A'' should also be chosen if all actions for A' or A'' are feasible:

EXPANSION: If action $a \in A$ is chosen for A' $(a = \gamma(A'))$ and for A'' $(a = \gamma(A''))$, it is also chosen for $A' \cup A''$ $(a = \gamma(A' \cup A''))$.

EXPANSION is also weaker than **WARP.**

9. Appendix: theorem IV.1

For a proof of the theorem, we assume two actions a and b where a is chosen at A' while $b \in A'$ and $a \in A''$ hold. Then, it cannot be the case that $b \in A_{sv}$ and $a \in A_{pa}$ (because then a would have been eliminated in the first round at A'). Three possibilities remain:

(1) $b \in A_{sv}$ and $a \in A_{sv}$. Then, both a and b survive the first round and $f(a) \succ_C f(b)$. In this case, $a \in A''$ cannot lose out against b in the second round.

(2) $b \in A_{pa}$ and $a \in A_{pa}$. Then, there is no other action c in A' with $c \in A_{sv}$ (otherwise, both a and b would have been eliminated). Therefore, we have $f(a) \succ_C f(b)$ and pursue as under 1.

(3) $b \in A_{pa}$ and $a \in A_{sv}$. Then, b is eliminated in the first round under A' as well as under A'' (if b belongs to A'').

This concludes the proof.

Accidentally, since both **WEAK WARP** and **EXPANSION** are weaker than **WARP**, γ_{sv} also fulfills these two axioms.

10. Solutions

Exercise IV.1

Yes. $\{1, 2\}$ and $\{2, 1\}$ are the same sets because they contain the same elements.

No. $\{1,2,3\}$ is not a subset of $\{1,2\}$ because one element (namely 3) is contained in $\{1,2,3\}$, but not in $\{1,2\}$.

Exercise IV.2

No. $(1,2,3)$ does not equal $(2,1,3)$ because the first entry is "1" in the first tuple and "2" in the second tuple.

No. $(1,2,2)$ and $(1,2)$ are not the same tuple because the first has three entries and the second two, only.

Exercise IV.3

According to the first rule (the preferences are called lexigraphic), the agent's preferences are as follows:

$$(1,3) \prec (2,2) \prec (3,2) \prec (3,5) \prec (4,2).$$

Thus, for every subset A' of A, the agent chooses the best bundle according to this strict (!) preference order.

The second rule implies that the two goods are perfect substitutes. For $A' := \{(1,3),(2,2)\}$, we have $\max(A';\succ) = A'$ and, hence, $\gamma(A') := \max(A';\succ)$ is not a choice function.

CHAPTER V

Monopoly theory and Kautilya's market tax

1. Introduction

[1]2000 years ago (give or take two or three hundred years), Kauṭilya wrote a manual on "wise kingship", the Arthaśāstra. Among other topics, this book deals with taxation, diplomacy, warfare, and the management of spies (see the survey by Boesche 2002). We like to concentrate on taxation and point to Ghoshal (1929) who deals with the history of the Hindu revenue system and Gopal (1935) who focuses on Mauryan public finance and hence on the Arthaśāstra.

In particular, we are interested in a peculiar market tax for foreign traders:

> **Kauṭilya on a market tax:** The Superintendent of Customs should set up the customs house along with the flag facing the east or the north near the main gate ... The traders should announce the quantity and the price of a commodity that has reached the foot of the flag: "Who will buy this commodity at this price for this quantity?" After it has been proclaimed aloud three times, he should give it to the bidders. If there is competition among buyers, the increase in price along with the customs duty goes to the treasury. (Olivelle (2013, p. 148; verses 1, 7-9))[2]

Olivelle (2013, p. 555) argues that Kauṭilya has an auction in mind. He interprets "increase in price" as follows: "This must refer to the increase beyond the asking price that was initially announced. Such an increase caused by the bidding process appears to go to the state rather than to the trader." The same interpretation is held by Rangarajan (1992, p. 239): "... He shall call out for bids three times and sell to anyone who is willing to buy at the price demanded. If there is competition among buyers and a higher price is realised, the difference between the call price and the sale price along with the duty thereon shall go to the Treasury."

An important point concerns the question whether Kauṭilya had an ascending or a descending auction in mind. (Auction theory is presented by McAfee & McMillan (1987).) In ascending auctions (also called English

[1]I like to thank Frank Hüttner and Kerstin Szwedek for insightful comments. Thanks are also due to Werner Knobl.

[2]Alternatively, one may consult Kangle (1972, pp. 141; verses 1, 7-9), Shamasastry (2005, pp. 217; verses 1, 7), or Rangarajan (1992, p. 340) and Rangarajan (1992, p. 239).

auctions), the auctioneer raises the price starting with some minimum price. The last bidder still upholding his wish to buy, gets the object. In a descending auction (Dutch auction), the auctioneer lowers the price starting with some maximum price. As soon as one bidder is prepared to pay, he obtains the object. Of course, "the increase in price" clearly points to the ascending auction. A second reason will be given below.

Since some of the goods were exempt from duty (Olivelle (2013, pp. 148; verse 18)), it is not obvious whether Kauṭilya proposes the market tax for dutiable and non-dutiable goods alike. Be that as it may, we deal with the market tax, exclusively. It is, of course, debatable whether "market tax" is a suitable term for Kauṭilya's tax. Obviously, Kauṭilya has in mind an indirect tax, i.e., a tax on transactions (like a value-added tax), in contrast to a direct tax which would affect income or property (see, for example, Schenk & Oldman 2001, pp. 12).

For simplicity, we assume that one unit of a good is to be imported and sold. To fix ideas, let us denote the call price by V (the value declared by the trader) and the sale price by p. Also, the trader's cost of buying, or producing, this good is denoted by C.

We need to distinguish two cases:

- First, the buyers' competition for the good drives up the highest bid p above V. Then, the price to be paid by the winning bidder is p, the tax authorities collect $p - V$ and the trader's revenue is

$$p - (p - V) = V,$$

  and, taking the cost into account also, his profit amounts to

$$V - C,$$

  possibly minus duty and/or transportation cost.
- Second, the highest bid obtained is below V. In that case, taxes and revenue are zero, and the trader needs to turn to another market place or come back another time.

A very honest trader might try $V := C$. His profit would be zero (or even negative) and the market tax $p - C$. This difference is known as the producer's rent. Therefore, Sihag (2009, p. 62) presents the market tax as evidence that the Arthaśāstra's author already knew about this concept. (Note, however, that p_w is defined as "cost plus some reasonable profit" on p. 60.)

We like to think that Kauṭilya is aware of the possibility that the trader's call price V may differ from the cost C. Indeed, the market tax presents the foreign trader with an optimization problem. On one hand, he would like to choose a relatively high valuation V in order to evade the market tax. On the other hand, a high valuation carries the risk of not selling the good and duty and transportation cost incurring once again.

This trade-off would not be present if the trader were allowed a descending auction. He could then quote a very high valuation and find out the highest bidder by successively lowering the price. In that case, there would be no danger of not finding a buyer and the market tax $p - V$ would be negative!

Before we delve into the market tax, we prepare the ground by presenting some simpler monopoly theory. Section 2 looks at a monopoly firm that fixes the price of its good, while section 3 does the same for a quantity-setting monopolist. In these sections, we assume that a single firm is a producer and does not fear entry by other firms. Governmental entry restrictions may be responsible for this state of affairs.

The trade-off driving the market tax model is presented in section 4. We then discuss additional provisions made by Kauṭilya in section 5. Finally, concluding remarks (on translational problems, on the lost-manuscript puzzle) are offered in section 6.

2. Monopoly: Pricing policy

2.1. The linear model. Assume the demand function X given by

$$X(p) = d - ep,$$

where d and e are positive constants and p obeys $p \leq \frac{d}{e}$.The reader is invited to go back to chapter II (exercise II.4, p. 14) where he can find the definitions for satiation quantity and prohibitive price.

EXERCISE V.1. *Find the satiation quantity and the prohibitive price of the above demand curve.*

DEFINITION V.1 (a monopolist's profit). *A monopoly's profit in terms of price p is given by*

$$\begin{aligned} \underbrace{\Pi(p)}_{profit} \quad : &= \underbrace{R(p)}_{revenue} - \underbrace{C(p)}_{cost} \\ &= pX(p) - C[X(p)] \end{aligned} \qquad \text{(V.1)}$$

where X is the demand function. In our linear model, we obtain

$$\Pi(p) = p(d - ep) - c(d - ep), p \leq \frac{d}{e},$$

where d, e and c are positive parameters. The profit-maximizing price (also called monopoly price) is denoted by p^M.

Note that cost is a function of the quantity produced, but that the quantity itself is determined by the price. In fig. 2, you see that the cost curve is downward-sloping (with the price on the abscissa!). c can be addressed as both marginal and average cost.

EXERCISE V.2. *What is the interpretation of $p^?$ in fig. 1?*

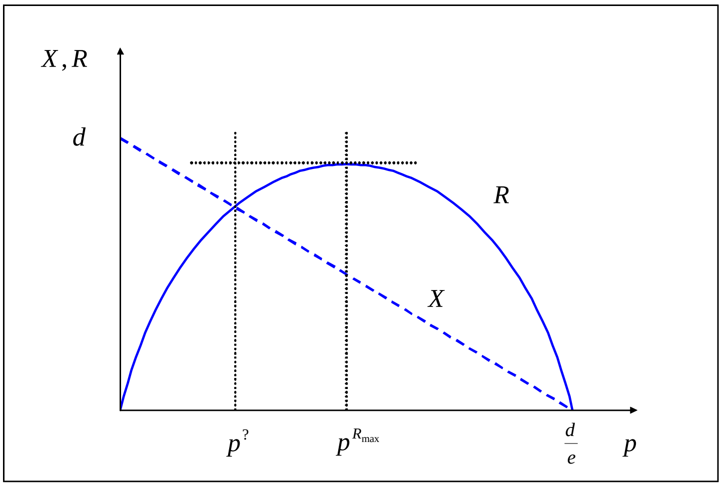

FIGURE 1. Find the economic meaning of the question mark!

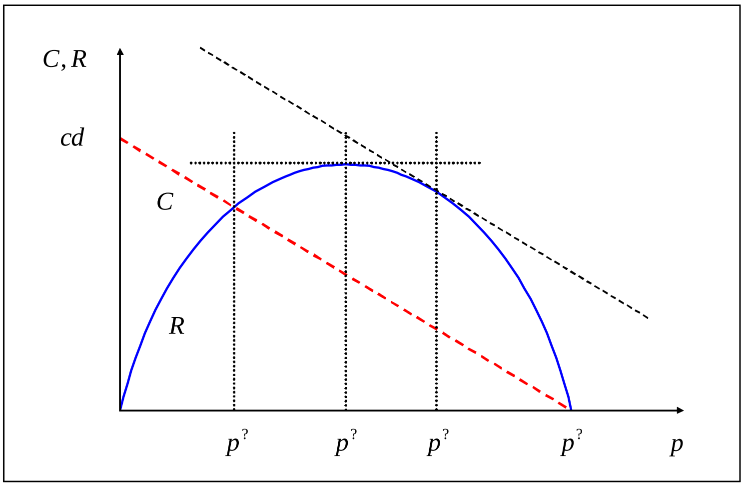

FIGURE 2. Same question, different answer

In order to make meaningful comparisons, units of measurement have to be the same. Prices are measured in $\frac{\text{monetary units}}{\text{quantity units}}$ while revenue (price times quantity!) is measured in

$$\begin{aligned} & \frac{\text{monetary units}}{\text{quantity units}} \cdot \text{quantity units} \\ = & \text{ monetary units.} \end{aligned}$$

EXERCISE V.3. *And how about the interpretation of the $p^?$ in fig. 2?*

2.2. Marginal revenue. Differentiating V.1 leads to marginal revenue with respect to price and to marginal cost with respect to price. The former is given by

$$\frac{dR(p)}{dp} = \frac{d\left[pX(p)\right]}{dp} = X(p) + p\frac{dX(p)}{dp}$$

and consists of two summands:

- A price increase by one Euro per unit of quantity increases revenue by X; for every unit sold the firm obtains an extra Euro.
- A price increase by one Euro per unit of quantity changes demand by $\frac{dX}{dp}$ (which is negative!) and hence revenue by $p\frac{dX}{dp}$.

Marginal cost with respect to price $\frac{dC}{dp}$ is related to marginal cost (with respect to quantity), $\frac{dC}{dX}$. Indeed, we have

$$\frac{dC}{dp} = \underbrace{\frac{dC}{dX}}_{>0} \underbrace{\frac{dX}{dp}}_{<0} < 0.$$

2.3. Profit maximization. The first-order condition for profit maximization is

$$\frac{dR}{dp} \stackrel{!}{=} \frac{dC}{dp}.$$

In our linear case, we find the profit-maximizing price (also called monopoly price)

$$p^M = \frac{d + ce}{2e} = \frac{1}{2}\left(\frac{d}{e} + c\right)$$

which is the mean of the prohibitive price and the marginal cost.

EXERCISE V.4. *Confirm the formula for the above monopoly price. Which price maximizes revenue? How does the monopoly price change if average cost c change?*

3. Monopoly: quantity policy

3.1. The linear model. The quantity setting monopoly presupposes an inverse demand function. Again, we work with a linear specification.

EXERCISE V.5. *Assume the inverse linear demand function p given by $p(X) = a - bX$, with positive constants a and b. Determine*

(1) *the slope of the inverse linear demand function,*
(2) *the slope of its marginal-revenue curve,*
(3) *satiation quantity and*
(4) *prohibitive price.*

DEFINITION V.2. *A monopoly's profit in terms of quantity $X \geq 0$ is given by*

$$\begin{aligned} \underbrace{\Pi(X)}_{\text{profit}} \quad : \quad &= \underbrace{R(X)}_{\text{revenue}} - \underbrace{C(X)}_{\text{cost}} \\ &= p(X)X - C(X) \end{aligned}$$

where p is the inverse demand function. In our linear model, we obtain

$$\Pi(X) = (a - bX)X - cX, X \leq \frac{a}{b},$$

where a, b and c are positive parameters. The profit-maximizing quantity X^M (also called monopoly quantity) is denoted by X^M.

3.2. Marginal revenue. In our linear model, marginal cost is simply c. Marginal revenue is more interesting. It is given by

$$MR(X) = p(X) + X\frac{dp(X)}{dX}$$

and consists of two summands:

- If the monopolist increases his quantity by one unit, he obtains the current price for that last unit sold.
- The bad news is that a quantity increase decreases the price by $\frac{dp}{dX}$. Without price discrimination, this price decrease applies to all units sold. Thus, in case of a negatively sloped inverse demand curve, revenue is changed by $X\frac{dp}{dX} \leq 0$.

3.3. Monopoly profit. The profit at some quantity $\bar{X}$ is given by

$$\begin{aligned}
\Pi\left(\bar{X}\right) &= p(\bar{X})\bar{X} - C\left(\bar{X}\right) \\
&= \left[p(\bar{X}) - AC\left(\bar{X}\right)\right]\bar{X} \text{ (average definition)} \\
&= \int_0^{\bar{X}} \left[MR(X) - MC(X)\right] dX \text{ (marginal definition)}
\end{aligned}$$

Graphically, this profit can be reflected in two different manners:

- Average viewpoint: For $\bar{X} > 0$, the monopoly profit is equal to average profit $\left(p(\bar{X}) - AC\left(\bar{X}\right)\right)$ times quantity $\bar{X}$. In fig. 3, the corresponding area is the rectangle EGHF.
- Marginal viewpoint: We add the marginal profit for the first, the second etc. units. Algebraically, we have the above integral, graphically, we address the area between marginal-revenue curve and marginal-cost curve, DCBA.

3.4. Profit maximization. The first-order condition for profit maximization is

$$MC \stackrel{!}{=} MR.$$

EXERCISE V.6. *Find the profit-maximizing quantity X^M for the inverse demand curve $p(X) = 24 - X$ and constant unit cost $c = 2$!*

The profit-maximizing rule "marginal revenue equals marginal cost" determines X^M (see fig. 4 for the linear case). The consumers have to pay the price $p^M = p\left(X^M\right)$. $M = \left(X^M, p^M\right)$ is sometimes denoted as Cournot point. Antoine Augustin Cournot (1801-1877) was a French philosopher, mathematician and economist. He is rightly famous for his 1838 treatise "Recherches sur les principes mathématiques de la théorie des richesses". There, in chapter 5, Cournot deals with the main elements of monopoly theory and chapter 7 contains oligopoly theory.

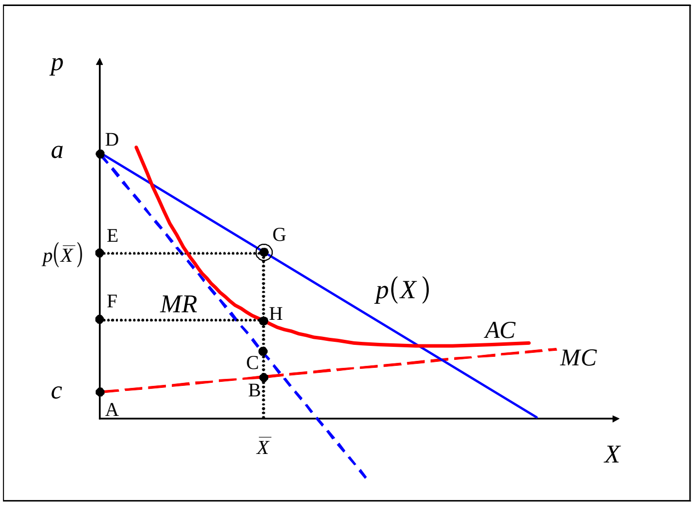

FIGURE 3. Average versus marginal profit

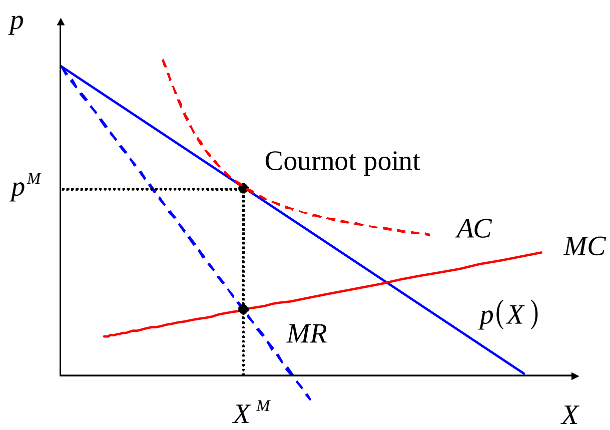

FIGURE 4. The Cournot monopoly

Assuming $c \leq a$, we obtain the optimal quantity

$$X^M = X^M(c, a, b) = \frac{1}{2}\frac{(a-c)}{b} \tag{V.2}$$

In our linear case, it is easy to do some comparative statics. For that purpose, we write down the equilibrium variables X^M, p^M and Π^M together with the partial derivatives with respect to a, b and c :

$$\begin{aligned}
X^M(a,b,c) &= \tfrac{1}{2}\tfrac{(a-c)}{b}, && \text{where } \tfrac{\partial X^M}{\partial c} < 0;\ \tfrac{\partial X^M}{\partial a} > 0;\ \tfrac{\partial X^M}{\partial b} < 0, \\
p^M(a,b,c) &= \tfrac{1}{2}(a+c), && \text{where } \tfrac{\partial p^M}{\partial c} > 0;\ \tfrac{\partial p^M}{\partial a} > 0;\ \tfrac{\partial p^M}{\partial b} = 0, \\
\Pi^M(a,b,c) &= \tfrac{1}{4}\tfrac{(a-c)^2}{b}, && \text{where } \tfrac{\partial \Pi^M}{\partial c} < 0;\ \tfrac{\partial \Pi^M}{\partial a} > 0;\ \tfrac{\partial \Pi^M}{\partial b} < 0.
\end{aligned} \tag{V.3}$$

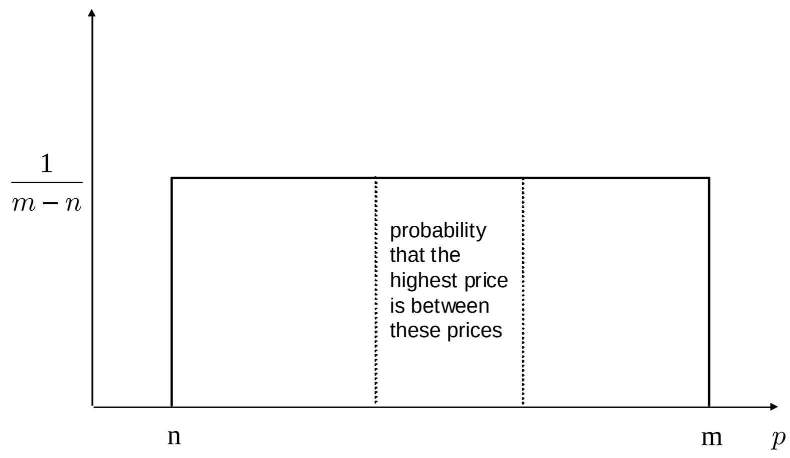

FIGURE 5. Density for the highest bids

4. The market tax

Let us assume that the foreign trader has some probability assessment of the highest bid he can obtain for his good. In particular, the highest bid p has the density distribution f given by

$$f(p) = \begin{cases} \frac{1}{m-n}, & p \in [n,m] \\ 0, & p \notin [n,m] \end{cases}$$

where n is the lowest possible value for the highest bid and $m > n$ the highest possible value. Between n and m the density is constant. See also fig. 5.

The trader's decision problem is to choose the valuation (or call price) V. It depends on

- C, the cost of producing one sole unit of the good,
- F, the cost of market entry (duty and/or transportation cost),
- δ, the discount factor which reflects the trader's time preference, with $0 < \delta < 1$,
- $T = p - V$, the market tax, where p is distributed as described above.

We work with two additional assumptions. First, we focus on

$$\underbrace{n}_{\text{lowest possible bid}} \leq \underbrace{C}_{\text{cost of production}} \leq \underbrace{m}_{\text{highest possible bid}}. \quad \text{(V.4)}$$

If we had $C > m$, the trader could never find a buyer prepared to pay at least C. The case of $C < n$ is excluded for reasons of simplicity. The second assumption will be dealt with later on.

Now, the derivation of the foreign trader's expected profit is based on the assumption that he chooses a constant valuation V in the current period and in every future period. This is a reasonable assumption if the model's parameters (such as n, m, and F) do not change. We can safely assume $n \leq V \leq m$, because any $V < n$ leads to a higher tax than $V = n$ while any $V > m$ has the same effect as $V = m$ (no sale).

Now, the foreign trader's expected profit at one specific market place is given by

$$\begin{aligned}\Pi &= -\underbrace{F}_{\substack{\text{cost of}\\\text{market entry}}} + \underbrace{\delta\Pi \underbrace{\int_n^V f(p)\,dp}_{\substack{\text{probability}\\\text{for bid below } V}}}_{\substack{\text{expected discounted profit}\\\text{in next period}}} + \underbrace{\int_V^m \underbrace{(p - C - T)}_{\substack{\text{unit profit after tax}\\\text{for bids above } V}} f(p)\,dp}_{\substack{\text{expected profit}\\\text{in current period}}} \\ &= -F + \delta\Pi \int_n^V f(p)\,dp + \int_V^m (V - C) f(p)\,dp.\end{aligned}$$

According to this formula, profit Π has three summands:

- First summand: The foreign trader has to pay the market entry cost F.
- Second summand: He encounters a highest bid below his valuation V with probability $\int_n^V f(p)\,dp$. Then he has to wait one period before offering the good again. Assuming the same valuation in the next period, he then obtains the profit Π which has to be discounted by $\delta < 1$.
- Third summand: He immediately manages to obtain a bid above his valuation with probability $\int_V^m f(p)\,dp$. The expression reports the expected profit in the current period.

Note that Π occurs on both sides of the equality sign. Solving for Π yields

$$\Pi(V) = \frac{-F + \int_V^m (V - C) f(p)\,dp}{1 - \delta \int_n^V f(p)\,dp} = \frac{(V - C)\frac{m-V}{m-n} - F}{1 - \delta \frac{V-n}{m-n}} \qquad \text{(V.5)}$$

where $\frac{m-V}{m-n}$ $\left(\frac{V-n}{m-n}\right)$ is the probability for selling the object in the current period (for not selling in the current period).

We now turn to a technical observation:

LEMMA V.1. $\Pi(V)$ *is non-negative for suitably chosen valuations* V *if and only if*

$$F \leq \frac{(m-C)^2}{4(m-n)} \tag{V.6}$$

holds. Also, this inequality implies

$$\delta F \leq m - n + (1-\delta)(C-n). \tag{V.7}$$

Its proof can be found in the appendix as can the proof of the following proposition:

PROPOSITION V.1. *Assume inequality V.6. The foreign trader maximizes his expected profit by choosing*

$$V^* = n + \frac{m-n-\sqrt{R}}{\delta}$$

where

$$R := (m-n)\left[(1-\delta)(m-n-\delta(C-n)) + \delta^2 F\right] > 0.$$

We have $\frac{\partial V^*}{\partial \delta} > 0$.

If the unsuccessful trader does not try again in the kingdom at hand, the expected tax payment is

$$\begin{aligned} \int_V^m (p-V) f(p)\, dp &= \frac{1}{2}\frac{(m-V^*)^2}{m-n} \\ &= \frac{1}{2}\frac{\left(\sqrt{R}-(1-\delta)(m-n)\right)^2}{(m-n)\,\delta^2} \end{aligned}$$

We comment on some comparative-statics results.

- $\frac{\partial V^*}{\partial C} > 0$: The seller's valuation has to be a positive function of the cost of producing (or buying) the good.
- Assume $\delta = 1$ and $F = 0$. In that extreme case, waiting for the next period has no cost. Indeed, we find

  $$R = 0 \text{ and } V^* = m$$

  and the trader can risk to ask for the highest possible bid.

Related to the second bullet item, we find

- $\frac{\partial V^*}{\partial F} < 0$: If the trader faces high entry costs, he is less prepared to risk the no-sales case.
- $\frac{\partial V^*}{\partial \delta} \geq 0$: If the discount factor is low, future profits are discounted heavily and the trader prefers not to risk waiting for the next period.

The last inequality is shown in the appendix. (The other comparative-statics results are straightforward.)

5. Revisiting the market tax in the *Arthaśāstra*

So far, we have made our life easy by focusing on the market tax. The main result is this: The trader has to find a call price or value V that is

- sufficiently high (so that the market tax can be reduced as much as possible)
- sufficiently low (so that at least one bidder can be found and the cost of market entry F is not to be paid again)

We now argue that Kauṭilya was aware of these conflicting interests and also of the unwillingness of traders to pay duties. His further provisions are to ensure that traders quote "correct" values, according to "weight, measure, or number" (Olivelle (2013, p. 148; verse 15)):

Kauṭilya's additional provisions: When a man, fearing customs duty, declares a lower quantity or price, the king shall confiscate the amount in excess of that; or he should pay eight times the customs duty. ...

Or, when a man, fearing competing buyers, increases the price beyond the normal price of a commodity, the king shall confiscate the increase in price or assess twice the customs duty ... . (Olivelle (2013, p. 148; verses 10, 11, 13))[3]

The first provision deals with dutiable goods. Apparently, these duties also depended on the valuation. This gave the trader (if he were to sell dutiable goods) an additional incentive to state a low V. According to Kauṭilya, a fine of "eight times the customs duty" should give him a proper disincentive.

The second quotation above ("or ... fearing competing buyers") has caused some puzzlement. The Sanskrit reads "pratikretṛ bhayād vā". Indeed, one may follow Olivelle (2013, p. 555) and ask why additional buyers should constitute a reason for fear. Therefore, Olivelle suggests that pratikretṛ stands for competing traders who "may sell their goods at a higher price than he". This interpretation is not impossible–a trader may be jealous of other traders who are more successful in obtaining a high price. However, the current author does not find this interpretation very plausible: If a trader thinks that other traders will compete with him, he will typically (for reasons of expected-profit maximization) reduce his price. In contrast, it is the absence of competitors that allows a trader to increase his price.

We suggest the following interpretation of "fear". If the trader expects many eager bidders, it would be in his interest to drive up V. Inversely, if he has chosen a low V, he may indeed fear many bidders that would make

[3]Similarly, Kangle (1972, pp. 142; verses 10, 11, 13), Shamasastry (2005, pp. 217; verses 8, 10), or Rangarajan (1992, pp. 341) (where the important link between the market tax and the additional provisions is made obscure by the translator's restructuring of the material).

him regret his decision. Consider this analogy: You take an umbrella with you, but "fear" it might not rain after all (in that case you would have taken the umbrella without good cause).

- Thus, "fearing competing buyers" is short for "fearing the regret of having taken the wrong decision in case of competing buyers".
- Similarly, "fearing good weather" may be shorthand for "fearing the regret of having carried an umbrella which was unnecessary because it happened not to rain".

(From the rhetorical point of view, we seem to have an example of "soloecismus per detractionem" (see the handbook by Lausberg, Orton & Anderson 1998, p. 235).)

Of course, Olivelle also has his doubts (after all, the translation above is his). He finally resorts to an interpretation similar to our one. However, we insist that the auction envisioned by Kauṭilya can only be an ascending one, upwards from V, not a descending one.

Finally, a comment on Kangle (1972, p. 142; fn 13) is called for. Kangle wonders about the difference between "competition among buyers" (quote in the introduction, Kangle uses "purchasers" instead of "buyers") and "fearing competing buyers" (quote in this section, Kangle: "fear of a rival purchaser").[4] In terms of our model, the first quote is about price increases above a given V. In contrast, the second quote is concerned with incentives to increase V above the correct level.

6. Conclusions

Kauṭilya's market tax is very unusual in basing the tax payments on a price or value declared by the seller. Our model shows that the tax "works" in the sense of giving the trader an incentive to quote a valuation V^* that leads to positive expected tax payments. It is not to be overlooked that the practical implementation of this tax should prove difficult. After all, the seller and the final buyer have a very clear motivation to report a lower bid to the tax authorities, for some side-payment from the buyer to the seller. While this problem holds for many taxes, it is very serious for the market tax because the trader's profit does not depend on p as long as p is at least as high as V^*. However, if the sale is to be effected near the "foot of the flag" (see the quote from the introduction), supervision of both seller and buyer may not be too difficult. Presumably, this tax has been applied a few times a day, maybe, but certainly not a thousand times per day.

Otherwise, for many traders, the supervision problems should be very serious. In that case, Kauṭilya may have hoped to deal with this problem by employing spies. In any case, collecting taxes would have been expensive for the government.

[4]The first part of the composite "pratikretṛ bhayād" can be understood as a singular or a plural. To our mind, Olivelle's plural is the better choice.

An ongoing debate on Kauṭilya's Arthaśāstra dwells on the question of whether it should be seen as a historical document (telling us a lot about actual diplomacy, spying and taxing etc.) or, rather, as a teaching manual on statecraft (see the discussion by Rangarajan 1992, pp. 31). Of course, both aspects may be relevant for different subject matters or, sometimes, even one and the same topic. While Olivelle (2013, p. 39) argues that the Arthaśāstra may be quite accurate with respect to the material culture, our analysis of Kauṭilya's market tax argues against the historical view. Indeed, the current author conjectures that Kauṭilya's market tax (if ever applied) was unique in human history. In any case, a suchlike tax has not been reported by tax historians Webber & Wildavsky (1986) who do not, contrary to the title of their book, restrict attention to the "western world".

The previous section may make the reader feel sorry for traders who had a hard time finding the correct valuation V. They will be punished if V is too low and also, if it is too high. One is reminded of modern-day competition policy: According to William Landes (see Kitch 1983, p. 193), Ronald Coase, a famous member of the Chicago school said "he had gotten tired of anti-trust because when the prices went up the judges said it was monopoly, when the prices went down, they said it was predatory pricing, and when they stayed the same, they said it was tacit collusion."

Indeed, Kauṭilya may have had the idea to supervise nearly all traders in the same way that competition theory sometimes suggests to deal with so-called natural monopolies. In network markets like the provision of electricity, gas, or water, many firms are unlikely to co-exist. Therefore, it is often thought that (i) competition will be defective in these markets and (ii) that the government should step in to ensure "efficiency"(see Braeutigam 1989). (It is no foregone conclusion that (ii) follows from (i).)

Kauṭilya, it seems, had no belief in market mechanisms and wanted traders of all goods (not just natural monopolies) to quote the correct or normal value. However, the requirement that prices are to be set according to "weight, measure, or number" (Olivelle (2013, p. 148; verse 15)) is not operational. Presumably, implementation rules were necessary to tell the customs officers how to proceed in practice. And those rules were written not just for the market tax but for practically all areas of the economy.

One may be tempted to link this assessment with the mystery of the lost Artha-śāstra. While the existence of the text was well-known through the centuries, a manuscript turned up "sometime before 1905", after a gap of about 1000 years (Olivelle (2013, pp. 1, 51-53)). We have to imagine that the Artha-śāstra and the accompanying implementation rules (many volumes) have been in use in those administrations that favored Kauṭilya's approach to economics. Private incentives were curbed, an armee of spies had to be paid for: an economy fashioned along Kauṭilya's ideas resembled the German Democratic Republic. (Without this bold comparison, Scharfe

(1968, pp. 144-289) discusses the civil servants, the spies and the economy in a Kauṭilya state.) Perhaps, the economies built on Kauṭilya's ideas have shared the GDR's fate with one exception. The demise was not peaceful but accelerated by exterior enemies, eager to exploit the economic weakness, quite in line with Kauṭilya's ideas on warfare. It is our guess that Kauṭilya's manuscripts perished along with the governments that used them.

As a final after-thought, while the market tax may not have been implemented (or may not have been a good idea) in Kauṭilya's times (in the case of many transactions, at least), auctioneering houses like Sotheby's or electronic trading platforms like ebay do not encounter these supervision problems since p is readily available for these market makers. Therefore, Kauṭilya's market tax (without punishment for too low or too high valuations) may still await realization in modern times.

7. Appendix

Proof of lemma V.1:

$\Pi(V) = \frac{(V-C)\frac{m-V}{m-n}-F}{1-\delta\frac{V-n}{m-n}}$ is positive if the nominator is positive. We now set the parabola $(V-C)\frac{m-V}{m-n}-F$, which opens downward (!), equal to zero and obtain two solutions,

$$\begin{aligned} V' &= \frac{C+m}{2} - \frac{1}{2}\sqrt{(m-C)^2 - 4F(m-n)} \text{ and} \\ V'' &= \frac{C+m}{2} + \frac{1}{2}\sqrt{(m-C)^2 - 4F(m-n)}. \end{aligned}$$

Therefore, we have $\Pi(V) \geq 0$ if and only if

- V' and V'' are real:
$$(m-C)^2 - 4F(m-n) \geq 0$$
and
- V is between V' and V'':
$$V' \leq V \leq V''.$$

We now obtain the first claim by solving $(m-C)^2 - 4F(m-n) \geq 0$ for F.

The second claim follows from

$$\begin{aligned} &\frac{(m-C)^2}{4(m-n)} - \frac{m-n+(1-\delta)(C-n)}{\delta} \\ =\ &\frac{1}{4}([C-n]+[m-n])\frac{-4(m-n)+\delta([C-n]+[m-n])}{\delta(m-n)} \end{aligned}$$

and

$$\delta < 1 < 2 = \frac{4(m-n)}{[m-n]+[m-n]} < \frac{4(m-n)}{[C-n]+[m-n]}$$

Proof of proposition V.1:

Deriving eq. V.5 with respect to V and setting equal to zero yields

$$\begin{aligned} 0 &= \frac{\partial\left(\frac{(V-C)\frac{m-V}{m-n}-F}{1-\delta\frac{V-n}{m-n}}\right)}{\partial V} \\ &= \frac{\delta\left(V^2 - 2Vn + mn\right) + (m-n)(C - 2V + m - C\delta - F\delta)}{(m-n-\delta(V-n))^2} \end{aligned}$$

with the two solutions

$$\begin{aligned} V_1 &= \frac{1}{\delta}\left(m - n + n\delta - \sqrt{R}\right), \\ &= \frac{1}{\delta}\left(m - n + n\delta - \sqrt{(m-n)\left((1-\delta)(m-n-\delta(C-n)) + \delta^2 F\right)}\right) \\ V_2 &= \frac{1}{\delta}\left(m - n + n\delta + \sqrt{R}\right), \end{aligned}$$

where

$$R := (m-n)\left[(1-\delta)(m-n-\delta(C-n)) + \delta^2 F\right] > 0$$

by $n < m, C < m$, $F > 0$ and $0 < \delta < 1$.

It is possible to show the following five claims:

(1) $\left.\frac{\partial^2 \Pi}{(\partial V)^2}\right|_{V_1} < 0$,
(2) $V_1 \geq n$,
(3) $V_1 \leq m$, and
(4) $\left.\frac{\partial \Pi}{\partial V}\right|_m < 0$.

Since Π is differentiable for $n \leq V \leq m$, these conditions imply that $V^* := V_1$ is the profit-maximizing valution. We now take up these claims in turn:

(1) The second derivative at V_1 is given by

$$\left.\frac{\partial \left(\frac{V^2\delta+Cm-Cn-2Vm+2Vn-mn+m^2-Cm\delta+Cn\delta-Fm\delta+Fn\delta-2Vn\delta+mn\delta}{(m-n-\delta(V-n))^2}\right)}{\partial V}\right|_{V_1}$$

$$= \left.\frac{-2R}{(m-n-V\delta+n\delta)^3}\right|_{V_1}$$

$$= \frac{-2R}{\left(\sqrt{R}\right)^3} < 0$$

(2) $V_1 \geq n$ means

$$\begin{aligned} V_1 - n &= \frac{1}{\delta}\left(m - n + n\delta - \sqrt{R}\right) - n \\ &= \frac{m - n - \sqrt{R}}{\delta} \geq 0. \end{aligned}$$

Now, observe

$$\begin{aligned} & m - n \geq \sqrt{R} \\ \Leftrightarrow\ & (m-n)^2 \geq R \\ \Leftrightarrow\ & (m-n)^2 - R \geq 0 \\ \Leftrightarrow\ & \delta\,(m-n)\,(m-n+(1-\delta)\,(C-n)-\delta F) \geq 0 \end{aligned}$$

where the last inequality follows from assumption V.7.

(3) $V_1 \leq m$ is equivalent to

$$m - V_1 = \frac{\sqrt{R} - (1-\delta)\,(m-n)}{\delta} \geq 0$$

where the nominator is non-negative by

$$\begin{aligned} & \sqrt{R} \geq (1-\delta)\,(m-n) \\ \Leftrightarrow\ & R \geq \left[(1-\delta)\,(m-n)\right]^2 \\ \Leftrightarrow\ & R - \left[(1-\delta)\,(m-n)\right]^2 \geq 0 \\ \Leftrightarrow\ & \delta\,(m-n)\left[(1-\delta)\,(m-C)+\delta F\right] \geq 0 \end{aligned}$$

(4) We find

$$\left.\frac{\partial \Pi}{\partial V}\right|_m = -\frac{(1-\delta)(m-C)+\delta F}{(m-n)(1-\delta)^2} < 0$$

In order to show $\frac{\partial V^*}{\partial \delta} \geq 0$, note

$$\begin{aligned} R'(\delta) &= (m-n)\left[-(m-n-\delta(C-n))+(1-\delta)(-1)(C-n)+2\delta F\right] \\ &= (m-n)\left[2n-m-C+2\delta(C+F-n)\right]. \end{aligned}$$

We then obtain

$$\begin{aligned} \frac{\partial V^*}{\partial \delta} &= \frac{\partial\left(n+\frac{m-n-\sqrt{R(\delta)}}{\delta}\right)}{\partial \delta} \\ &= -\frac{m-n}{\delta^2} - \frac{\frac{1}{2}R(\delta)^{-\frac{1}{2}}R'(\delta)\delta - R(\delta)^{\frac{1}{2}}}{\delta^2} \\ &= -\frac{m-n+\frac{1}{2}R(\delta)^{-\frac{1}{2}}\left((m-n)\left[2n-m-C+2\delta(C+F-n)\right]\right)\delta - R(\delta)^{\frac{1}{2}}}{\delta^2} \end{aligned}$$

which is equal to, or larger than, zero iff

$$\begin{aligned} 0 &\geq R(\delta)^{\frac{1}{2}} + \frac{1}{2}\left(\left[-m+n+(2\delta-1)(C-n)+2F\delta\right]\right)\delta - R(\delta):(m-n) \\ &= R(\delta)^{\frac{1}{2}} + \frac{\delta}{2}\left(\left[-(m-n)+(2\delta-1)(C-n)+2F\delta\right]\right) \\ &\quad - \left[(1-\delta)(m-n-\delta(C-n))+\delta^2F\right] \\ &= R(\delta)^{\frac{1}{2}} + (m-n)\left(-\frac{\delta}{2}-(1-\delta)\right) + (C-n)\left(\frac{\delta}{2}(2\delta-1)+\delta(1-\delta)\right) + F\cdot 0 \\ &= \underbrace{R(\delta)^{\frac{1}{2}}}_{>0} - \underbrace{\left[(m-n)\left(1-\frac{1}{2}\delta\right)-(C-n)\left(\frac{1}{2}\delta\right)\right]}_{>0}. \end{aligned}$$

Equivalently, we obtain the inequality

$$\left[(m-n)\left(1-\frac{1}{2}\delta\right)-(C-n)\left(\frac{1}{2}\delta\right)\right]^2 \geq R(\delta)$$

or

$$\begin{aligned} &\left[(m-n)\left(1-\frac{1}{2}\delta\right)-(C-n)\left(\frac{1}{2}\delta\right)\right]^2 \\ \geq\ & (m-n)\left[(1-\delta)(m-n-\delta(C-n))\right]+(m-n)\delta^2F \end{aligned}$$

or

$$\begin{aligned} F &\leq \frac{\left[(m-n)\left(1-\frac{1}{2}\delta\right)-(C-n)\left(\frac{1}{2}\delta\right)\right]^2-(m-n)\left[(1-\delta)(m-n-\delta(C-n))\right]}{(m-n)\delta^2} \\ &= \frac{(m-C)^2}{4(m-n)}. \end{aligned}$$

Since this last inquality is assumption V.6, we have shown $\frac{\partial V^*}{\partial \delta} \geq 0$.

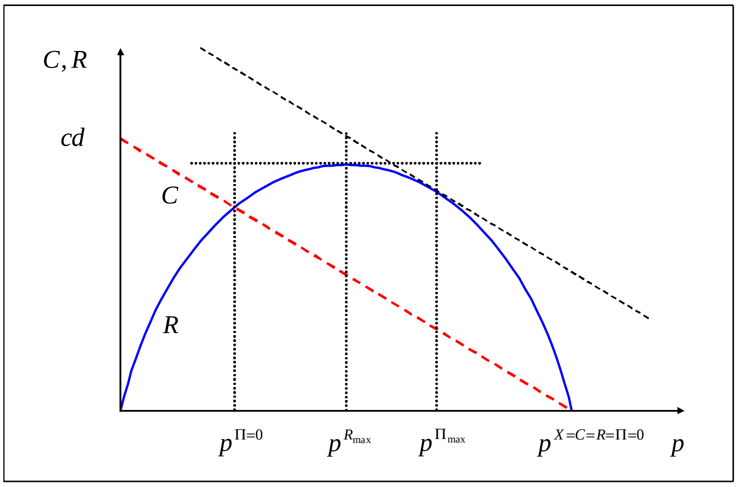

FIGURE 6. Solution

Finally, the expected tax payments follows from straightforward calculations.

8. Solutions

Exercise V.1

The satiation quantity is $X(0) = d$, the prohibitive price is implicitly defined by $X(p) = 0$ and hence equal to $\frac{d}{e}$.

Exercise V.2

Sorry, there is none.

Exercise V.3

The answer is provided in fig. 6. Note that we have to shift the cost curve upwards in order to find the price at which the difference between revenue and cost is maximal.

Exercise V.4

The monopolist's profit function is given by

$$\begin{aligned}\Pi(p) &= X(p)\,p - cX(p)\\ &= (d - ep)\,p - c(d - ep)\\ &= dp - ep^2 - cd + cep.\end{aligned}$$

Setting the derivative equal to zero and solving for p yields

$$\begin{aligned}\frac{d\Pi}{dp} &= d - 2ep + ce \stackrel{!}{=} 0 \text{ and, indeed,}\\ p^M &= \frac{d + ce}{2e} = \frac{d}{2e} + \frac{c}{2}.\end{aligned}$$

The revenue-maximizing price is $\frac{d}{2e}$ (just let $c = 0$).

Finally, we find

$$\frac{\partial p^M}{\partial c} = \frac{1}{2}.$$

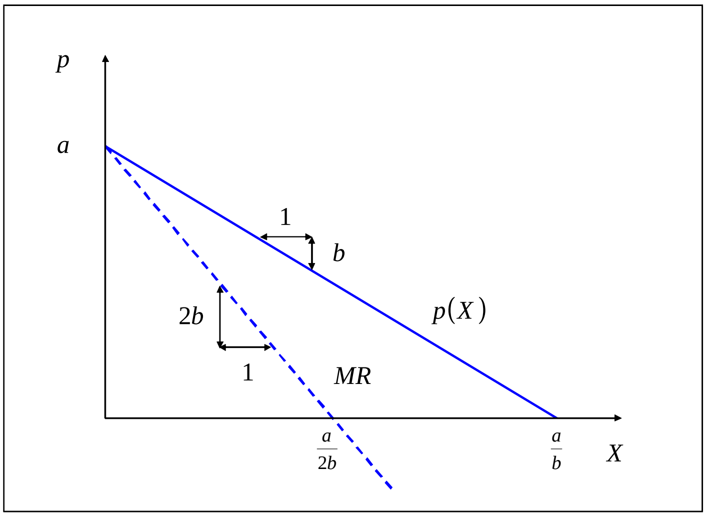

FIGURE 7. Demand curve and marginal-revenue curve

Thus, an increase of the unit cost by one unit leads an price increase by $\frac{1}{2}$.

Exercise V.5

(1) The slope of the inverse demand curve is $\frac{dp}{dX} = -b$.
(2) Revenue is given by $R(X) = p(X)X = aX - bX^2$ so that we obtain the marginal revenue
$$\frac{dR(X)}{dX} = a - 2bX.$$
The slope of the marginal revenue curve is $-2b$ and hence the marginal revenue curve is twice as steep as the demand curve itself (see fig. 7).
(3) The satiation quantity is $\frac{a}{b}$.
(4) a is the prohibitive price.

Exercise V.6

The monopoly quantity is $X^M = 11$.

Part B

Game theory

CHAPTER VI

Noncooperative games

1. Introduction

The most exciting event from the Pañcatantra or the Hitopadeśa is the moment when the lion and his friend, the bull, approach each other, after having been made suspicious of each other by the jackal, Damanaka. We will explain the highly suspenseful nail-biting confrontation by way of non-cooperative game theory. Therefore, we need to subject the reader to some game theory. The interested reader can consult one of the many textbooks on game theory, for example parts 1 and 2 in Gibbons (1992) or chapter 3 in Dixit & Skeath (1999).

2. Bimatrix games

Let us look at some simple and prominent examples of bimatrix games. We first consider the "stag hunt":

		hunter 2	
		stag	hare
hunter 1	stag	$5, 5$	$0, 4$
	hare	$4, 0$	$4, 4$

The first number in each field indicates the payoff for player 1 (hunter 1) and the second number is the payoff for player 2 (hunter 2). The two hunters cannot hunt both animals at the same time but have to focus on one of them. Stag hunting requires communal effort while the hare can be brought down by a single hunter. Hunter 1 is willing to take part in the stag hunt if hunter 2 also engages in stag hunting. If hunter 2 chases the hare, hunter 1 would just waste his effort trying to catch the stag.

The "stag hunt" is an instance of a *bimatrix* game. Indeed, we need *two* matrixes to describe the payoffs of the two players – hence the term *bi*-matrix.

Our second example of a bimatrix game is called "matching pennies" or "head or tail". Two players have the choice between head or tail. Player 1 wins one Euro if both choose head or both choose tail. Player 2 wins if their choices differ. We obtain the following game matrix:

		player 2	
		head	tail
player 1	head	$1, -1$	$-1, 1$
	tail	$-1, 1$	$1, -1$

The third game is the "battle of the sexes". A couple argues about how to spend the evening. Player 1, she, would like to go to the theatre, player 2, he, prefers football. However, both have a preference for spending the evening together. The following matrix captures these preferences:

		he	
		theatre	football
she	theatre	$4, 3$	$2, 2$
	football	$1, 1$	$3, 4$

Another famous game is called the "game of chicken". Two car drivers head towards each other. Their strategies are "continue" and "swerve". Whoever swerves is a chicken (a coward) and obtains a low payoff. However, continuing is a risky business:

		driver 2	
		continue	swerve
driver 1	continue	$0, 0$	$4, 2$
	swerve	$2, 4$	$3, 3$

We finally consider the following game:

		player 2	
		deny	confess
player 1	deny	$4, 4$	$0, 5$
	confess	$5, 0$	$1, 1$

It is known as the "prisoners' dilemma". Two prisoners sit in separate cells. They either confess their crime or deny it. If both confess, they will have to go to jail for some moderate time (payoff 1). If one confesses while the other denies, the first will obtain favorable terms (payoff 5) while the second will go to jail for a long time (payoff 0). If both deny, punishment will be small, due to the problem to prove them guilty (payoff 4). Sometimes, the first strategy is called cooperative where the cooperation refers to the other criminal, not the court.

Bimatrix games (you have just seen some examples) belong to the class of games in strategic form. They are described by

- the set of players, often $N = \{1, ..., n\}$ with $n = |N|$ ($|N|$ is the cardinality of N and denotes the number of elements in N),
- the strategy sets S_i for each player $i \in N$,
- all possible strategy combinations $s = (s_1, s_2, ..., s_n)$ (i.e., one strategy for each player) that are assembled in the set S, and
- the payoff functions $u_i : S \to \mathbb{R}$ for every player $i \in N$ (here $\mathbb{R}$ is the set of real numbers)

For example, the battle of the sexes has $N = \{she, he\}$, $S_{she} = S_{he} = \{\text{theatre,football}\}$ and the bimatrix reveals that u_{she} is given by

$$\begin{aligned} u_{she}\,(\text{theatre, theatre}) &= 4, \\ u_{she}\,(\text{theatre, football}) &= 2, \\ u_{she}\,(\text{football, theatre}) &= 1, \\ u_{she}\,(\text{football, football}) &= 3. \end{aligned}$$

3. Dominance

Similar to the definition of dominance in chapter III, we sometimes find that a strategy s_i of some player i dominates another strategy s_i' of that same player. That means that s_i yields a higher payoff for player i than s_i' in the presence of any strategies chosen by the other player(s). On p. 92 we present the "prisoners' dilemma". It is easy to see that both players have a strictly dominant strategy, confess. It may seem that the solution is very obvious and the game a boring one. Far from it! The "prisoners' dilemma" is one of the most widely discussed two-person games in economics, philosophy and sociology. Although the choice is obvious for each individual player, the overall result is unsatisfactory, viz., Pareto inferior. Pareto inferiority means that it is possible to make one player better off without making the others worse off. Here, all players can be made better off: $u_i\,(\text{confess, confess}) = 4 > 1 = u_i\,(\text{deny, deny})$ for $i = 1, 2$.

This contradiction between individual rationality (choose the dominant strategy!) and collective rationality (avoid Pareto-inferior payoffs) is not atypical. For example, tax payers like others to pay for public expenditures while shying away from the burden themselves. However, they (often) prefer the paying of taxes by everybody to the paying of taxes by nobody.

As another example consider two firms with the strategies of charging a high price or a low price:

		firm 2	
		high	low
firm 1	high	$4, 4$	$0, 5$
	low	$5, 0$	$1, 1$

A high price is the nice, cooperative strategy and corresponds to the strategy of denying in the original game.

In the literature, there are attempts to soften the dilemma. For example, it has been suggested that players promise each other to stick to the cooperative strategy (to choose deny). However, such a promise does not help; confess is still a dominant strategy.

The twin argument is also popular and builds on the symmetry of the prisoners' dilemma game. According to the twin argument, the players (should) deliberate as follows: Whatever reason leads one player to choose strategy 1 over strategy 2, is also valid for the other player. Therefore, the players' choice reduces to the diagonal payoffs and they choose the cooperative strategy by $4 > 1$. The problem with this argument is that players choose simultaneously and independently. The choice by one player 1 does not influence the other player's choice of strategy.

Exercise VI.1. *Is the stag hunt solvable by dominance arguments? How about head or tail, chicken, or the battle of the sexes.*

4. Revisiting the lion and the bull

In chapter III, pp. 40, we have told the Pañcatantra and Hitopadeśa story how the jackal Damanka manages to sow distrust between two former friends, the lion-king Piṅgalaka and the bull Saṃjīvaka. Finally, they approach each other, suspicious the other might want to attack first. We argue for this game matrix:

		Bull: attack (A)	Bull: not attack (NA)
Lion	attack (A)	NF_L, NF_B	V_L, D_B
	not attack (NA)	D_L, V_B	F_L, F_B

where both players have two strategies ("attack" or "not attack") and where the lion's payoffs obey

F_L (the lion's value for friendship)
$>$ V_L (the lion's payoff for victory over the bull)
$>$ NF_L (the lion's assessment for the loss of friendship and possible death(s))
$>$ D_L (the lion's payoff for death)

and analogously for the bull, with index B.

5. Best responses and Nash equilibria

The "prisoners' dilemma" is solvable by applying dominance. Many other games are more difficult. Then, one may resort to the Nash equilibrium. The Nash equilibrium is a strategy combination so that no player, by himself, can do any better. Differently put, at a Nash equilibrium, no player $i \in N$ has an incentive to deviate unilaterally, i.e., to choose another strategy if the other players stick to their part of the equilibrium combination.

EXERCISE VI.2. *Determine the equilibria of the following game:*

		player 2: *left*	***player 2***: *right*
player 1	*up*	$4,4$	$4,4$
	down	$0,0$	$4,4$

We have a simple procedure that helps to determine all the equilibria in matrix games. For every row (strategy chosen by player 1) we determine the best strategy or the best strategies for player 2 and mark the corresponding field or fields by a "2". You remember the R-procedure introduced in chapter III? For every column, we find the best strategy or the best strategies for player 1 and put a "1" into the corresponding field. Any field with two marks points to an equilibrium strategy combination. No player wins by deviating unilaterally. The stag hunt provides an example of two equilibria:

		hunter 2: stag	hunter 2: hare
hunter 1	stag	$5,5$ 1 2	$0,4$
	hare	$4,0$	$4,4$ 1 2

Exercise VI.3. *Using the marking technique, determine the Nash equilibria of the following three games:*

		player 2: *left*	*player 2*: *right*
player 1	*up*	$1,-1$	$-1,1$
	down	$-1,1$	$1,-1$

		player 2: *left*	*player 2*: *right*
player 1	*up*	$4,4$	$0,5$
	down	$5,0$	$1,1$

		player 2: *left*	*player 2*: *right*
player 1	*up*	$1,1$	$1,1$
	down	$1,1$	$0,0$

How about equilibria in the lion-bull game? Using the marking technique yields

		Bull: attack (A)	Bull: not attack (NA)
Lion	attack (A)	NF_L, NF_B L B	V_L, D_B
	not attack (NA)	D_L, V_B	F_L, F_B L B

Thus, no animal has a dominant strategy. However, we find two equilibria: Both animals attack, or both animals do not attack.

6. Risk dominance

The NA equilibrium payoff-dominates the A equilibrium, i.e., F is better than NF for both animals. On the other hand, "attack" has the advantage of avoiding the worst outcome D. In that sense NA is risky. One can calculate whether one equilibrium risk-dominates the other. This is done by looking at the product of the payoff differences for individual deviations. In our example, the A equilibrium risk-dominantes the NA equilibrium if

$$(NF_L - D_L)(NF_B - D_B) > (F_L - V_L)(F_B - V_B)$$

holds. Thus, an attack might happen from risk-dominance deliberations, whenever a suitable combination of the following holds:

- If the opponent attacks, the gain from attacking also (NF minus D for both animals) is large.
- If the opponent does not attack, the gain from not attacking also (F minus V) is small.

However, if both bull and lion would take the attitude destribed by Olivelle (2006, p. 191) ("If he is killed, to heaven he will go."), D_L and D_B would be relatively large and hence the NA equilibrium might be the risk-dominant one.

Exercise VI.4. *Both of these two games have two equilibria. Is one of them payoff-dominant or risk-dominant?*

		driver 2 *continue*	*swerve*
driver 1	*continue*	$0,0$	$6,1$
	swerve	$2,4$	$3,3$

		he *theatre*	*football*
she	*theatre*	$5,3$	$2,1$
	football	$1,1$	$3,4$

7. Signals

Aumann (2000) discusses whether pre-play communication may help agents to achieve the payoff-dominant equilibrium. Could not both animals signal or say "I will not attack" and help to coordinate on the better equilibrium?

So it seems. But, quoting Aumann (with the appropriate changes in the names of players, strategy labels, and payoffs), the lion might reason as follows: "Suppose the bull doesn't trust me, and so will attack in spite of our agreement. Then he would still want *me* not to attack, because that way he will get V_B rather than NF_B. And of course, also if he does not attack, it is better for him that I do not attack. Thus he wants me to refrain from attacking no matter what. So he wants the agreement not to attack in any case; it doesn't bind him, and might increase his chances of my not attacking. That doesn't imply that he will necessarily attack, but he may; since he wants the agreement no matter what he does, the agreement conveys no information about his acting. In fact, he may well have signed it without giving any thought as to how actually to act. Since he can reason in the same way about me, neither of us gets any information from the agreement; it is as if there were no agreement. So I will choose now what I would have chosen without an agreement, namely attacking."

Aumann points out that his argument works for his example (and also for the lion-bull fable), but not for the "battle of the sexes":

		he	
		theatre	football
she	theatre	$4, 3$	$2, 2$
	football	$1, 1$	$3, 4$

Indeed, if the two agents agree on theatre, then they are motivated to stick to the agreement. In Aumann's words: "It is not that she takes the agreement as a direct signal that [he] will keep it. Rather ... she realizes that by signing the agreement, [he] is signalling that he wants *her* to keep it. But ... here the fact that he wants her to keep it implies that he intends to keep it himself. So for her, too, it is worthwhile to keep it. Similarly for him. *This* agreement is self-enforcing."

Skyrms (2004, chapter 5) reports some experimental evidence according to which costless pre-play communication increases the probability of the good outcome and also discusses signals in an evolutionary setting.

8. Solutions

Exercise VI.1

In none of these games does any player have a dominant strategy.

Exercise VI.2

There are three Nash equilibria, the strategy combinations (up, left), (up, right) and (down, right).

Exercise VI.3

After marking, the three matrices look like fig. 1. The first does not have an equilibrium, the second has exactly one and the third has three equilibria.

Exercise VI.4

player 2

player 1		left	right
	up	$1,-1$ [1]	$-1,1$ [2]
	down	$-1,1$ [2]	$1,-1$ [1]

player 2

		left	right
	up	$4,4$	$0,5$ [2]
	down	$5,0$ [1]	$1,1$ [1][2]

player 2

player 1		left	right
	up	$1,1$ [1][2]	$1,1$ [1][2]
	down	$1,1$ [1][2]	$0,0$

FIGURE 1. Matrices and markings

The equilibria of this "game of chicken" are

$$\begin{aligned}&(\text{continue, swerve}) \text{ with payoffs } (6,1) \text{ and}\\&(\text{swerve, continue}) \text{ with payoffs } (2,4)\,.\end{aligned}$$

Neither dominates the other with respect to payoffs because the first equilibrium is preferred by driver 1 while the second one is preferred by driver 2. The first equilibrium risk-dominates the second one by

$$(6-3)(1-0)=3>2=(2-0)(4-3)\,.$$

The second game is the "battle of the sexes" where we have the equilibria

$$\begin{aligned}&(\text{theatre, theatre}) \text{ with payoffs } (5,3) \text{ and}\\&(\text{football, football}) \text{ with payoffs } (3,4)\,.\end{aligned}$$

Again, there is no payoff domination (she prefers the theatre equilibrium and he the football equilibrium). The first equilibrium risk-dominates the second one by

$$(5-1)(3-1)=8>3=(3-2)(4-1)$$

CHAPTER VII

Backward induction

The tragedy that follows a wrong plan,
The triumph that results from the right plan,
To the rules of Polity both are linked;
so the wise can point them out,
as if displayed in advance.
(from the Panchatantra, translated by Olivelle 2006, p. 77, verse 1.60)

1. Introduction

The famous indologist Heinrich Zimmer (1969, p. 89) observes that Indian political thought was characterized by "cold-blooded cynical realism and sophistication". He also finds that "ancient Hindu political wisdom" brings about "the cold precision of a kind of political algebra, certain fundamental natural laws that govern political life, no matter where" (p. 90).

Meanwhile, Zimmer's political algebra has been developed by economists and mathematicians under the heading of "game theory". The aim of this chapter is to show that the reasoning employed by human and animal actors in some Indian fables can be analyzed by a powerful method developed by game theorists, backward induction. Sometimes these actors employ backward induction and sometimes, very much to their detriment, they fail to do so. In the stories presented in this chapter, the didactic purpose of teaching forward-looking behavior seems very obvious. Thus, we may credit Indian political thought with the early invention and application of backward induction.

It is not an easy question whether or not the Indians share this achievement (independent invention and application of backward induction) with other ancient cultures. For example, when Brams (2003) analyzes stories from the Hebrew bible, he also uses backward induction. In our mind, this does not necessarily mean that the bible authors also apply backward induction. In contrast to the Indian fable tellers, their focus is not on strategic thinking, but rather on telling the history of the Israelis and on the relationship between God and His people. (Of course, the fact that Brams (2003) and, in more detail, Brams (2011), apply the Theory of Moves developed by that author to biblical stories, does not imply that biblical story tellers had any idea about this recent branch of game theory.)

Apart from biblical stories, Brams (2011) shows how non-cooperative game theory can be used to analyze, inter alia, jury selection, Aritophanes's play Lysistrata, Shakespeare's Macbeth, or the Cuban Missile Crisis. Similarly, in an as yet unpublished manuscript, Chwe (2010) argues that "folk game theory" can "take the perspective of outsiders", such as slaves or Jews. To the best of our knowledge, this essay is the first to provide a game-theoretic analysis of some Indian fables. However, in the context of the *Artha-śāstra*, Sihag (2007) claims that Kauṭilya already "knew" about game-theoretic niceties such as time inconsistency and asymmetric information.

We will explain the political algebra of game theory by way of three animal tales, (i), the tiger and the traveller, (ii), the lion, the mouse, and the cat, and (iii), the cat and the mouse. Zimmer himself cites the second and third fable. While the Indian fable tellers did not have the formal instrument of backward induction at their disposal, the stories and morals of the stories clearly show that they understood backward induction very well. This is obvious from all three stories although only the last one has the players act according to backward induction. In the first two examples, backward induction is violated and it is this very violation that the fable tellers want to point out to their readers.

2. Backward induction

In this section, we present as much game theory as needed for the purposes of this chapter. Have a look at this game.

		player 2	
		left	right
player 1	up	$10,5$ $\boxed{1}\boxed{2}$	$0,3$
	down	$8,1$	$9,4$ $\boxed{1}\boxed{2}$

With the instruments learned in the previous chapter, you know that we have two equilibria in this game. Consider, now, the sequential version of this game in fig. 1. You see that some nodes are indexed by the players' names (1 or 2). At these nodes players 1 and 2 have to make a choice. Player 1 moves first, at the initial node (the leftmost node). He chooses "up" or "down". Next, it is player 2's turn who chooses between "left" and "right". When both players have chosen their actions, they obtain the appropriate payoffs or "utilities". The payoff information is noted at the terminal nodes (the rightmost nodes). The first number indicates the payoff for player 1 and the second number is the payoff for player 2. For example, if player 1 chooses "up" and player 2 chooses "right", player 1 obtains the payoff of 0 and player 2 the payoff of 3.

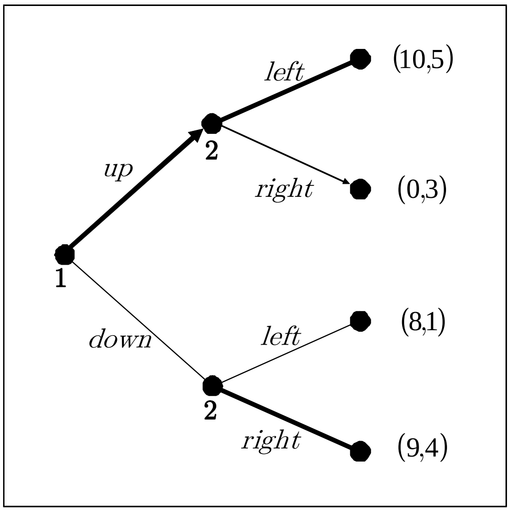

FIGURE 1. A game tree

Backward induction means "looking ahead" by "proceeding backwards". Before player 1 can decide on his move, he needs to know how player 2 will react to "up", or "down", chosen by player 1. Thus, backward induction starts with the players that move last. Consider the node where player 2 has to make a decision after player 1 chose "up". Comparing the payoffs 5 and 3, player 2 chooses "left". The corresponding edge has been reinforced. In contrast, player 2 will choose "right" if he learns that player 1 has chosen "down" (this follows from $4 > 1$).

Now, after knowing the choices of player 2, we can look at player 1's decision. If he chooses "up", player 2 will choose "left" so that player 1 obtains a payoff of 10. If, however, player 1 chooses "down", player 2 will choose "right" so that player 1 obtains 9. Comparing 10 and 9, it is obvious that player 1 should choose "up".

Thus, player 1 choosing "up" and player 2 choosing "left" is the predicted outcome. However, this may not be the observed outcome. For example, player 1 choosing "up" and player 2 choosing "right" is indicated by the arrows. In that sequence of events, player 2 would have made a mistake. By $5 > 3$ he could have done better.

EXERCISE VII.1. *Find the backward-induction solution for the game of chicken (p. 92).*

3. The tiger and the traveller

The first example is the tale of the tiger and the traveller known from the Hitopadeśa collection of fable-based advice (see, for example, Kale & Kale (1967, pp. 7-9) or the comic book by Chandakant & Fowler (1975, pp. 14-18)).

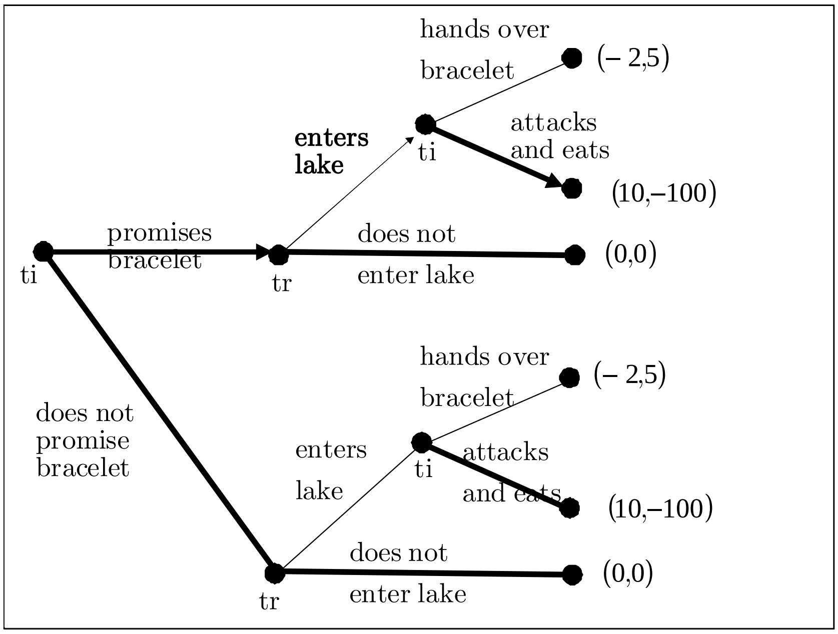

FIGURE 2. The tiger and the traveller

This is the story: A tiger that finds himself on one side of a lake sees a traveller passing by on the opposite side. The tiger attempts to catch and eat the traveller by offering a golden bracelet to him. Since the traveller is suspicious of the tiger's intentions, the tiger argues that he would not (he claims to have profoundly changed his former evil behavior) and could not (he claims to be old and weak) do any harm to the traveller. Finally, the traveller is convinced, gets into the murky waters where he gets stuck. Immediately, the tiger takes advantage of the traveller's misfortune and kills him as planned.

Consider the payoffs in figure 2. The first number at the final nodes refers to the tiger, the second one to the traveller. The tiger's payoffs are -2 for giving away the bracelet and not eating the traveller, 10 for keeping the bracelet and enjoying a good meal, and 0 for the status quo of keeping the bracelet but staying hungry. The corresponding traveller's payoffs are 5, -100, and 0.

The tragic sequence of events sketched above is indicated by the arrows. The tiger (ti) moves first by promising the bracelet (upper branch). The traveller (tr) enters the lake (upper branch) and then the tiger kills the traveller (lower branch).

The game tree of this story has three stages. First, the tiger offers the bracelet and talks about his guru who has convinced him to lead a more virtuous life or the tiger refrains from offering the bracelet and/or from talking convincingly. Then, the traveller needs to decide on whether or not to accept the tiger's invitation to join him by crossing the lake. Finally, the tiger fulfills his promise or reneges on it.

One may of course speculate why the traveller is so "stupid". Did "greed cloud the mind" or did he act on some probability assessment about the tiger telling the truth? Indeed, the tiger claims to have studied the Vedas to lend credibility to his peaceful intentions. However, it seems obvious that the fable writer does not think of this example under the heading of "better safe than sorry". Instead he argues that the tiger's preferences being as they are the traveller should have known his fate in advance. Before being killed, the traveller has time for some wise insights to share with the readers (see Kale & Kale 1967, p. 8):

> That he reads the texts of religious law and studies the Vedas, is no reason why confidence should be reposed in a villain: it is the nature that predominate [sic] in such a case: just as by nature the milk of cows is sweet.

Knowledge of backward induction would also have led the traveller to avoid the lake. By $10 > -2$, he should have foreseen his being eaten after entering the lake so that keeping clear of the lake is best by $0 > -100$.

Interestingly, the traveller should refrain from entering the lake independent of whether or not the tiger talks about his guru who advised the tiger to pursue a more virtuous life. In game-theory parlance, the tiger's arguments, the first step in our game tree, are just "cheap talk". Both a mischievous and a benevolent tiger could claim their benevolence without any cost. Therefore, this claim is not credible.

Pious appearances are also used by the cat in an animal tale from the Pañca-tantra (see, for example Olivelle 2006, pp. 393-399). The cat is chosen as a judge in a dispute between a partridge and a hare. Although wary of the danger, the two contestants finally approach the cat who kills them without much ado.

4. The lion, the mouse, and the cat

The second animal tale is also taken from the Hitopadeśa (see Kale & Kale 1967, p. 51). A lion that lives in a cave is infuriated by a mouse that also lives in his cave. The mouse regularly gnaws at the sleeping lion's mane. Since the lion does not succeed in catching the mouse, he invites a hungry and desperate cat to live and eat in his cave.

The arrangement between the lion and the cat works out well. The mouse does not dare to show up while the cat is present. Therefore, the lion is happy to share his food with the cat as promised although he does not particularly like the cat's company by itself. One day, the mouse is detected by the cat who catches and kills it. The lion does not see any reason to extend his hospitality and makes the cat leave his cave. Soon, the cat returns to her former miserable state.

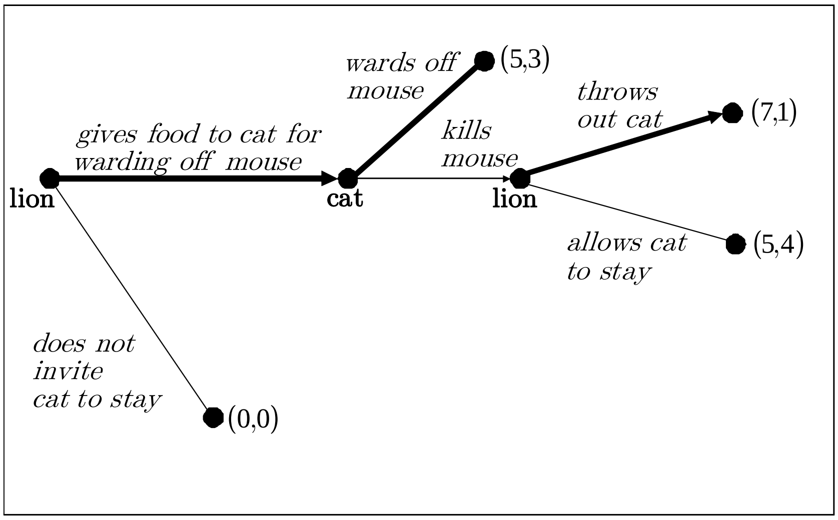

FIGURE 3. The lion, the mouse, and the cat

The moral to be drawn from this fable is obvious: Do your work but see to it that you are also needed in the future.

The reader is invited to have a look at figure 3. The first number at the final nodes refers to the lion, the second to the cat. Both players obtain a payoff of 0 if the lion does not invite the cat to stay so that the lion's mouse problem is not solved and the cat cannot eat the food provided by the lion. The lion's payoff is 5 if the mouse does not annoy him and increases up to 7 if, on top, the cat does not stay in the cave. The cat in the cave has a payoff of 3 if she can stay in the cave and an increased payoff of 4 for eating the mouse and staying in the cave.

The arrows indicate the story as told in the Hitopadeśa. This is not the backward-induction result which, again, is indicated by the thickened lines. The wise cat would foresee that it is in the best interest of the lion to get rid of it after the mouse is killed ($7 > 5$). Therefore, the cat should have kept on warding off the mouse (payoff 3) rather than killing the mouse and be thrown out of the convenient cave (payoff 1). Working backwards one final step, we see that the lion was right to invite the cat into his cave ($5 > 0$). Indeed, because of the cat's mistake, the lion is even better off obtaining 7 rather than 5.

Again, one may ask the question whether there are defensible reasons for the violation of backward induction. Did the cat think that another mouse would show up promptly so that the lion would need the cat's services again? It seems that the fable's author did not think along these lines, but had the more straight-forward didactic aim of teaching the forward-looking behavior the cat did not master.

A second possibility comes to mind: The cat may have entertained the hope that the lion would show thankfulness to the cat for freeing the lion of the mouse for good. However, in line with the cynical realism observed by Zimmer, we would rather not follow this line of thought, but insist on the lesson that friendship has no worth and that the behaviors of humans or animals are dictated by future gains and losses, rather than by friendly acts in the past.

5. The cat and the mouse

In the previous animal tale, the lion profited from the opponent's mistake. Sometimes, however, players hope that opponents react rationally. To show this, we finally present a fable from book 12 of the grand epic Mahabharata (see Fitzgerald 2004, pp. 513-518). A he-cat is caught in a net laid out by a trapper. The mouse is happy to see her enemy in this difficult situation when she realizes that an owl is about to attack from above and a mongoose is sneaking up on her. She offers the cat to destroy the net if the cat gives her shelter. The mouse realizes that her plan needs a good deal of rationality and foresight on the cat's part (p. 514):

> So I will make use of my enemy the cat. I shall contribute to his welfare ... And now may this enemy of mine happen to be smart.

Fortunately, the cat agrees to the bargain. When seeing the mouse under the cat's protection, owl and mongoose look out for other prey. The cat is dismayed to find that the mouse is in no hurry to fulfill her promise. Indeed, the mouse realizes that freeing the cat immediately makes her an easy victim of the cat. In a long dialogue, the logic of the situation is explicitly spelled out. As the mouse remarks (p. 517):

> No one is really an ally to anyone, no one is really a friend to anyone ... When a job has been finished, no one pays any attention to the one who did it; so one should make sure to leave all his tasks with something still to be done. At just the right time, sir, you will be filled with fear of the [trapper] and intent on getting away, and you won't be able to capture me.

Thus, the mouse waits until the trapper approaches. At the very last moment, the mouse liberates the cat that now has better things to do than mouse hunting. Both manage to find a safe place to hide, but certainly not the same.

Figure 4 shows the game tree of this animal tale. The first payoff accrues to the mouse (m), the second one to the cat. The mouse obtains 0 for escaping unharmed and suffers the payoff of -100 for being killed by owl, mongoose, or cat. The cat's payoff is zero for escaping unharmed, 2 for escaping and eating the mouse, -50 for being killed by the trapper and -48 for being killed by the trapper after eating the mouse.

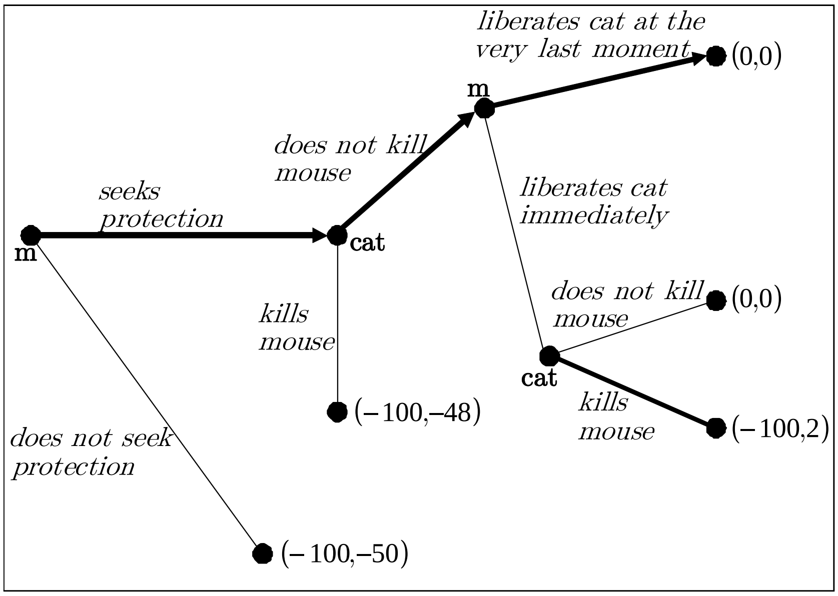

FIGURE 4. The cat and the mouse

Foreseeing that the cat will kill the mouse if liberated well before the trapper arrives ($2 > 0$), the mouse prefers to wait until the trapper approaches ($0 > -100$). The cat is clever enough not to kill the mouse before he is liberated ($0 > -48$). Thus, indeed, the mouse made a clever move to seek the cat's protection ($0 > -100$).

Unlike the first two stories, in this story, the sequence of events is the one predicted by backward induction. Neither the mouse nor the cat makes a mistake.

Now a question that is totally unrelated to the above story, but an interesting application of backward induction:

EXERCISE VII.2. *Consider the centipede game depicted in fig. 5! The players 1 and 2 take turns in choosing between "finish" (action f) or "go on" (action g). For every player, a strategy is a 99-tuple. For example,* $\lfloor g, g, g, g, f, ..., f \rfloor$ *is the strategy according to which a player chooses "go on" at his first four decision nodes and chooses "finish" at all the others.*

(1) *Which strategy would you choose if you were player 1? Does your answer depend on who takes on 2's role?*
(2) *Solve the centipede game by backward induction!*
(3) *Do you want to reconsider your answer to the first question?*

6. Conclusions

As noted in the introduction, Indian political thought was cold-blooded and cynical. From the point of view of virtue ethics (see, for example, McCloskey 2006, pp. 63), one may note that Indian fables and also a

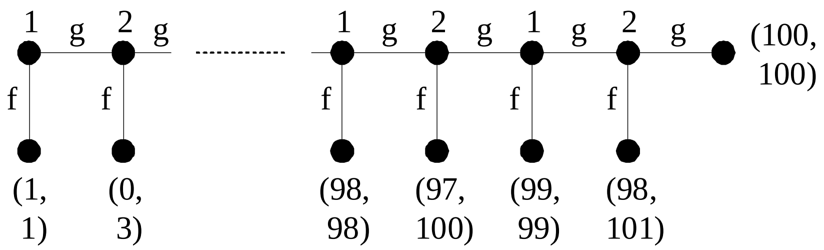

FIGURE 5. The centipede game

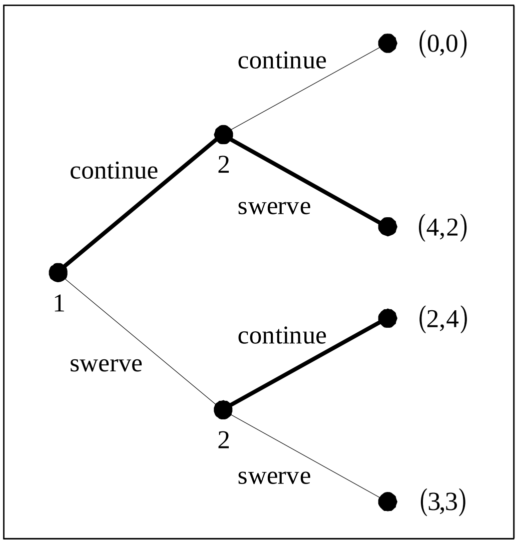

FIGURE 6. Backward induction for the game of chicken

good deal of economics stress the virtue of prudence at the expense of other virtues, such as justice, hope, love, faith, etc. Indeed, Indian animal tales often have a clear didactic purpose – to teach future kings how to exercise prudence by paying heed to basic tricks in strategic thinking.

Schwalbe & Walker (2001) trace the "early (sic) history of game theory" and note (on p. 126) that the first time "a proof by backward induction is used seems to be in von Neumann and Morgenstern (1953)". We do not mean to contradict these authors when we say that the application (rather than the use for a proof) is definitely much older, at least going back to some hundred years BCE, in India and may-be also in other ancient cultures.

7. Solutions

Exercise VII.1

Driver 1 has a first-mover advantage in the game of chicken. He chooses "continue" so that driver 2 is forced to swerve. The game tree and its backward-induction solution is depicted in fig. 6.

Exercise VII.2

(1) You will probably choose "go on" for a while?
(2) Backward induction implies that player 1 finishes the game immediately. (You should have thickened the "finish" actions everywhere.)
(3) Maybe, if your opponent knows backward induction you will choose "finish" rather sooner than later? Or, perhaps, game theory is not always sufficient to make a good decision?

CHAPTER VIII

The mandala theory

1. Introduction

There can be no doubt that the Indian Arthaśāstra, which was probably written by Kauṭilya (roughly 2000 years ago), belongs to the major treatises of political science (see the survey by Boesche 2002). In this chapter, we are concerned with the maṇḍala theory which explains how a king should manage war and peace with direct and indirect neighbours. Kauṭilya's main idea was simple and intriguing. War can only be waged with direct neighbours (local warfare). Therefore, neighbours tend to be enemies and the enemies of enemies tend to be friends. While this Kauṭilya conjecture surely has a lot of intuitive appeal, we are not aware of any formal model that confirms or disproves it.

Providing the building blocks for analyzing Kauṭilya's conjecture is one main aim of this chapter. We formally define important concepts, such as neighbourhood structure, fighting structure, (steadfast) friend, and (steadfast) enemy. We also provide a game-theoretic analysis. We admit, of course, that our simple definitions and models do not do justice to the thoughtful analysis provided by this eminent Indian social scientist. (See the conclusions for further remarks.)

A recent article by Schetelich (???a) draws a line from the maṇḍala model to an Indian board game called four-handed chess or *catūrājī*. (A discussion of its history can be found in Petzold (1986, pp. 17-40) and, more recently and in more detail, Bock-Raming (1996) and Bock-Raming (2001), while related dice games are treated by Lüders (1907).) A second aim of this chapter is to lend additional support to Schetelich's argument by analyzing a 4-country model which might be seen as a stylized version of Indian four-handed chess. In broad terms, Kauṭilya's conjecture is confirmed within this 4-country model and hence, in our view, the link argued for by Schetelich.

We present Kauṭilya's model and Schetelich's argument in the next section. Section 3 then provides all the formal definitions. Section 4 analyzes some simple examples. Finally, section 5 concludes.

2. Maṇḍala theory and Indian four-player chess

2.1. Maṇḍala theory. Schetelich (1997, pp. 213) explains Kauṭilya's maṇḍala theories. A king considers himself in the center of a ringlike structure, the rājamaṇḍala ("maṇḍala of kings"). In the next section, we define an important aspect of the maṇḍala of kings, the neighbourhood structure. It summarizes all the direct neighbours for every kingdom. We assume that only direct neighbours can be attacked (local warfare).

For our chapter, two quotations from the Arthaśāstra (as translated by Rangarajan (1992, p. 555)) are of central importance:

Kauṭilya on antagonist and enemy: "Any king, whose kingdom shares a common border with that of the conqueror is an *antagonist*. Neighbouring kings, who are deemed to be antagonists, are of different kinds: (i) a powerful antagonistic neighbour [having excellent personal qualities, resources and constituents] is an *enemy* ...".

Kauṭilya on ally: "A king whose territory has a common boundary with that of an antagonist [i.e., one whose territory has no common border with that of the conqueror] is an *ally*."

In our mind, it is not quite clear whether the quotations for "antagonist", "enemy", or "ally" are definitions or claims. The same holds for the motto "The enemy of my enemy is my friend" (see, for example, Schetelich ???b, p. 7). For the purposes of this chapter, we shall understand both quotations as conjectures. Therefore, we refer to the ally-quote as Kauṭilya's conjecture.

2.2. Maṇḍala model and four-handed chess. It has been obvious to many chess historians (see, for example, Bock-Raming 1996, p. 7) that Indian chess is closely related to warfare. Schetelich (??) takes up Lüders ()' idea that Indian four-handed chess is a board-game version of Kauṭilya's maṇḍala model. She presents the following arguments:

- Kauṭilya teaches kings to fight for supremacy where "diplomatic action and weakening an enemy by indirect action as well as by economic and political pressure from inside and outside should always be preferred to open armed conflicts. The dominant strategies in *catūrājī* - obstructing the pieces of the rival king instead of capturing them at any cost - matches perfectly with this tendency." (Schetelich (???a, p. 6))
- According to the Muslim polymath Al Birunī (who lived around 1000 CE), the maximal points achievable by the winner in four-handed chess is 54. Schetelich (???a, pp. 2, 9, 10) points to the intriguing fact that this number can be understood as 3 times 18 where 3 is the number of opponents in *four*-handed chess and $18 = 3 \times 6$ results from Kauṭilya's maṇḍala model: Each king is backed by an ally and the ally's ally (thus, three) and each kingdom has 6

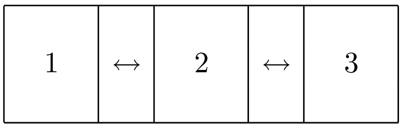

FIGURE 1. An asymmetric three-country neighbourhood structure

constituent elements: the king himself, the minister, the country, the fort, the treasury, and the army.

- As additional support, Schetelich (???a, pp. 6) offers the following comment: "The most simple *rājamaṇḍala* has three constituent elements: a leader-king (also called would be-emperor, *vijigīṣu*), his enemy and his friend while the *maṇḍala* most common in political practice has four (the leader and his enemy, each of them having a *mitra*)". Here, *mitra* means someone with whom I have entered into a contractual relation, a friend. In line with this judgement, we concentrate on 3- and 4-country neighbourhood structures in section 4 after providing the formal apparatus for the general case in the next section.

3. Fighting involving friends and enemies

3.1. Neighbourhood structures and fighting structures. We now define neighbourhood structures and fighting structures. Both are undirected graphs.

DEFINITION VIII.1. *Let* $N = \{1, ..., n\}$ *be a set of* n *countries. A neighbourhood structure* $\mathcal{N}$ *on* N *is a subset of* $N^{(2)} := \{i \leftrightarrow j : i, j \in N, i \neq j\}$ *with* $i \leftrightarrow j \in \mathcal{N} \Leftrightarrow j \leftrightarrow i \in \mathcal{N}$. *A fighting structure* F *on* N *is a subset of* $\mathcal{N}$. *The set of fighting structures is denoted by* $\mathfrak{F}$.

Thus, fighting is possible between (direct) neighbours, only. As a special example consider three countries $i = 1, 2, 3$ in a row where country 2 has two direct neighbours, 1 and 3 (see figure 1). Four fighting structures are possible for this neighbourhood structure: F= $\emptyset$ (no fighting), F= $\{1 \leftrightarrow 2\}$ (1 and 2 fight), F= $\{2 \leftrightarrow 3\}$ (2 and 3 fight), and F= $\mathcal{N}$ (every country fights every neighbour).

A second (symmetric!) example concerns four countries (figure 2).

EXERCISE VIII.1. *Determine all fighting structures for* $N = \{1, 2\}$*!*

3.2. Friends and enemies. Before turning to definitions of "friend" and "enemy", some remarks on notation are in order. First, we use preference relations $\succsim_i$ on $\mathfrak{F}$ for country $i \in N$ in the same sense as in chapter II, but not as in chapter IV. I.e., preference relations are reflexive, complete, and transitive.

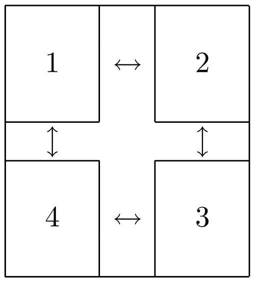

FIGURE 2. A symmetric four-country neighbourhood structure

Note also: Whenever we use expressions like $\mathcal{F} \cup \{i \leftrightarrow k\} \succ_i \mathcal{F}$, it is assumed that $\mathcal{F} \cup \{i \leftrightarrow k\}$ is a fighting structure, also.

The outcomes of fighting obey the following assumptions:

- The size of each country is 1. This is the basic payoff and fighting power.
- A fight ensues whenever one of two neighboring countries so wishes.
- If a country wins against another country, the winning country takes over the territory of the losing one. If several winning countries are involved, the losing country's territory is split evenly between the winning ones. This assumption is squarely in line with Kauṭilya's recommendation to "induce a neighbouring ruler to undertake a simultaneous expedition, each attacking the enemy from a different direction, [on the understanding that] the spoils will be divided equally" (Rangarajan (1992, p. 575)).
- If a country is involved in two or more fights, its fighting power is split evenly.
- The fighting powers of several attackers are added. The relative fighting power determines the winner. If the fighting power is the same, the outcome is a "draw".
- Fighting is costly. For each fight, the fighting power is reduced by $\delta > 0$ (which should be thought of as a small number).
- Basically, utility equals fighting power. But each country prefers weaker neighbours to stronger ones. ε stands for the advantage of being stronger than neighbours while $-\varepsilon$ represents the disadvantage of being weaker. While ε is also small, we assume $\varepsilon > \delta$.

We are now in a position to define friendship and enmity:

DEFINITION VIII.2. *Let N be a set of countries with fighting structure $\mathcal{F}$. For three countries i, j, and k assume*

$$\begin{aligned} i &\leftrightarrow j \notin \mathcal{F}, \\ j &\leftrightarrow k \in \mathcal{F}, \\ i &\leftrightarrow k \notin \mathcal{F}. \end{aligned}$$

Country i is called a friend of j against k at $\mathcal{F}$ if

$$\mathcal{F} \cup \{i \leftrightarrow k\} \succ_i \mathcal{F}$$

holds. Country i is called a steadfast friend of j if i is a friend of j against every k at every $\mathcal{F}$ with the above properties.

DEFINITION VIII.3. *Let N be a set of countries with fighting structure $\mathcal{F}$. Country i is called an enemy of country j at $\mathcal{F}$ if one of two conditions hold:*

- *either i fights against j ($i \leftrightarrow j \in \mathcal{F}$),*
- *or, if i does not fight j, she would like to do so ($i \leftrightarrow j \notin \mathcal{F} \Rightarrow \mathcal{F} \cup \{i \leftrightarrow j\} \succ_i \mathcal{F}$).*

Country i is called a steadfast enemy of j if i is an enemy of j at every $\mathcal{F}$.

We close this section with the following two observations: In both fighting structures shown above (the neighbouring structures in figures 1 and 2), (i) country 1 is never a friend of country 2. This is due to the fact that 1 cannot attack 3. For the same reason, (ii) country 1 is never an enemy of country 3. While these claims are surely in line with Kauṭilyan thought (see below), they are trivial in the light of our definitions of friendship and enmity (they hold for any preferences on $\mathcal{F}$).

Observation (i) is obviously in the spirit of Kauṭilya's definitions of antagonist and enemy given in the section on maṇḍala theory. Also, Kauṭilya's definition on allies seems to fit well with (ii).

3.3. Strategies and equilibria. We find it natural to assume that a fighting relation $i \leftrightarrow j \in \mathcal{F}$ comes about whenever i alone or j alone or both attack. Since a country cannot attack herself, country i's strategy s_i is a tuple of $n-1$ attacking decisions:

DEFINITION VIII.4. *Let N be a set of countries. Player i's strategy s_i is a tuple $\left(a_i^j\right)_{j \in N, i \neq j}$ with two properties:*

- *For any $j \neq i$, a_i^j can take two values, value "y" for "yes, i attacks j" and value "n" for "no, i does not attack j".*
- *We have $a_i^j = n$ whenever $i \leftrightarrow j \notin \mathcal{N}$ (local warfare).*

Let $s = (s_1, ..., s_n)$ be a tuple of strategies (also known as strategy combination), one strategy for each country. Then, the induced fighting structure $\mathcal{F}(s)$ is given by

$$i \leftrightarrow j \in \mathcal{F} :\Leftrightarrow a_i^j = y \text{ or } a_j^i = y.$$

By s_{-i} we denote the strategy combination for all players except player i, i.e., $s_{-i} = (s_1, ..., s_{i-1}, s_{-i+1}, ..., s_n)$. Let S denote the set of strategy combinations.

Definition VIII.5. *Let $u_i : \mathfrak{F} \to \mathbb{R}$ be a utility function on $\mathfrak{F}$. We define a utility function $U_i : S \to \mathbb{R}$ on S by $U_i(s) = u_i(\mathcal{F}(s))$. Let $U = (U_1, ..., U_n)$. Then, (N, S, U) is a game in the sense of non-cooperative game theory. A strategy combination s^* obeying*

$$U_i\left(s_i^*, s_{-i}^*\right) \geq U_i\left(s_i, s_{-i}^*\right)$$

for all players $i \in N$ is called an equilibrium of this game. The strategy combination given by $a_i^j = y$ for all $i, j \in N$ with $i \leftrightarrow j \in \mathcal{N}$ is called the trivial equilibrium.

Thus, an equilibrium s^* is defined by the following property: No player $i \in N$ gains by not choosing s_i^* if the other players choose s_{-i}^*. Differently put, unilateral deviation does not pay.

Note that the trivial equilibrium is an equilibrium. Indeed, if only one player deviates, the fighting structure is not changed.

Exercise VIII.2. *For $N = \{1, 2\}$, determine all strategies for country 1!*

4. Identifying friends and enemies

4.1. The asymmetric 3-country example. We now turn to non-trivial results. In this section, we deal with the asymmetric 3-country example, in the next section with the symmetric 3-country example and, in the section after that, with the symmetric 4-country example.

Reconsider three countries $i = 1, 2, 3$ with the neighbourhood structure of figure 1. To our mind, the payoff assumptions listed are reflected in the following payoffs for country 1 (which are analogous to those of country 3):

$$u_1(\mathcal{F}) = \begin{cases} 1, & \mathcal{F} = \emptyset & \text{no fighting} \\ 1 - \delta, & \mathcal{F} = \{1 \leftrightarrow 2\} & \text{1 fights} \\ 1 + \varepsilon, & \mathcal{F} = \{2 \leftrightarrow 3\} & \text{the other countries fight} \\ \frac{3}{2} - \delta, & \mathcal{F} = \mathcal{N} & \text{all neighbours fight} \end{cases}$$

and country 2:

$$u_2(\mathcal{F}) = \begin{cases} 1, & \mathcal{F} = \emptyset & \text{no fighting} \\ 1 - \delta - \varepsilon, & \mathcal{F} = \{1 \leftrightarrow 2\} \text{ or } \mathcal{F} = \{2 \leftrightarrow 3\} & \text{2 fights with one neighbour} \\ 0, & \mathcal{F} = \mathcal{N} & \text{2 fights with both neighbours} \end{cases}$$

where we assume $\varepsilon > \delta > 0$ (see the list of payoff assumptions) and (by smallness of δ and ε) $\varepsilon + \delta < \frac{1}{2}$.

Some comments on the payoffs are in order. The first lines refer to no fighting at all. In that case, no asymmetry develops and fighting costs are absent. Therefore, all countries obtain a payoff of 1. If countries 1 and 2 fight (second lines), there is a draw between them and they suffer the costs of fighting δ. Also, country 2 is then weaker than her direct neighbour 3 so that we have $-\varepsilon$ for country 2, but not for country 1. In the third line for u_1, country 1 is not involved in any fighting. It preserves the worth of its territory but gains ε because it is now stronger than country 2 because of that country's cost of fighting. If all neighbours fight, country 2 loses the war and obtains payoff 0 while the winning countries 1 and 3 share 2's territory $(+\frac{1}{2})$ and suffer the costs of fighting $(-\delta)$.

Straightforward comparisons yield the following theorem:

THEOREM VIII.1. *Assume the asymmetric 3-country neighbouring structure together with the utilities given above. Then, country* 1 *is a steadfast friend of country* 3*, but not a steadfast enemy of country* 2*. In particular, country* 1

- **a):** *is a friend of country* 3 *against* 2 *at* $\{2 \leftrightarrow 3\}$ *by* $\varepsilon + \delta < \frac{1}{2}$,
- **b):** *is an enemy of country* 2 *at* $\mathcal{F} = \{2 \leftrightarrow 3\}$ *by* $\varepsilon + \delta < \frac{1}{2}$,
- **c):** 1 *is not an enemy of country* 2 *at* $\mathcal{F} = \emptyset$ *by* $\delta > 0$.

Analogous results hold for country 3*. Fighting is never in country* 2*'s interest. Indeed, country* 2 *is an enemy to* 1 *(or* 3*) if and only if* $1 \leftrightarrow 2 \in \mathcal{F}$ *(*$2 \leftrightarrow 3 \in \mathcal{F}$ *) holds.*

Is the enemy of my enemy my friend? Let us consider country 1 and the fighting structure $\{2 \leftrightarrow 3\}$. Then, 2 and 3 are enemies and 1 is a friend of 3 against 2 (see a) in the above theorem). The fighting structure $\{1 \leftrightarrow 2\}$ is analogous and the structures $\mathcal{F} = \emptyset$ and $\mathcal{F} = \mathcal{N}$ cannot be used to check the definition of friendship. Thus, Kauṭilya's conjecture is confirmed in this 3-country example.

EXERCISE VIII.3. *Determine* u_1 *for* $N = \{1, 2\}$*! Is country* 1 *an enemy of country* 2 *at* $\mathcal{F} = \emptyset$ *or at* $\mathcal{F} = \{1 \leftrightarrow 2\}$.

We now turn to the equilibria in our 3-country example. They lead to the empty fighting structure (if no country attacks any other) or to all-out fighting (where 2 is fighting on both fronts):

THEOREM VIII.2. *In the asymmetric 3-country example, we have five equilibria:*

- **a):** *the no-attack equilibrium* s^* *given by* $\mathcal{F}(s^*) = \emptyset$ *(or, if you prefer,* $a_i^j = n$ *for* $i, j = 1, 2, 3,\ i \neq j$*),*
- **b):** *the four equilibria* s^* *leading to* $\mathcal{F}(s^*) = \mathcal{N}$ *and given by* $s_1^* = \left(a_1^2 = y,\ a_1^3 = n\right)$, $s_3^* = \left(a_3^1 = n,\ a_3^2 = y\right)$ *and any* s_2^* *(four different strategies exist).*

In this example and the two following, not attacking always forms an equilibrium. This is due to the cost of fighting δ and well in line with Kauṭilya: "If ... the conqueror is superior, the campaign shall be undertaken; otherwise not." (Rangarajan (1992, p. 627))

EXERCISE VIII.4. *For* $N = \{1, 2\}$*, determine all equilibria!*

4.2. The symmetric 3-country example. We now turn to a symmetric 3-country example where every country has common borders with the other two countries. By symmetry, it is sufficient to note the payoffs for country 1 :

$$u_1(\mathcal{F}) = \begin{cases} 1, & \mathcal{F} = \emptyset & \text{no fighting} \\ 1-\delta-\varepsilon, & \mathcal{F} = \{1 \leftrightarrow 2\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 3\} & \text{1 fights} \\ 1+\varepsilon, & \mathcal{F} = \{2 \leftrightarrow 3\} & \text{the other c. fight} \\ \frac{3}{2}-\delta, & \mathcal{F} = \{1 \leftrightarrow 2, 2 \leftrightarrow 3\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 3, 2 \leftrightarrow 3\} & \text{two c. against one} \\ 0, & \mathcal{F} = \{1 \leftrightarrow 2, 1 \leftrightarrow 3\} & \text{two c. against one} \\ 1-\delta, & \mathcal{F} = \mathcal{N} & \text{all-out fighting} \end{cases}$$

where we assume $\varepsilon > \delta > 0$ (see the list of payoff assumptions) and (by smallness of δ and ε) $\varepsilon + \delta < \frac{1}{2}$.

Again some comments on the payoffs: The second line means that 1 is involved in a balanced fight with either 2 or 3. 1 then suffers fighting costs of δ and has less fighting power than the country that is not involved in fighting. If 1 herself does not fight while the other two so do, 1 has more fighting power ($+\varepsilon$). The fourth and fifth line refer to the cases where two countries fight against a third, but not against each other. Then, the losing country obtains the payoff of 0 and the winning countries share the losing country's territory.

Comparing payoffs leads to the following theorem:

THEOREM VIII.3. *Assume the symmetric 3-country neighbouring structure together with the utilities given above. Then, no country is a steadfast friend or a steadfast enemy of any other country. In particular, country* 1

- **a):** *is a friend of country* 3 *(*2*) against* 2 *(*3*) at* $\{2 \leftrightarrow 3\}$ *by* $\varepsilon + \delta < \frac{1}{2}$,
- **b):** *is an enemy of country* 2 *(*3*) at* $\mathcal{F} = \{2 \leftrightarrow 3\}$ *by* $\varepsilon + \delta < \frac{1}{2}$,
- **c):** 1 *is not an enemy of country* 2 *(*3*) at* $\mathcal{F} = \emptyset$ *by* $\varepsilon + \delta > 0$.

Somewhat similar to the asymmetric 3-country case, 1 likes to attack 2 at the fighting structure $\{2 \leftrightarrow 3\}$. However, due to the symmetric neighbourhood structure, 1 also likes to attack 3 at this fighting structure.

We now turn to the equilibria in our 3-country example. They lead to the empty fighting structure (if no country attacks any other) or to all-out fighting:

THEOREM VIII.4. *In the symmetric 3-country example, we have 14 equilibria:*

- **a):** *the no-attack equilibrium* s^* *given by* $\mathcal{F}(s^*) = \emptyset$,
- **b):** *the trivial equilibrium* s^* *leading to* $\mathcal{F}(s^*) = \mathcal{N}$,
- **c):** *the four equilibria* s^* *leading to* $\mathcal{F}(s^*) = \{1 \leftrightarrow 2, 2 \leftrightarrow 3\}$ *and given by* $s_1^* = (a_1^2 = y, a_1^3 = n)$, $s_3^* = (a_3^1 = n, a_3^2 = y)$ *and any* s_2^*

together with the $2 \times 4 = 8$ *analogous equilibria* s^* *leading to the fighting structures* $\mathcal{F}(s^*) = \{1 \leftrightarrow 3, 2 \leftrightarrow 3\}$ *and* $\mathcal{F}(s^*) = \{1 \leftrightarrow 2, 1 \leftrightarrow 3\}$.

In both theorems, fighting structures such as $\{1 \leftrightarrow 2, 2 \leftrightarrow 3\}$ are important. For country 1, country 3 is a "friendly neighbour" in Kauṭilya's sense as "one undertaking a campaign simultaneously with the conqueror with the same objective" (Rangarajan (1992, p. 556)).

4.3. The 4-country example. We now turn to four countries $i = 1, ..., 4$ where each country has two neighbours (see figure 2). The 4-country example is meant as a stylized model of Indian four-handed chess.

The payoff assumptions lead to these payoffs for country 1 (with analogous payoffs for the other countries):

$$u_1(\mathcal{F}) = \begin{cases} 1, & \mathcal{F} = \emptyset & \text{(i)} \\ 1+\varepsilon, & \mathcal{F} = \{2 \leftrightarrow 3\} \text{ or } \mathcal{F} = \{3 \leftrightarrow 4\} & \text{(ii)} \\ 1-\varepsilon, & \mathcal{F} = \{2 \leftrightarrow 3, 3 \leftrightarrow 4\} & \text{(iii)} \\ 1-\delta, & \mathcal{F} = \{1 \leftrightarrow 2, 3 \leftrightarrow 4\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 4, 2 \leftrightarrow 3\} \text{ or } \mathcal{F} = \mathcal{N} & \text{(iv)} \\ 1-\delta-\varepsilon & \mathcal{F} = \{1 \leftrightarrow 2\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 4\} & \text{(v)} \\ \frac{3}{2}-\delta+\varepsilon, & \mathcal{F} = \{1 \leftrightarrow 2, 2 \leftrightarrow 3\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 4, 3 \leftrightarrow 4\} & \text{(vi)} \\ 0, & \mathcal{F} = \{1 \leftrightarrow 2, 1 \leftrightarrow 4\} & \text{(vii)} \\ 0 & \mathcal{F} = \{1 \leftrightarrow 2, 1 \leftrightarrow 4, 2 \leftrightarrow 3\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 2, 1 \leftrightarrow 4, 3 \leftrightarrow 4\} & \text{(viii)} \\ 2-\delta & \mathcal{F} = \{1 \leftrightarrow 2, 2 \leftrightarrow 3, 3 \leftrightarrow 4\} \text{ or } \mathcal{F} = \{1 \leftrightarrow 4, 2 \leftrightarrow 3, 3 \leftrightarrow 4\} & \text{(ix)} \end{cases}$$

where we assume $\varepsilon > \delta < \frac{1}{2}$ and (i) through (ix) can be seen from

(i)	no fighting
(ii)	two others fight
(iii)	3 loses against 2 and 4
(iv)	all neighbours fight
(v)	1 fights against 2 or 4
(vi)	1 joins 3 to win against 2 or 4
(vii)	1 loses against 2 and 4
(viii)	1 and 2 lose, or 1 and 4 lose
(ix)	1 and 4 win, or 1 and 2 win

For example, consider the second line where two other countries (2 and 3, or 3 and 4) fight. There is a draw between them and they suffer the costs of fighting δ. Hence, player 1 is stronger than the fighting pair of countries and his future fighting power is $1+\varepsilon$. In line (vi), country 1 joins country 3 to attack country 2 (or country 4). 1 and 3 then share country 2 's (or country 4's) territory. In the last line, both countries 2 and 3 (or both countries 3 and 4) lose so that country 1 obtains one extra full territory.

Theorem VIII.5. *Assume the symmetric 4-country structure together with the utility* u_1 *given above. Then, country* 1 *is not a steadfast friend of* *any other country nor a steadfast enemy of any other country. In particular, country* 1

- **a):** *is a friend of country* 3 *against* 2 *(against* 4*) at* $\{2 \leftrightarrow 3\}$ *(at* $\{3 \leftrightarrow 4\}$*) by* $\delta < \frac{1}{2}$,
- **b):** *is an enemy of country* 2 *(of country* 4*) at* $\{2 \leftrightarrow 3\}$ *(at* $\{3 \leftrightarrow 4\}$*) by* $\delta < \frac{1}{2}$,
- **c):** *might be, but need not be, a friend of country* 3 *against* 2 *(against* 4*) at* $\{2 \leftrightarrow 3, 3 \leftrightarrow 4\}$ *(*1 *turns against either* 2*, or* 4*),*
- **d):** *might be, but need not be, an enemy of* 2 *(of* 4*) at* $\{2 \leftrightarrow 3, 3 \leftrightarrow 4\}$ *(*1 *turns against either* 2*, or* 4*),*
- **e):** *is not a friend of country* 3 *against* 2 *at* $\{1 \leftrightarrow 4, 2 \leftrightarrow 3, 3 \leftrightarrow 4\}$ *(or against* 4 *at* $\{1 \leftrightarrow 2, 2 \leftrightarrow 3, 3 \leftrightarrow 4\}$*),*
- **f):** *is not an enemy of country* 2 *(or country* 4*) at* $\mathcal{F} = \emptyset$.

Is the enemy of my enemy my friend? Consider the fighting structure $\{2 \leftrightarrow 3\}$. Then, 1 is a friend of 3 against 2 (see a)) and an enemy of 2 (see b)). In that situation, the enemy of country 1's enemy is a friend. We can argue in a similar manner for the fighting structure $\{3 \leftrightarrow 4\}$. However, e) shows an example where Kauṭilya's conjecture does not hold:

- 4 is 1's enemy by $1 \leftrightarrow 4$,
- 3 is 4's enemy by $3 \leftrightarrow 4$, but
- 3 is not 1's friend because 1 is not prepared to attack 2.

Turn now to c) versus d) at fighting structure $\mathcal{F} = \{2 \leftrightarrow 3, 3 \leftrightarrow 4\}$. If country 1 is a friend of country 3 against 2 at $\mathcal{F}$, country 1 is an enemy of country 2. Again, the enemy of country 1's enemy is her friend.

Let us now report the equilibria for the symmetric 4-country example, some of which are asymmetric:

Theorem VIII.6. *In the symmetric 4-country example, we have* 20 *equilibria:*

- **a):** *the trivial equilibrium* s^* *leading to* $\mathcal{F}(s^*) = \mathcal{N}$,
- **b):** *the no-attack equilibrium* s^* *given by* $\mathcal{F}(s^*) = \emptyset$,
- **c):** *the equilibrium* s^* *with fighting pairs* $1 \leftrightarrow 2$ *and* $3 \leftrightarrow 4$*, i.e.,*

$$\begin{aligned} s_1^* &= \left(a_1^2 = y, a_1^3 = n, a_1^4 = n\right), \\ s_2^* &= \left(a_2^1 = y, a_2^3 = n, a_2^4 = n\right), \\ s_3^* &= \left(a_3^1 = n, a_3^2 = n, a_3^4 = y\right), \\ s_4^* &= \left(a_4^1 = n, a_4^2 = n, a_4^3 = y\right) \end{aligned}$$

  together with the analogous equilibrium s^* *with two fighting pairs* $1 \leftrightarrow 4$ *and* $2 \leftrightarrow 3$,
- **d):** *the* $2 \times 2 = 4$ *asymmetric equilibria* s^* *given by* $\mathcal{F}(s^*) = \{1 \leftrightarrow 2, 2 \leftrightarrow 3, 3 \leftrightarrow 4\}$ *and* $a_2^3 = y$ *and* $a_3^2 = y$*, where* $1 \leftrightarrow 2$ *(and also* $3 \leftrightarrow 4$*) may come about by both countries attacking or country* 1 *attacking country* 2 *(country* 4 *attacking country* 3*).*

together with the analogous $3 \times 4 = 12$ *equilibria with no fighting between* 1 *and* 2, 2 *and* 3, *or* 3 *and* 4, *respectively.*

We want to stress asymmetric equilibria s^* leading to fighting structures such as $\mathcal{F}(s^*) = \{1 \leftrightarrow 2, 2 \leftrightarrow 3, 3 \leftrightarrow 4\}$. Country 1, together with country 4, manages to overwhelm countries 2 and 3. Maybe, country 1's king (advised by Kauṭilya's Arthaśāstra) was clever enough to make countries 2 and 3 fight against each other, giving countries 1 and 4 the decisive advantage. Indeed, in a somewhat different context, Kauṭilya advises to "sow dissension between the enemy and the enemy's allies" (Rangarajan (1992, p. 564)).

5. Conclusions

In our mind, this chapter achieves two aims. First of all, it provides a formal structure within which we can discuss Kauṭilya's maṇḍala theory. Loosely speaking, Kauṭilya's conjecture holds most of the time.

Second, this chapter is meant to provide further evidence to the interesting thesis supported by Schetelich (???a) that Indian four-handed chess reflects Kauṭilya's maṇḍala model. We therefore take Schetelich's side against Thieme (1994, p.19) who considers four-handed chess an "unrealistic construction".

Four-handed chess is as "unrealistic" as models in economics and elsewhere tend to be. It was deemed to be realistic enough so that princes could be taught the tricks of coping with friendly and unfriendly neighbouring countries. In a similar manner, Wiese (2012) claims that Indian princes were meant to learn backward induction by way of animal tales.

Our simple formal model necessarily falls short of Kauṭilya's maṇḍala theory in many respects, some of which we like to point out:

- Our model does not give advice to any specific king. However, some of the equilibria are asymmetric even in the symmetric models. Then, it is quite natural to interpret the equilibria in the sense of strategic advice, as we have done at the end of subsections 4.2 and 4.3.
- Second, Kauṭilya's maṇḍala theory is clearly dynamic in nature. We try to capture the gist of his theory by way of a static model, where ε reflects the dynamic aspects. Still, a formal dynamic model would do more justice to Kauṭilya and might bring out results that our static version suppresses. In particular, a dynamic model might show that friendship (even the steadfast variety) may turn into enmity. As noted by Schetelich (???a, p. 6): "In the long run, the friendship *per definitionem* has to be sacrificed by the leader king on the altar of his leadership in his "Circle of Kings"."
- Third, while we define a general framework within which Kauṭilya's maṇḍala theory can fruitfully be analyzed and discussed (or so we flatter ourselves), we surely do not cover all important aspects even

in a static model. For example, middle and neutral kings play an important role in Kauṭilya's thinking. Hopefully, the current author or some other authors will take up the challenge to analyze neighbourhood structures with uneven fighting power.

6. Solutions

Exercise VIII.1

In case of two countries, there are only two fighting structures: F= $\emptyset$ (no fighting), F= $\{1 \leftrightarrow 2\}$.

Exercise VIII.3

Country 1's payoff is given by

$$u_1(\mathcal{F}) = \begin{cases} 1, & \mathcal{F} = \emptyset & \text{no fighting} \\ 1-\delta, & \mathcal{F} = \{1 \leftrightarrow 2\} & \text{1 fights} \end{cases}$$

Thus, 1 is 2's enemy at $\mathcal{F} = \{1 \leftrightarrow 2\}$ (actual fighting takes place), but not at $\mathcal{F} = \emptyset$ (1 prefers not to fight).

Exercise VIII.2

Since country 1 has one neighboring country, only, she has two strategies:

- attack country 2 (strategy $s_1 = \left(a_1^2\right) = \text{y}$)
- do not attack country 2 (strategy $s_1 = \left(a_1^2\right) = \text{n}$)

Exercise VIII.4

There exist two equilibria:

- the no-attack equilibrium given by $s_1 = \left(a_1^2\right) = \text{n}$ and $s_2 = \left(a_2^1\right) = \text{n}$ and
- the fighting-equilibrium given by $s_1 = \left(a_1^2\right) = \text{y}$ and $s_2 = \left(a_2^1\right) = \text{y}$

$a_1^2 = \text{y}$ and $a_2^1 = \text{n}$ does not constitute an equilibrium. In that case the attacking country could deviate by not attacking and increase its payoff from $1-\delta$ to 1.

Part C

Others

CHAPTER IX

Pareto optimality, general equilibrium theory, and asymmetric information

1. Introduction

This is a large and somewhat over-ambitious chapter. It deals with many central economic concepts. Allocation of goods (who gets what) takes place in two different modes:

(1) Person-to-person, depicted by the Edgeworth box (see the next section)
(2) Impersonal trading based on prices, expounded by General Equilibrium Theory (GET), the subject matter of section 3

Both models are utopian in the sense that theft or cheating is absent. Actors behave according to *dharma* as in the First book of the *Mahābhārata* (see, for example, von Simson 2011, p. 47) where businessmen do not sell their produce by using wrong weights.

2. Pareto optimality and Edgeworth box

2.1. Pareto improvements. Economists are somewhat restricted when it comes to judgements on the relative advantages of economic situations. The reason is that ordinal utility (rank orders, only) does not allow for comparison of the utilities of different people.

However, situations can be ranked according to their Pareto efficiency (Vilfredo Pareto, Italian sociologue, 1848-1923). Situation 1 is called a Pareto superior to situation 2 if no individual is worse off in the first than in the second while at least one individual is strictly better off. Then, the move from 2 to 1 is called a Pareto improvement. Situations are called Pareto efficient, Pareto optimal or just efficient if Pareto improvements are not possible.

Exercise IX.1. *Define Pareto optimality by way of Pareto improvements.*

Exercise IX.2. *a) Is the redistribution of wealth a Pareto improvement if it reduces social inequality?*
b) Can a situation be efficient if one individual possesses everything?

This chapter rests on the premise that bargaining leads to an efficient outcome under ideal conditions. As long as Pareto improvements are available, there is no reason (so one could argue) not to "cash in" on them.

However, the existence of Pareto improvements does not make their realization a forgone conclusion. This is obvious from the prisoners' dilemma.

2.2. Exchange Edgeworth box. We consider agents or households that consume bundles of goods. A distribution of such bundles among all households is called an allocation. In a two-agents two-goods environment, allocations can be visualized via the Edgeworth box. Exchange Edgeworth boxes allow to depict preferences by the use of indifference curves.

The analysis of bargaining between consumers in an exchange Edgeworth box is due to Francis Ysidro Edgeworth (1845-1926). Edgeworth is the author of a book with the beautiful title "Mathematical Psychics" (1881). Fig. 1 represents the exchange Edgeworth box for goods 1 and 2 and individuals A and B. The exchange Edgeworth box exhibits two points of origin, one for individual A (bottom left corner) and another one for individual B (top right).

Every point in the box denotes an allocation: how much of each good belongs to which individual. One possible allocation is the (initial) endowment. For all allocations $\left(x^A, x^B\right)$ with $x^A = \left(x_1^A, x_2^A\right)$ for individual A and $x^B = \left(x_1^B, x_2^B\right)$ for individual B we have

$$\begin{aligned} x_1^A + x_1^B &= \omega_1^A + \omega_1^B \text{ and} \\ x_2^A + x_2^B &= \omega_2^A + \omega_2^B. \end{aligned}$$

Individual A possesses an endowment $\omega^A = \left(\omega_1^A, \omega_2^A\right)$, i.e., ω_1^A units of good 1 and ω_2^A units of good 2. Similarly, individual B has an endowment $\omega^B = \left(\omega_1^B, \omega_2^B\right)$.

EXERCISE IX.3. *Do the two individuals in fig. 1 possess the same quantities of good* 1*, i.e., do we have* $\omega_1^A = \omega_1^B$*?*

EXERCISE IX.4. *Interpret the length and the breadth of the Edgeworth box!*

Seen from the respective points of origin, the Edgeworth box depicts the two individuals' preferences via indifference curves. Refer to fig. 1 when you work on the following exercise.

EXERCISE IX.5. *Which bundles of goods does individual A prefer to his endowment? Which allocations do both individuals prefer to their endowments?*

The two indifference curves in fig. 1, crossing at the endowment point, form the so-called exchange lens which represents those allocations that are Pareto improvements to the endowment point. A Pareto efficient allocation

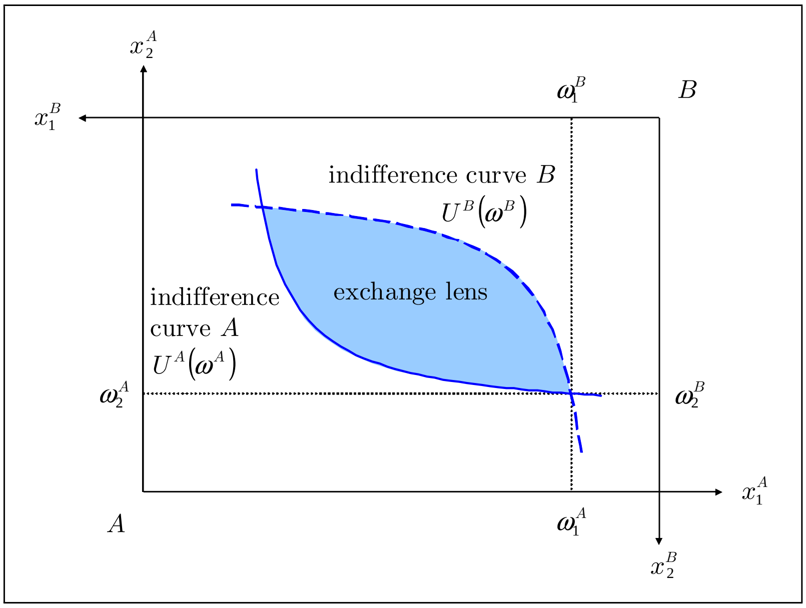

FIGURE 1. The exchange Edgeworth box

is achieved if no further improvement is possible. Then, no individual can be made better off without making the other worse off. Oftentimes, we imagine that individuals achieve a Pareto efficient point by a series of exchanges. As long as a Pareto optimum has not been reached, they will try to improve their lots.

EXERCISE IX.6. *Sketch an inequitable Pareto optimum in an exchange Edgeworth box. Is the relation "allocation x is a Pareto improvement over allocation y" complete (see definition II.1, p. 9)?*

Finally, we turn to the equality of the marginal rates of substitution. Consider an exchange economy with two individuals A and B where the marginal rate of substitution of individual A is smaller than that of individual B:

$$(3 =) \left| \frac{dx_2^A}{dx_1^A} \right| = MRS^A < MRS^B = \left| \frac{dx_2^B}{dx_1^B} \right| (= 5)$$

We can show that this situation allows Pareto improvements. Individual A is prepared to give up a small amount of good 1 in exchange for at least MRS^A units (3, for example) of good 2. If individual B obtains a small amount of good 1, he is prepared to give up MRS^B (5, for example) or less units of good 2. Thus, if A gives one unit of good 1 to B, by $MRS^A < MRS^B$ individual B can offer more of good 2 in exchange than individual A would require for compensation. The two agents might agree on 4 units so that both of them would be better off. Thus, the above inequality signals the possibility of mutually beneficial trade.

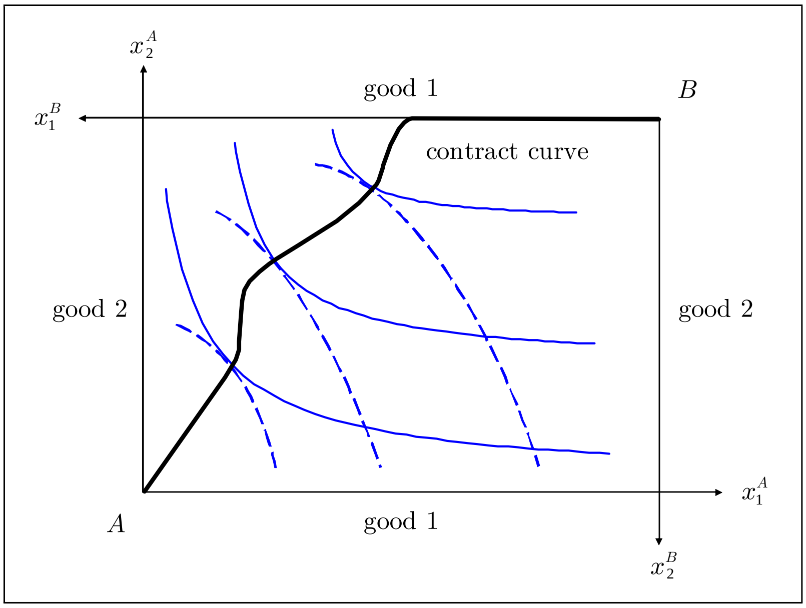

FIGURE 2. The contract curve

Differently put, Pareto optimality requires the equality of the marginal rates of substitution for any two agents A and B and any pair of goods 1 and 2. The locus of all Pareto optima in the Edgeworth box is called the contract curve or exchange curve (see fig. 2).

3. General equilibrium theory

3.1. Introductory remarks. General equilibrium theory (GET) envisions a market system with perfect competition. This means that all agents (households and firms) are price takers. The aim is to find prices such that

- all actors behave in a utility, or profit, maximizing way and
- the demand and supply schedules can be fulfilled simultaneously.

In that case, we have found a Walras equilibrium. Note that the price-finding process is not addressed in GET. A careful introduction into the General Equilibrium Theory is presented by Hildenbrandt & Kirman (1988).

Finding equilibrium prices for the whole economy is an ambitious undertaking. We need an elaborated mathematical mechanism and a list of restrictive assumptions:

- The goods are private and there are no external effects.
- The individuals interact via market transactions only.
- The individuals take prices as given.
- There are no transaction costs.
- The goods are homogeneous but there can be many goods.
- The preferences are monotonic and convex (and, of course, transitive, reflexive, and symmetric).

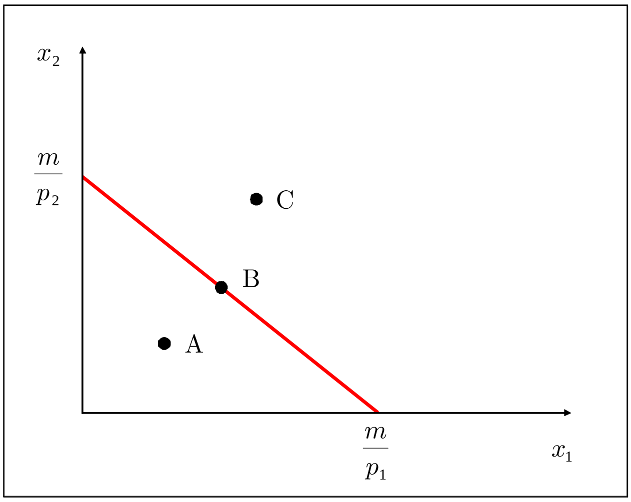

FIGURE 3. The budget for two goods

Following Hildenbrandt & Kirman (1988), it is helpful to differentiate between

- the implications of Pareto efficiency on the one hand (this is the Edgeworthian theme of cooperation) and
- the implications of individual utility and profit maximization for markets (the Walrasian theme of decentralization).

3.2. Household theory.

3.2.1. *Money budget.* We first assume that the household has some monetary amount m at his disposal. The budget is the set of good bundles that the household can afford, i.e., the set of bundles whose expenditure is not above m. The expenditure for a bundle of goods $x = (x_1, x_2)$ at (a vector of) prices $p = (p_1, p_2)$ is given by

$$p_1 x_1 + p_2 x_2$$

DEFINITION IX.1 (money budget). *For the money income m, the money budget is defined by*

$$B(p, m) := \{(x_1, x_2) : p_1 x_1 + p_2 x_2 \leq m\}$$

where

$$\{(x_1, x_2) : p_1 x_1 + p_2 x_2 = m\}$$

is called the budget line.

If the household does not consume good 1 ($x_1 = 0$), he can consume up to m/p_2 units of good 2. (Just solve the inequality for x_2.) In fig. 3, the household can afford bundles A and B, but not C.

EXERCISE IX.7. *Verify that the budget line's slope is given by $-\frac{p_1}{p_2}$ (in case of $p_2 \neq 0$).*

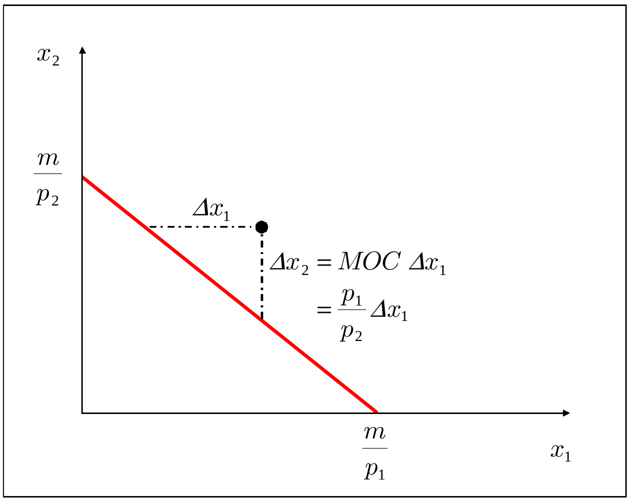

FIGURE 4. The opportunity cost of one additional unit of good 1

If both prices are positive, the budget line is negatively sloped.

DEFINITION IX.2. *If prices are non-negative and the price of good* 2 *is positive, the marginal opportunity cost of consuming one unit of good* 1 *in terms of good* 2 *is denoted by* $MOC(x_1)$ *and given by*

$$MOC(x_1) = \left|\frac{dx_2}{dx_1}\right| = \frac{p_1}{p_2}.$$

Thus, if the household wants to consume one additional unit of good 1, he needs to forgo MOC units of good 2 (see also fig. 4). Note that we use the absolute value of the budget line's slope – very similar to the definition of the marginal rate of substitution on pp. 20.

3.2.2. *Endowment budget.* In the previous section, the budget is defined by some monetary income m. We now assume that the household has some endowment (ω_1, ω_2) which it can consume or, at the prevailing prices, use to buy another bundle. In any case, we obtain the following definition:

DEFINITION IX.3. *For the endowment* (ω_1, ω_2)*, the endowment budget is defined by*

$$B(p, \omega) := \{(x_1, x_2) : p_1x_1 + p_2x_2 \leq p_1\omega_1 + p_2\omega_2\}$$

where

$$\{(x_1, x_2) : p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2\}$$

is the budget line.

The budget line is depicted in fig. 5.

3.3. The household's decision situation and problem. The household aims to find the highest indifference curve attainable with his budget.

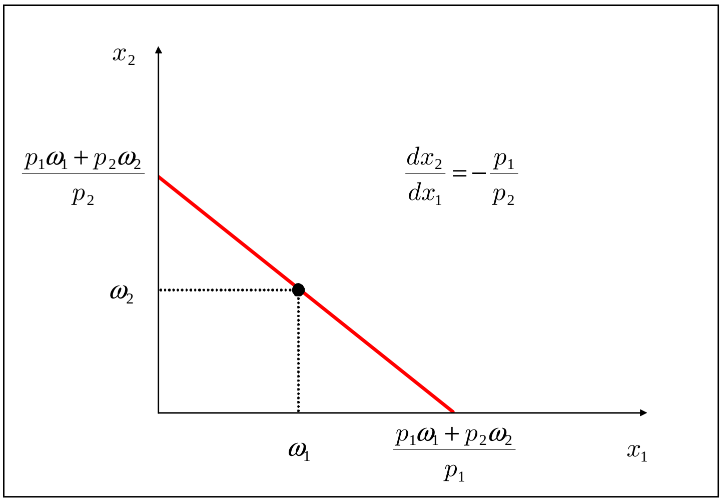

FIGURE 5. The endowment budget

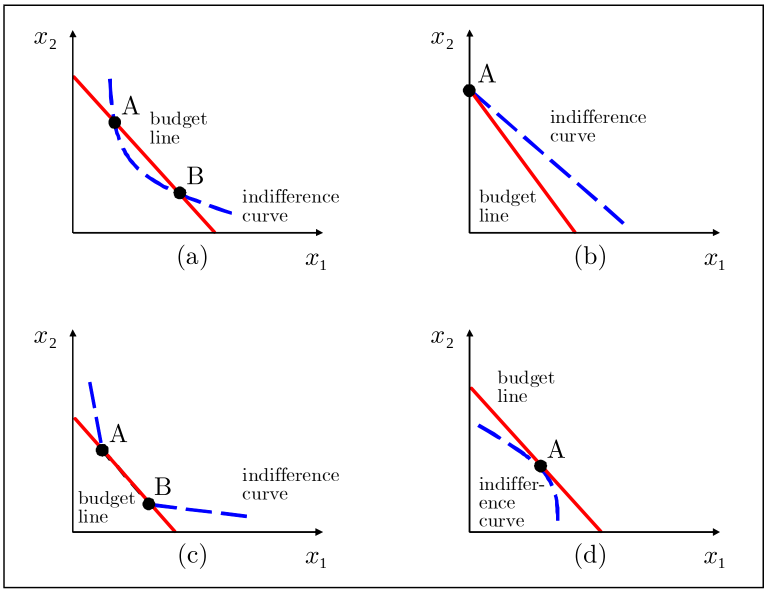

FIGURE 6. Household optima?

EXERCISE IX.8. *Look at the household situations depicted in fig. 6. Assume monotonicity of preferences. Are the highlighted points A or B optima?*

3.4. MRS versus MOC. A good part of household theory can be couched in terms of the marginal rate of substitution and the marginal opportunity cost. Consider fig. 7. We can ask two questions:

- What is the household's willingness to pay for one additional unit of good 1 in terms of units of good 2? The answer is MRS units of good 2.

Marginal willingness to pay:	$MRS = \left\| \frac{dx_2}{dx_1} \right\|$
If the household consumes one additional unit of good 1, how many units of good 2 can he forgo so as to remain indifferent.	movement on the indifference curve
Marginal opportunity cost:	$MOC = \left\| \frac{dx_2}{dx_1} \right\|$
If the household consumes one additional unit of good 1, how many units of good 2 does he have to forgo so as to remain within his budget.	movement on the budget line

FIGURE 7. Willingness to pay and opportunity cost

- What is the household's cost for one additional unit of good 1 in terms of units of good 2? The answer: MOC units of good 2.

Now, the interplay of the marginal rate of substitution MRS and marginal opportunity cost MOC helps to find the household optimum. Consider the inequality

$$MRS = \underbrace{\left|\frac{dx_2}{dx_1}\right|}_{\substack{\text{absolute value}\\ \text{of the slope of}\\ \text{the indifference}\\ \text{curve}}} > \underbrace{\left|\frac{dx_2}{dx_1}\right|}_{\substack{\text{absolute value}\\ \text{of the slope of}\\ \text{the budget line}}} = MOC.$$

If, now, the household increases his consumption of good 1 by one unit, he can decrease his consumption of good 2 by MRS units and still stay on the same indifference curve (compare fig. 8). However, the increase of good 1 necessitates a decrease of only $MOC < MRS$ units of good 2. Therefore, the household needs to give up less than he would be prepared to. In case of strict monotonicity, increasing the consumption of good 1 leads to a higher indifference curve.

Thus, the optimal bundle is given at

$$MRS \stackrel{!}{=} MOC$$

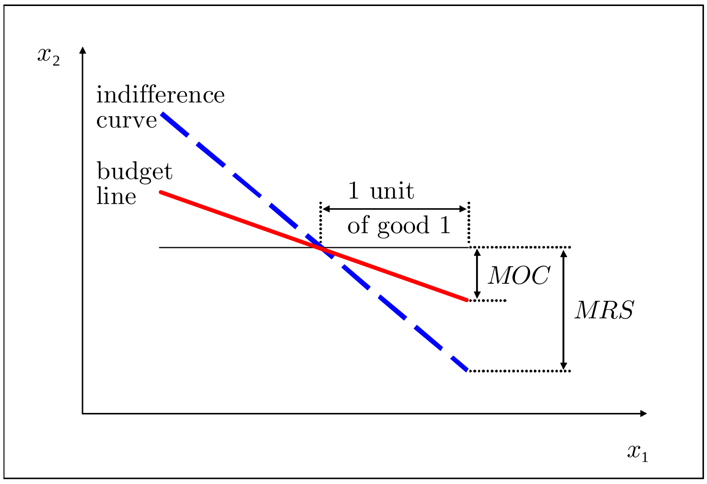

FIGURE 8. Not optimal

The MRS- versus-MOC rule can help to derive the household optimum for Cobb-Douglas utility functions (see chapter II, p. 19) that are given by $U(x_1, x_2) = x_1^a x_2^{1-a}$ with $0 < a < 1$. They lead to the marginal rate of substitution

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{a}{1-a}\frac{x_2}{x_1} \overset{!}{=} \frac{p_1}{p_2}.$$

Together with the budget line, we obtain the household optimum

$$x_1(m, p) = a\frac{m}{p_1},$$
$$x_2(m, p) = (1-a)\frac{m}{p_2}.$$

3.5. Exchange Edgeworth box: prices and equilibria. It is possible to add price information into Edgeworth boxes. If household A buys a bundle (x_1^A, x_2^A) with the same value as his endowment, we have

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A.$$

Starting from an endowment point, positive prices p_1 and p_2 lead to negatively sloped budget lines for both individuals. In fig. 9, two price lines with prices $p_1^l < p_1^h$ are depicted. The indifference curves indicate which bundles the households prefer.

Of course, we would like to know whether these prices are compatible in the sense of allowing both agents to demand the preferred bundle. If that is the case, the prices and the bundles at these prices constitute a Walras equilibrium.

The low price p_1^l is not possible in a Walras equilibrium, because there is excess demand for good 1 at this price:

$$x_1^A + x_1^B > \omega_1^A + \omega_1^B.$$

Do you see that? How about good 2?

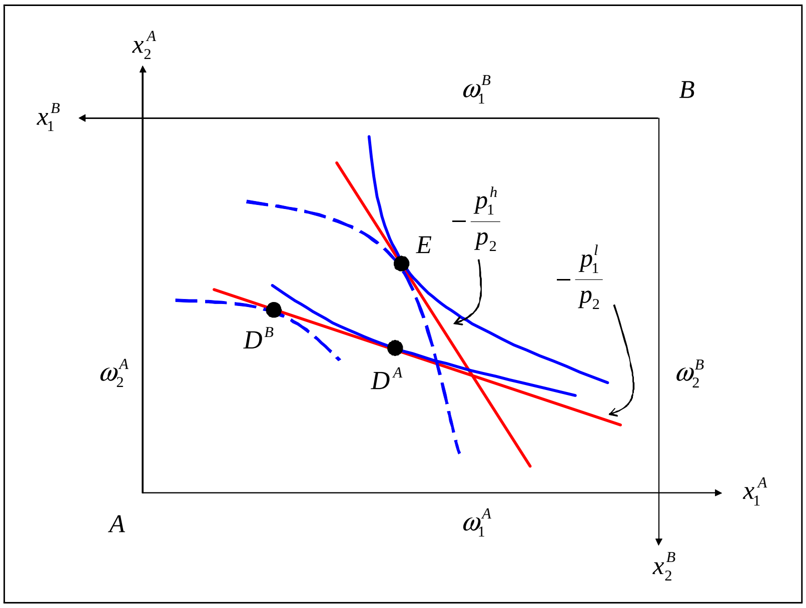

FIGURE 9. Walras equilibrium

3.6. Results. GET produces two important results:

- Under certain conditions, a price vector can be found where
  - all actors behave in a utility, or profit, maximizing way and
  - the demand and supply schedules can be fulfilled simultaneously.
- The allocation resulting from that price vector is Pareto efficient. This is called the first welfare theorem and one reason why economists often think that markets are wonderful.

3.7. Remarks on GET. The general equilibrium analysis is an important part of economic theory. Using GET we can analyze any number of markets simultaneously. We mention these important points:

- GET checks whether equilibrium prices exist but does not explain how prices are formed. Thus, arbitrageurs or real estate agents (as in "real" markets) have no role to play.
- The quality of goods is no problem tackled by GET. Indeed, when people are prepared to pay a certain price for a good, they can be sure to obtain the quality agreed upon. Thus, all the problems with which the principal-agent theory deals are assumed away.
- Contracts in GET are complete and simple: Goods are exchanged for other goods or against money. In contrast, contracting in everyday life is seldom done on the basis of complete contracts. Bowles (2004, p. 10) claims that norms and power replace contracts. "An employment contract does not specify any particular level of effort, but the employee's work ethic or fear of job termination or

peer pressure from workmates may accomplish what contractual enforcement cannot."

- The strong point of the economic system, envisioned by GET, is the impersonal nature of economic transactions. Agents just have to observe the price vector and do not need to do complicated deals with other (possibly several) agents. Bowles (2004, p. 208) calls this a "utopian capitalism" in spite of the negligence of distributive justice. Indeed, GET depicts a utopian state of affairs in many respects: no theft, no quality problems, no market concentration.
- Pareto optimality and extreme inequality of consumption goods or income can go hand in hand.

4. Mistrust in fable collections

4.1. Introductory remark. In contrast to the utopian state of affairs depicted by GET, in the Pañcatantra, in the Hitopadeśa, and in the Arthaśāstra, it is noted at several places that people have reason to mistrust others. Indeed, Olivelle (2006, pp. 40) presents many examples that show the Pañcatantra's "central message": "craft and deception constitute the major art of business". Olivelle then says: "Deception, of course, is a double-edged sword; it is important to use it against others, but just as importantly one must guard against its use by others against oneself. So, in a sense, even the losers provide counter-examples; don't be like the bull Sanjivaka [see chapter VI, pp. 94, HW] ... and let others practice deception on you." (In a similar manner, backward induction is taught by pointing to actors who do not practice it, see chapter VII).

4.2. A wicked person's way of thinking. It is difficult to guess the intentions of other people:

> If you have to cross an impassable ocean, you have a boat;
> when darkness comes, you have a lamp;
> if there is no breeze, you have a fan;
> and if you have to calm maddened rut-blinded elephants, you have a goad (Treibstock, HW).
> Thus there is no problem in the world for which the Creator has not carefully invented some solution.
> But when it comes to countering a wicked person's way of thinking,
> it seems to me that even the Creator has failed in his efforts.
> (Torzsok (2007, p. 323))

4.3. Money incentives. If servants are not rewarded generously, they may not act in the interest of their king. Thus, the Hitopadeśa tells us that money incentives matter:

> "Whereever the king is, there too must the treasury be; there is no kingship without the treasury. And the king should give some of it to his servants. For who would not fight for a generous patron?
> For,
> A man is never a servant to another man, he is a servant to money, O king. Whether you are considered important or not also depends on money or lack of it."
> (Torzsok (2007, p. 393))
> "The servants of someone greedy will not fight, for he never shares the profit. And he who has greedy servants will be killed by them once they are bought by the enemy's gifts."
> (Torzsok (2007, p. 481))

In a similar vein, the Panca-tantra warns the readers:

> 'He is my friend!'–is that any reason to trust a scoundrel?
> 'I have done him a great many favors!'–that counts for nothing!
> 'This man is my very own relative!'–that's an old folk tale!
> People are driven by money alone, no matter how small.
> (Olivelle (2006, p. 271))

4.4. Selecting good servants. With respect to employing the servant Best-Hero:

> Your Majesty, employ him at this salary [an unusually high one, HW] for four days to ascertain his nature and whether he deserves the payment or not.
> (Torzsok (2007, p. 403))

We will see in chapter X (p. 147) that Best-Hero was worth every penny. Whom to employ:

> A brahmin, a warrior or a relative should never be appointed as treasurer. A brahmin would not be able to keep even the money that has already been obtained, however hard he tries. If a warrior were entrusted with money, he would surely wave his sword at you; and a relative would seize all your possessions on the grounds that they belong to the family.
> (Torzsok (2007, p. 271))

5. Mistrust in the Arthaśāstra

5.1. Sons. Kauṭilya and the writers he discusses were well aware of cheating: Even wives and sons can pose a danger to a king. This is how Kauṭilya sees the matter:

> The king can protect the kingdom only when he is protected from those close to him and from enemies, but first of all from his wives and sons. ...
>
> One who has a keen intellect, one whose intellect needs to be prodded [anspornen, HW], and one who has an evil intellect: These are the three varieties of sons. The one with a keen intellect, when he is being taught, understands and follows Law and Success [as a translation of *dharmārthau*, i.e., *dharma* and *artha,* HW]. The one whose intellect needs to be prodded understands but does not follow. The one with an evil intellect constantly pursues evil and detests Law and Success.
>
> If the latter is his only son, the king should try to get him to have a son, or get a "female-son" to bear sons. ... Never should he, however, install an only son who is undisciplined over the kingdom.
>
> (Olivelle (2013, pp. 88-90))

5.2. Hostages. In order to understand Kauṭilya's sophistication, let us quote his remarks on making a peace pact, in the translation of Olivelle (2013, pp. 323-324):

> Peace, pact, and hostage [Geisel, HW]; these have the same meaning, given that peace, pact, and hostage all create confidence in kings.
>
> "Truth or oath constitutes an unstable pact. A surety [Bürgschaft, HW] or a hostage constitutes a stable pact," so state the teachers. "No," says Kauṭilya. "Truth or oath constitutes a stable pact here and in the hereafter, while a surety or a hostage, depending on strength, is of use only here."
>
> ...
>
> The taking of a kinsman or a chief constitutes a hostage. In this event, the one who gives a traitorous minister or a traitorous offspring is the one who outwits.
>
> ...
>
> In giving an offspring as a hostage, however, as between a daughter and a son, the man who gives a daughter is the one who outwits; for a daughter is not a heir, is intended only for others, and cannot be tortured. A son has the opposite characteristics.
>
> Even between two sons, the man who gives a son who is legitimate, intelligent, brave, skilled in the use of arms, or a single son is the one who is outwitted.

6. Asymmetric information

6.1. Adverse selection. In microeconomic theory, trust and truth are dealt with under the heading of "asymmetric information". One agent knows something the other does not. As we have seen above, contracts or agreements (to choose one point in the Edgeworth box rather than another, to buy and sell for given prices) make all agents better off in the absence of cheating.

Thus, cheating is seen as a threat to mutually beneficial contracts and to efficiency. This aspect seems not to be present in the fable collection or in Kauṭilya.

6.2. Adverse selection. Two different model classes exist. In the first one, informational asymmetries are already present before the players decide whether or not to accept the contract or which contract to accept. The so-called adverse-selection models deal with these problems. For example,

- the ability of a worker is known to the worker (agent) but not to the firm (principal) who considers to hire the worker (see Best-Hero on p. 136 above),
- the car driver (agent) is better informed than the insurance company (principal) about the driver's accident-proneness, and, finally,
- the owner of a used car for sale (agent) may have a very good idea about the quality of that car while the potential buyer (principal) has not (somewhat similar to the bad and good hostages in Kauṭilya).

The problem of adverse selection is this: for a given wage, a given insurance premium, or a given price for a used car, the badly qualified workers, the high-risk insurees and the owners of bad cars are more eager to enter into a contract than the opposite types of agents. For the qualified workers have alternative employment possibilities, the low-risk insurees do not need the insurance as badly, and the good cars are of use to their owners. At first sight, the informational asymmetry is a problem for the badly informed party, the principal. However, the principal's problem immediately turns into a problem for the agent. It is the agent who needs to convince the principal that he is of a "good type".

Screening (or signalling) models show how good types can be distinguished from bad ones. For example, workers send signals by obtaining a university degree or other qualifications.

6.3. Hidden action. We now turn to hidden action. Here, the agent is to perform some action for the principal. For example, the insuree (agent) is careless about the insured object once he has obtained the insurance from the insurance company (principal). Another example: workers (or managers) do not exert the high effort that the manager (or the owners) expect. Thus, the asymmetry of information (has the worker exerted sufficient effort) occurs after the agent has been employed. This constellation is called a principal-agent situation or principal-agent problem.

Indeed, the problem arises because the output is assumed to be a function of both the agent's effort and of chance. Since the effort is not observable, the payment to the agent (as specified in the contract) is a function of the output, but not of effort.

Normally, the principal-agent problem is described as the principal's maximization problem subject to two conditions:

- The first is the agent's participation constraint. He enters into the contract only if he expects a payoff higher than his reservation utility. Once employed, the agent chooses among several actions.
- The action the principal would like to induce has to be a best action given the contract and the probability distribution dictated by nature. This second side condition is called incentive compatibility.

7. Social gods

7.1. A few important gods. The Indians have been worshipping many gods (see, for example, Gonda 1960), with changing characteristics, over the millennia. According to the Ṛgveda 4.92.4, there are 33 gods, 11 in heaven, earth, and water, respectively.

A detailed account on Vedic gods is provided by Oberlies (2012, chapter 5). Indra is one of the most important gods. He is especially known as the slayer of the demon Vṛta by which act the waters were freed. *Indra*'s world is a raw, unfinished business. It is *Varuṇa* who then determines the sun's orbit and the rivers' paths.

Analogously, *Indra* and *Varuṇa* are involved in the Vedic clans' living. Oberlies stresses the phases of *yoga* (yoking the horses in order to move to new areas in fighting mode) and *kṣema* (peaceful settlement). Indra is associated with *yoga*. He is invoked by the Vedic clans that hope for victory. In contrast, *Varuṇa* and other related gods see to the orderly function *kṣema*, of settled human society.

One of the most renowned indologist of the previous century was Paul Thieme (1905-2001). He is especially well-known for his work on (what might be called) social gods, i.e., gods that stand for social values. In particular, Thieme (1938) deals with the words for foreigner, enemy, and guest, while Thieme (1957) is concerned with contracts and truth-telling.

Thus, while the animal fables and the Arthaśāstra stress and even value "craft and deception", other parts of the Indian literature seem to be more modern in focusing on the advantages of telling the truth and of keeping contracts.

7.2. Mitra and Varuṇa. In classical Sanskrit, *mitram* is a neuter (!) noun meaning friend. Thieme (1957, p. 18) clearly sides with Antoine Meillet who claims that, in Vedic times, the meaning of *mitram* was "contract" from which the meaning of friendship and then friend developed. Thieme (the English citations are from the 1957 article) cites the Ṛgveda to support Meillet's and his claim:

- RV 3.59.1a:
  Contract, when named, makes peoples array (arrange) themselves [with regard to each other] (='causes them to make mutual arrangements') (Thieme, p. 39)

According to Thieme (p. 40), "Also other gods may receive this qualification: God Fire (Agni), the fire being invoked as a witness at the conclusion of certain contracts ...":

- RV 8.102.12c:
  [Fire,] who causes people to make mutual arrangements like Contract. (Thieme, p. 41)
- Or God Varuṇa, that is the personified Oath ... or, as I should prefer, the personified True Speech. (Thieme, p. 41)

Mitra and Varuṇa are often mentioned together:

- RV 5.72.2ab:
  You two (Mitra and Varuṇa, i.e., Contract and True-Speech) are of firm peace through vow (= you secure peace by seeing to it that vows are kept), you cause people to make mutual agreements through firmness (= you make contractual agreements desirable as establishing firm relations) (Thieme, p. 41??)
- RV 5.62.3:
  You two, king Contract and king True-Speech, made firm earth and heaven by your greatness. Cause plants to grow, cause cows to swell [with milk], send down rain, you of live wetness! (Thieme, p. 43)

Thieme (p. 43) comments: "The original motivation for their creating prosperity is, of course, that Contract and True-Speech secure peace." From an economic point of view, one may also add that contracts allow mutual gains from trade.

Of course, there must be some sanctions if somebody does not keep a contract:

- RV 10.152.1b-d:
  [Thou, o Indra, art] a miraculous crusher of those without contracts (who do not know or keep contracts) ... (Thieme, p. 45)
- RV 10.36.12bc:
  May we be without guilt against Contract and True-Speech, so that well-being prevail. (Thieme, p. 52)

Also, detection is important:

- RV 7.65.3ab:

> These two (Contract and True-Speech) have many slings (in which to catch a cunning transgressor), they are fetterers of untruth, difficult for the deceitful mortal to circumvent. (Thieme, p. 52)

7.3. Aryaman. "In classical Sanskrit arí is an unambiguous, very common term for 'enemy'." (Thieme 72) However, in the Ṛgveda, Thieme argues, arí is sometimes used in the sense of "guest". In his "Der Fremdling im Ṛgveda" (1938), Thieme claims "stranger" as the original underlying meaning of both enemy and, in RV, guest.

The god Aryaman means god Hospitality: "In my Fremdling 141-4 I have shown that the figure of God Aryaman in the RV becomes clear and consistent on the hypothesis that he is the personified and deified hospitality. He is the god who rewards the host, protects the guest, punishes those who act disgracefully (against guests) and watches over truth." (Thieme, p. 82)

Finally, it is interesting to take note of Thieme's claim that ār(i)ya (in English: aryan) was the term used by the Old Indians to describe themselves as people who are being hospitable to strangers.

8. Solutions

Exercise IX.1

A situation is Pareto optimal if no Pareto improvement is possible.

Exercise IX.2

a) A redistribution that reduces inequality will harm the rich. Therefore, such a redistribution is not a Pareto improvement.
b) Yes. It is not possible to improve the lot of the have-nots without harming the individual who possesses everything.

Exercise IX.3

No, obviously ω_1^A is much larger than ω_1^B.

Exercise IX.4

The length of the exchange Edgeworth box represents the units of good 1 to be divided between the two individuals, i.e., the sum of their endowment of good 1. Similarly, the breadth of the Edgeworth box is $\omega_2^A + \omega_2^B$.

Exercise IX.5

Individual A prefers all those bundels x_A that lie to the right and above the indifference curve that crosses his endowment point. The allocations preferred by both individuals are those in the hatched part of fig. 1.

Exercise IX.6

For the first question, you should have drawn something like fig. 10. Fig. 11 makes clear that we can have bundles A and B where A is no Pareto improvement over B and B is no improvement over A. Thus, the relation is not complete.

Exercise IX.7

Solving $p_1x_1 + p_2x_2 = m$ for x_2 yields $x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$ so that the derivative of x_2 (as a function of x_1) is $-\frac{p_1}{p_2}$.

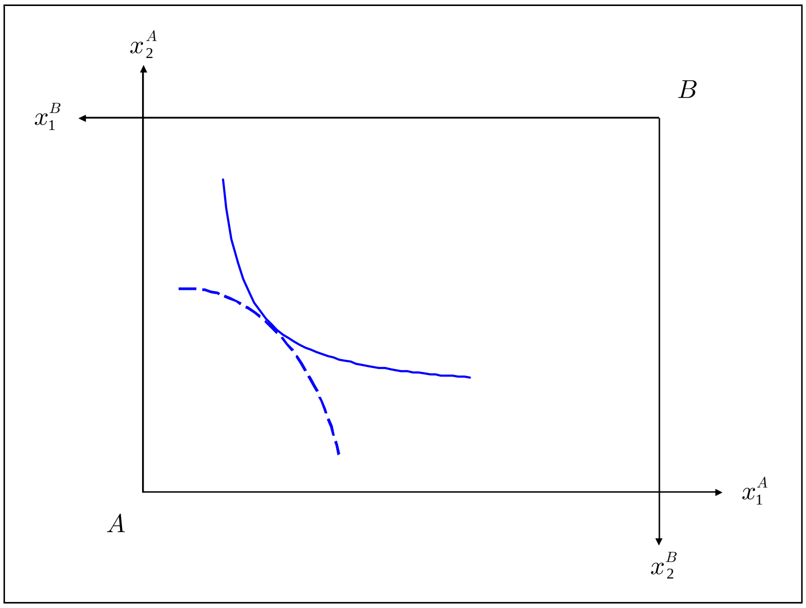

FIGURE 10. Pareto optimality and equality

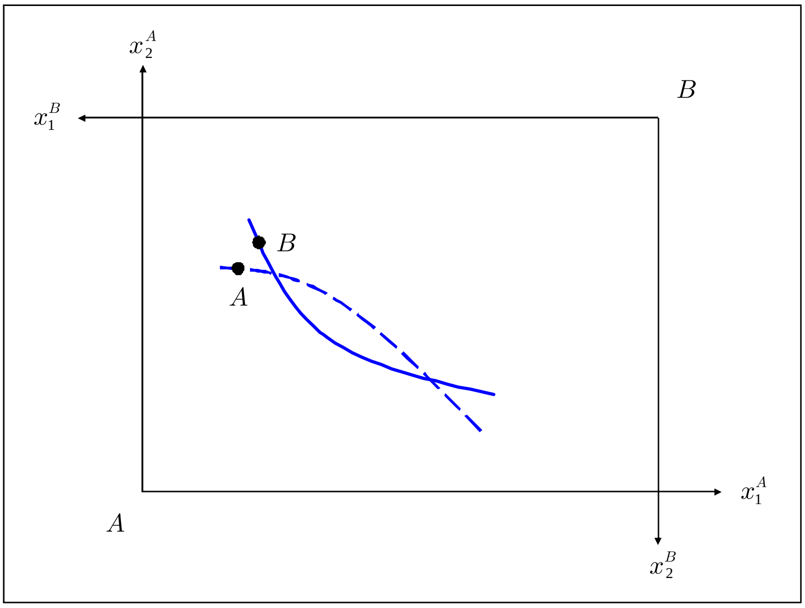

FIGURE 11. Incompleteness

Exercise IX.8

In subfigure (a), points A and B do not correspond to an optimum. The preferences are strictly convex and every point between A and B is better than A or B. Subfigure (b) depicts perfect substitutes. Point A is the household optimum. In subfigure (c), points A and B are optima but so are all the points in between. Turning to subfigure (d), the point of tangency A is the worst bundle of all the bundles on the budget line. There are two candidates for household optima in this case of concave preferences, the two extreme bundles $\left(\frac{m}{p_1}, 0\right)$ and $\left(0, \frac{m}{p_2}\right)$.

CHAPTER X

Altruism

1. Introduction

In a sense, altruism and hedonism stand for opposing attitudes or behaviors. In the next section, we briefly deal with the *Cārvāka* philosophy. We then turn to an isolated outbreak of altruism in the Hitopadeśa before turning to the Buddha's birth stories. Finally, we spell out in detail a simple economic theory of altruism.

2. *Cārvāka* philosophy

2.1. Much Ado About Religion. *Cārvāka* philosophy is often characterized as atheistic, non-Vedic, materialist, and hedonist (see chapter I). In the second part of Act three of the satirical play "Much Ado About Religion" (roughly 1100 years old), a Cārvāka hedonist argues for his points of view:

- Kings intent on staying in power should concentrate on worldly prosperity:

  "What a great disaster-making king Shankara·varman is, letting Vedic priests, hermits, renouncers, ..., Buddhists and the rest consume the great riches of his kingdom without check. ... I am going to take this opportunity to do away with God, set aside the world-to-come, demolish the validity of the Vedas, and thereby turn the king back from this wrong path and establish him on the right track, so that concentrating on worldly prosperity he can enjoy his kingship for a long time." (Dezsö (2009, pp. 151-153))
- Hedonistic plea against asceticism:

  "Why do you live so miserably because of hundreds of useless torments? Asceticism is just a variety of torture; self-restraint is just a way to cheat yourself of pleasures ..." (Dezsö (2009, p. 153))

However, a Shaiva abbot and his pupil argue convincingly (from the point of view of the writer of the play) for the existence of God and the authoritative nature of the Veda.

2.2. The Rise of the Wisdom Moon. In another play, "The Rise of the Wisdom Moon" (nearly 1000 years old), a *Cārvāka* philosopher also has the opportunity to advertise his belief:

- Political Science and Business Administration before the Vedas:

"As you know, Poli Sci's what's worth knowing, my boy! Business belongs here as well. But the three Vedas are crooks' patter [the babble of criminals, HW]!" (Kapstein (2009, p. 69))

- Worldly pleasures, including sexual ones, are to be preferred to penance:

  "DISCIPLE: Teacher! If eating and drinking were indeed man's highest truth, then why is it that these cultists reject worldly pleasures and wear themselves out with the horrible troubles of penance, self-mortification [Kasteiung, HW], fasting and what not?

  HEDONIST: The dopes [fools, HW] duped by cults contrived by crooks are satisfied with cakes from Never Never Land! Look, look,

  How can fools' emaciating [ausmergelnd, HW] restrictions–alms [Almosen, HW], fasting, rites of contrition [remorse, HW], and mortification under the sun–

  Compare with the embrace, arms pressed by arms entwined, of a sloe-eyed [mit Augen wie die Schlehe, HW] lass [girl, HW], pleasing you with firm, swollen nipples?" (Kapstein (2009, p. 71))

2.3. Mahābhārata (The book of peace). Interestingly, *Cārvāka* makes his appearance in the 12. book of the Mahābhārata (see Heera 2011, pp.19). He does not talk about pleasure, but seems to side the early Árjuna (see chapter IV). *Cārvāka* blames Yudhiṣṭhira for the Kurukṣetra battle: "What have you gained by destroying your own people and murdering your own elders?" Finally, *Cārvāka* is considered a demon in disguise and burned to ashes.

2.4. The birth-story of Brahma. In one of the Buddha's birth-stories, the birth-story of Brahma, the *Bodhi-sattva* shows pity towards a king with "extremely false views" which were of the *Cārvāka* variety:

"The king had reached the conclusion that the next world did not exist, let alone the ripening of the fruit of pure or impure deeds. As a result, any longing he may have felt for religious practices had been squashed. Turning his back on moral behavior such as giving or virtue, he felt a deep contempt for the pious and his lack of belief meant he held harsh opinions on religious teaching. Ridiculing stories about the next world, he showed little politness, courtesy, respect or honor toward ascetics or brahmins and was devoted to sensual pleasures. (all quotes in this subsection are from Meiland 2009b, pp. 269, 271).

The discussion between this king and the future Buddha has been dealt with in chapter III, pp. 43.

3. Altruism in the Hitopadeśa

Somewhat strangely, the Hitopadeśa has an isolated story about altruism, too (see the story and the quotes in Torzsok 2007, pp. 403-411). King *Śūdraka* once had a very well-paid servant called Best-Hero. The latter once discovers a crying woman, the king's fortune. She tells him that she can stay only if Best-Hero were to sacrifice his son to the Goddess of All Blessings. Best-Hero tells his wife and his son about the conversation. The latter is ready for the sacrifice:

"How fortunate I am that I can be of use in saving our master's kingdom! So what are we waiting for, father? It is praiseworthy to use this body of mine for such a noble cause when there is occasion to do so. For,

The wise do not hesitate to abandon their wealth and life for someone else; it is better to give them up for a good cause, for their loss is inevitable anyway."

After cutting of his son's head, the father does not see any reason for living himself and kills himself. Then, his wife follows their example. Luckily again, there is a happy end and the Goddess of all Blessings restores the family to life.

4. Altruism in Buddhist *jātaka*s

4.1. Introduction. One strand of the Old Indian literature consists of the *jātaka* stories (literally "birthstory"). In these stories, the Buddha is presented in his past lives that helped him to gain enlightment. Roughly speaking, these stories are concerned with (often extreme forms of) giving, virtue (for example, chapter III, pp. 43), and forbearance (see the related concept of endurance discussed in chapter II on p. 26). In this chapter, we concentrate on giving. Let us give a few examples:

- The birth-story of the tigress deals with a brahmin who gives himself as food for a young tiger mother (Meiland (2009a, pp. 7-25)).
- Similarly, an elephant commits suicide by falling down a mountain in order to give his flesh to a group of humans (Meiland (2009b, pp. 299-325)).
- The birth-story of Vishvan·tara is about a prince who engages in "heroic giving" that does not even stop at giving his children and his wife as a present. (Meiland (2009a, pp. 201-259))
- The birth story of the hare is expounded in the following subsection.

Virtue may not always mean altruism as becomes clear from the birth-story of the childless ascetic. The reasons for renouncing a householder's life may be virtuous, but certainly not devoid of egoism:

> Realizing the household life hindered virtue
> and involved the hardship of seeking wealth,
> whereas ascetic groves offered great happiness,

he remained unattached to household pleasures.

...

With its constant toil of acquiring and protecting wealth,
it is a prime target for murder, captivity, and misfortune.
Even a king is never satisfied by riches,
just as the ocean is never sated with showers of rain.

Why, how, or when is there any happiness in this world
for someone who has no desire for wisdom,
ignorantly believing happiness lies in pursuing desire,
like a person scratching a wound to make it heal?

(Meiland (2009a, pp. 415, 423))

4.2. The birth story of the hare. The motto of the birth story of the hare is given right in the beginning:

> Even as animals, Great Beings have displaced generosity to the best of their ability. Who then that is human would not give gifts?

This quote from the "Garland of the Buddha's Past Lives, Volume One" has been taken from Meiland (2009a, p. 111) and will henceforth be cited by "Past Lives, p. 111". The hare was an amazing creature:

His goodness, perfect beauty,
eminent strength, and abundant energy meant cruel animals
gave him no thought
as he roamed fearlessly with the grace of a king of beasts.

Using his skin as a deer hide,
and his fur as a bark garment,
he looked as glorious as an ascetic,
content with blades of grass.

His actions in mind, speech, and body
were so cleansed by kindness
that animals normally crooked and evil
became mild as disciples.
(Past Lives, p. 111)

When preaching to his friends (an otter, a jackal, and a monkey), he stresses the value of giving:

Strive to increase your merit
through giving, the ornament of virtue.
For merit is the best support for creatures
who wander the perils of rebirth.
(Past Lives, p. 115)

But then, he muses:

> The others can honor
> any guest that arrives
> with this or that offering.
> But my situation is pitiful.
>
> I cannot give a guest
> blades of bitter grass,
> chewed off by the tips of my teeth!
> How utterly powerless I am!
> ...
> I have something that belongs to myself
> and that is readily available.
> Inoffensive, it can be used to honor a guest.
> That possession is my body.
> (Past Lives, pp. 117, 119)

The gods liked this idea: "When Shakra, the lord of the gods, heard of it, his mind brimmed with astonishment and curiosity and he desired to test the Great Being's character." (Past Lives, p. 121) He pretends to be a travelling brahmin in distress. The hare's friends offer what they can, the otter seven carp, the jackal a lizard and a bowl of milk, and the monkey some mangoes. The hare, true to his intentions tells the false brahmin:

> A hare raised in the forest
> has no beans, sesame seeds, or grains of rice.
> But here is my body to cook on a fire.
> Enjoy it today and reside in this ascetic forest.
>
> At the joyous occasion of a beggar's arrival,
> one gives a possession to cater to their needs.
> I have no possessions other than my body.
> Please accept it. It is everything I own.
> (Past Lives, p. 125)

After the brahmin utters some protest, the hare insists:

> Giving is a duty and my heart wishes to give.
> And it is apt when I have a guest such as you.
> An opportunity like this cannot easily be gained.
> I rely on you to ensure my gift is not in vain.
> (Past Lives, p. 127)

And then the hare jumps into the fire. Luckily, the god rescues him. He praises the hare:

> Look you gods who dwell in heaven! And rejoice in the astonishing feat of this Great Being!
>
> See how, in his love of guests,
> this creature gave up his body without attachment,
> while those of unsturdy nature cannot discard
> even a used garland without quivering!
> And it is apt when I have a guest such as you.
> An opportunity like this cannot easily be gained.
>
> His noble generosity and sharp mind
> seem so contradictory to his animal birth!
> His deed is a clear rebuke to both gods and men
> who have weak regard for merit.
>
> To proclaim the Great Being's exceptional deed ... Shakra then adorned an image of the hare ... on the disc of the moon.
>
> (Past Lives, p. 129)

Indeed, *śaśas* means "hare" and *śaśī* (with stem *śaśin*) is another word for the "moon".

4.3. Efficiency in altruism. One last remark: the hare begged the traveller to ensure that his "gift is not in vain". From an economic point of view, a vain gift would be inefficient. A similar idea crops up in the birth story of the elephant. After the former *Buddha* had killed himself to offer his flesh to the destitute travelers, some of them have this noble idea:

> Who could possibly eat the flesh of this virtuous being, who was so determined to help us that he sacrificed his very life for our benefit, showing us greater affection than a loving relative or friend? We should instead repay our debt to him by honoring him with a cremation and due rites of worship.(Meiland (2009b, p. 321))

Others, obviously in consent with the narrator, argue against this view:

> For it was to save us that
> this unknown kinsman
> sacrificed his body,
> his guests dearer to him still.
>
> We should then fulfill his wishes,
> or his efforts will be in vain.
>
> Such was the affection he gave
> all he had as his guest-offering.

Who would invalidate this act
of honor by not accepting it?

(Meiland (2009b, p. 323))

5. The Stark 1993 model of altruism

5.1. The basic setup. In order to make some sense out of the hare's behavior, we build on the simple framework provided by Stark (1993). Consider two agents who are labeled father and son and who consume "corn" denoted by C_F and C_S, respectively. The consumption leads to direct pleasure V (called felicity by Stark) which is a function of consumption $C \geq 0$. However, the agents do not only care about their own consumption but also about the other agent's consumption:

$$\begin{aligned} U_F(C_F, C_S) &= \beta_F V_F(C_F) + \alpha_F V_S(C_S) \text{ and} \\ U_S(C_F, C_S) &= \beta_S V_S(C_S) + \alpha_S V_F(C_F). \end{aligned}$$

We assume

$$\begin{aligned} \frac{dV}{dC} &> 0 \text{ and} \\ \beta_F, \beta_S &> 0, \end{aligned}$$

i.e., preferences are strictly monotonic (in Stark's terminology, masochism is excluded). The β are called felicity factors.

α_F (and α_S) express the level of altruism felt by the father for the son (and vice versa). Indeed, let us call preferences with

- $\alpha > 0$ altruistic or benevolent,
- $\alpha < 0$ malevolent (in Stark's words: envious),
- $\alpha = 0$ neutral.

Thus, the typical microeconomic model (with $\alpha = 0$) does not represent vicious or especially egotistic preferences, but the neutral case. The biblical command to "love thy neighbor as yourself" implies

$$\beta = \alpha$$

(and the possibility to compare utilities).

We follow Stark in assuming

$$\begin{aligned} V_F(C_F) &= \ln(C_F), \\ V_S(C_S) &= \ln(C_F). \end{aligned}$$

We also assume that overall consumption of corn is given by C. Thus, the two agents have to decide on how to divide $C = C_F + C_S$ among themselves. Thus, we have

$$U_F(C_F) = \beta_F \ln(C_F) + \alpha_F \ln(C - C_F)$$

for the father's utiliy.

We define a conflict measure

$$conf = \frac{C_F^* + C_S^*}{C}$$

where the optimal values are indicated by *. We define

$$conf \begin{cases} < 1, & \text{altruistic conflict} \\ = 1, & \text{agreement} \\ > 1, < 2 & \text{mild egoistic conflict} \\ = 2, & \text{extreme egoistic conflict} \end{cases}$$

5.2. Results. It is obvious from inspecting

$$U_F(C_F) = \beta_F \ln(C_F) + \alpha_F \ln(C - C_F)$$

that $\alpha_F \leq 0$ implies $C_F = C$ is the utility maximizing consumption. The benevolent case is more difficult:

LEMMA X.1. *The first derivative of U_F with respect to C_F is given by*

$$\begin{aligned} \frac{dU_F(C_F)}{dC_F} &= \frac{\beta_F}{C_F} + \frac{\alpha_F}{[C - C_F]}(-1) \\ &= \frac{\beta_F}{C_F} - \frac{\alpha_F}{C - C_F} \\ &= \beta_F C_F^{-1} - \alpha_F (C - C_F)^{-1} \end{aligned}$$

and we have

$$\begin{aligned} \frac{dU_F(C_F)}{dC_F} &= 0 \Leftrightarrow C_F^* = \frac{\beta_F}{\alpha_F + \beta_F} C \\ &\Leftrightarrow \left(\frac{C_F^*}{C_S}\right)_F = \frac{\beta_F}{\alpha_F} \end{aligned}$$

The second derivative is

$$\begin{aligned} \frac{d^2U_F(C_F)}{(dC_F)^2} &= -\frac{\beta_F}{C_F^2} - (-)\frac{\alpha_F}{(C - C_F)^2}(-1) \\ &= -\frac{\beta_F}{C_F^2} - \frac{\alpha_F}{(C - C_F)^2} \end{aligned}$$

which is smaller than zero in case of $\alpha_F \geq 0$.

Applying the above lemma to an altruistic son means

$$\left(\frac{C_F}{C_S^*}\right)_S = \frac{\alpha_S}{\beta_S}.$$

LEMMA X.2. *For $\alpha_F > 0$ and $\alpha_S > 0$, we obtain*

$$\begin{aligned} \left(\frac{C_F^*}{C_S}\right)_F &> \left(\frac{C_F}{C_S^*}\right)_S \Leftrightarrow \frac{\beta_F}{\alpha_F} > \frac{\alpha_S}{\beta_S} \\ &\Leftrightarrow \beta_F \beta_S > \alpha_S \alpha_F \\ &\Leftrightarrow conf > 1 \end{aligned}$$

Proof: $\beta_F \beta_S > \alpha_S \alpha_F$ implies

$$\begin{aligned} conf &= \frac{C_F^* + C_S^*}{C} = \frac{\beta_F}{\alpha_F + \beta_F} + \frac{\beta_S}{\alpha_S + \beta_S} \\ &= \frac{1}{\frac{\alpha_F \beta_S}{\beta_F \beta_S} + 1} + \frac{\beta_S}{\alpha_S + \beta_S} \\ &> \frac{1}{\frac{\alpha_F \beta_S}{\alpha_S \alpha_F} + 1} + \frac{\beta_S}{\alpha_S + \beta_S} = 1 \end{aligned}$$

and $conf > 1$ leads to

$$1 < \frac{C_F^* + C_S^*}{C} = \frac{C_F^*}{C} + \frac{C_S^*}{C}$$

and hence

$$C_F^* > C - C_S^*.$$

Thus, the father wants more for himself than the son is prepared to offer. This also implies that the father will offer the son less than what the son wants for himself.

Proposition X.1. *In case of benevolent preferences, we find*

- *that son and father agree on increasing the father's share in case of* $\frac{C_F}{C_S} < \min\left(\left(\frac{C_F}{C_S^*}\right)_S, \left(\frac{C_F^*}{C_S}\right)_F\right)$,
- *that son and father agree on decreasing the father's share in case of* $\max\left(\left(\frac{C_F}{C_S^*}\right)_S, \left(\frac{C_F^*}{C_S}\right)_F\right) < \frac{C_F}{C_S}$,
- *However, we have conflict if* $\frac{C_F}{C_S}$ *is in between the two optimal consumption ratios. In particular,*
  - *if the product of the felicity factors outweighs the product of the altruistic factors, we have* $\left(\frac{C_F}{C_S^*}\right)_S < \frac{C_F}{C_S} < \left(\frac{C_F^*}{C_S}\right)_F$ *and both agents would like to increase their own consumption shares to the material detriment of the other agent, i.e., we have a mild egoistic conflict with*

    $$conf > 1 \text{ and } conf < 2$$

  - *if the product of the felicity factors is smaller than the product of the altruistic factors, we have* $\left(\frac{C_F^*}{C_S}\right)_F < \frac{C_F}{C_S} < \left(\frac{C_F}{C_S^*}\right)_S$ *and both agents would like to reduce their own consumption share to the material benefit of the other agent, i.e., we have an altruistic conflict with*

    $$conf < 1$$

We present two figures that summarize the results obtained above. Fig. 1 focuses on altruistic preferences. In contrast, fig. 2 depicts the results for both benevolent and malevolent preferences. From the discussion of the hare with god Shakra, we may infer that these two were involved in an altruistic conflict.

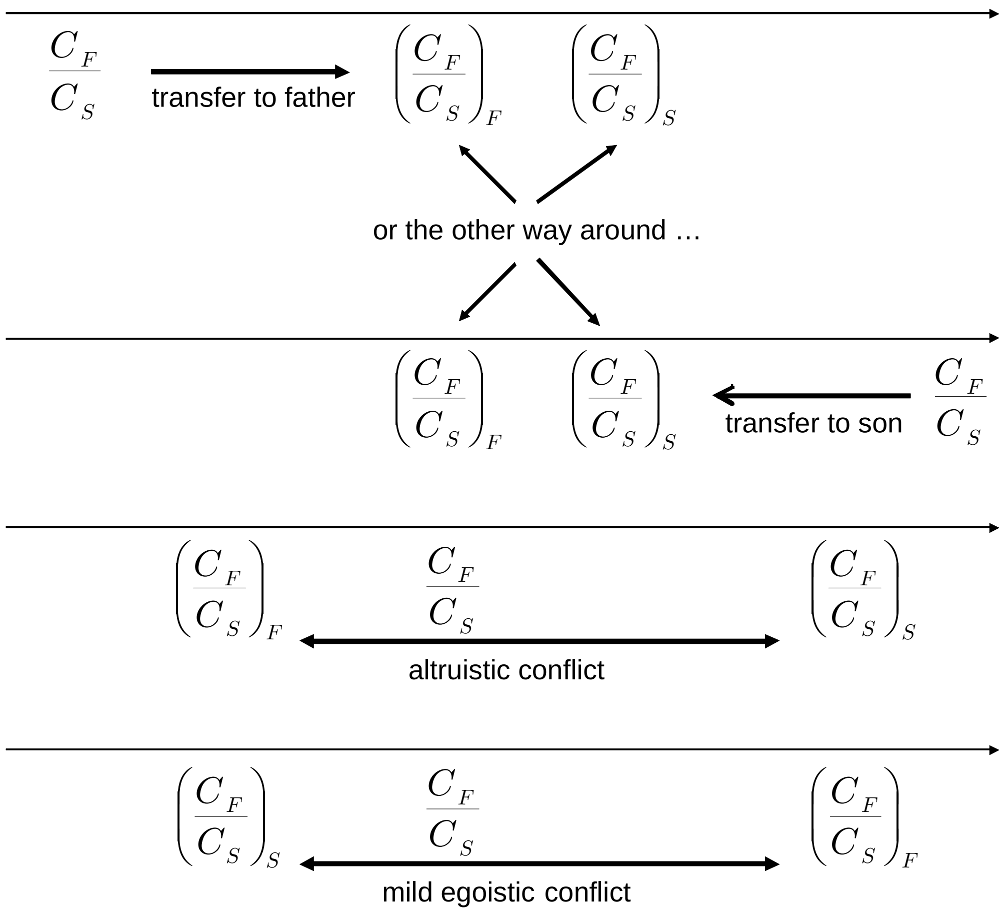

FIGURE 1. Conflict may arise for altruistic preferences, too

EXERCISE X.1. *Assume* $C = 100$, $\alpha_F = \alpha_S = 1$ *and* $\beta_F = \beta_S = \frac{1}{2}$. *Determine* C_F^*, C_S^*, *and the conflict measure* $conf$. *What kind of conflict do we have? Leaving the other parameters the same, how about* $\beta_F = 2$ *and* $\beta_S = 3$?

6. Other aspects of economic theories of altruism

In the economic literature, many articles are devoted to altruism. Zamagni (1995) is a collection of major contributions up to 1994. We comment on three aspects.

6.1. Altruism and the prisoners' dilemma. Stark (1989) shows how altruism might help people to get out of the prisoners' dilemma. In

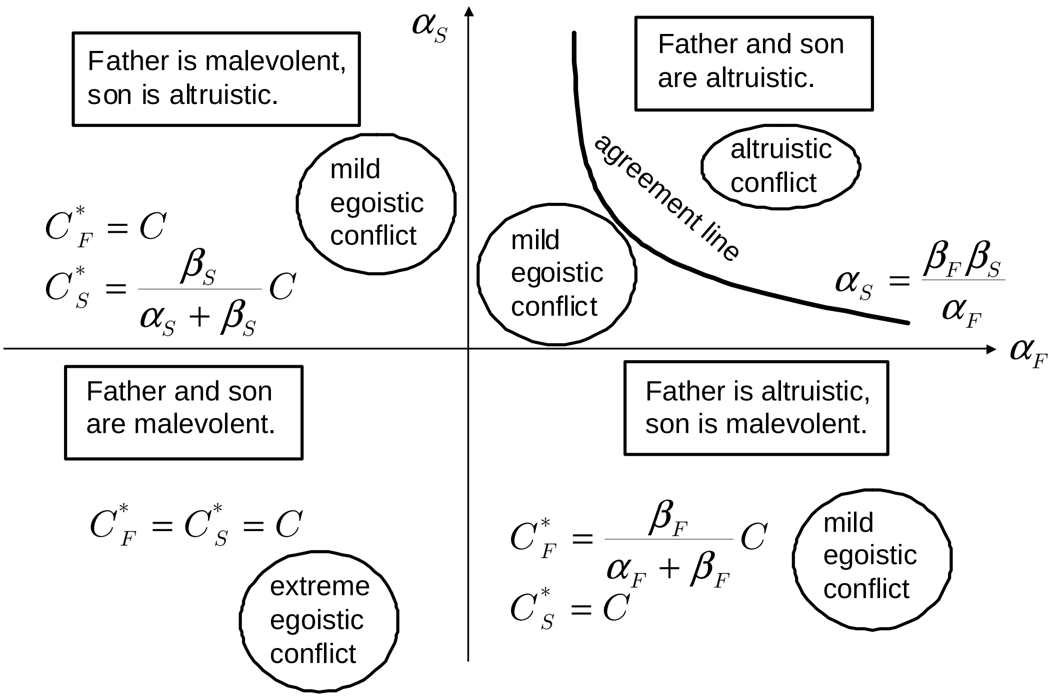

FIGURE 2. Results for altruistic and malevolent preferences

chapter VI, we have presented this payoff matrix:

		player 2 deny	confess
player 1	deny	$4,4$	$0,5$
	confess	$5,0$	$1,1$

There, we have pointed out the contradiction between

- individual rationality (confess is a dominant strategy for each player) and
- collective rationality (the payoff combination $(1,1)$ is Pareto-inferior to the payoff combination $(4,4)$.

Assume now that both agents are altruistic in the sense of "loving the other player as oneself". Then, for the strategy combination (confess, deny), the payoffs would be

$$\begin{aligned} U_1^{\text{altruistic}}(\text{confess, deny}) &= \frac{1}{2}u_1(\text{confess, deny}) + \frac{1}{2}u_2(\text{confess, deny}) \\ &= \frac{1}{2}\cdot 5 + \frac{1}{2}\cdot 0 \\ &= 2\frac{1}{2} \end{aligned}$$

and

$$\begin{aligned} U_2^{\text{altruistic}}(\text{confess, deny}) &= \frac{1}{2}u_2(\text{confess, deny}) + \frac{1}{2}u_1(\text{confess, deny}) \\ &= \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 5 \\ &= 2\frac{1}{2} \end{aligned}$$

Observe that these payoffs are feasible. We then obtain the new payoff matrix

player 2

		deny	confess
player 1	deny	$4,4$	$2\frac{1}{2},2\frac{1}{2}$
	confess	$2\frac{1}{2},2\frac{1}{2}$	$1,1$

EXERCISE X.2. *Can you see that altruism solves the prisoners' dilemma?*

6.2. Exploitation of the altruist. Several authors have discussed the problem that an altruist may be exploited by a needy person who forsees that the altruist will help him. For example, Lindbeck & Weibull (1988, pp. 284) write:

> "The source of the inefficiency is the recipient's strategic incentive to "squander" [verschwenden, HW] in an early period in order to subsequently receive more resources from the other agent, that is to "free-ride" on his concern. A "threat" by a potential donor not to give additional support to an agent because he squanders is not credible if the recipient knows that, ex post, it will be in the donor's (altruistic) interest to give such additional support."

6.3. Evolution of altruism. Biologists and economists have asked the question whether altruism can survive or even expand its domain in an evolutionary setting. In particular, the interdependencies of individual and group selection play an important role. Altruism reduces an individual's success but increases his group's survival probability. The reader is invited to consult the articles in Zamagni's (1995) collection.

7. Solutions

Exercise X.2

As in the prisoner's dilemma, both players have a dominant strategy, which is, however, the strategy "deny". The resulting equilibrium (deny, deny) payoff-dominates the strategy combination (confess, confess). Thus, we do not have a prisoners' dilemma any more.

Exercise X.1

We obtain

$$\begin{aligned} C_F^* &= \frac{\beta_F}{\alpha_F+\beta_F}C = \frac{\frac{1}{2}}{1+\frac{1}{2}}\cdot 100 = \frac{100}{3}, \\ C_S^* &= \frac{\beta_S}{\alpha_S+\beta_S}C = \frac{\frac{1}{2}}{1+\frac{1}{2}}\cdot 100 = \frac{100}{3}, \end{aligned}$$

The conflict measure is

$$conf = \frac{C_F^*+C_S^*}{C} = \frac{\frac{100}{3}+\frac{100}{3}}{100} = \frac{2}{3}$$

i.e., we have an instance of altruistic conflict.

For $\beta_F = 2$ and $\beta_S = 3$, we find

$$\begin{aligned} C_F^* &= \frac{\beta_F}{\alpha_F+\beta_F}C = \frac{2}{1+2}\cdot 100 = \frac{200}{3}, \\ C_S^* &= \frac{\beta_S}{\alpha_S+\beta_S}C = \frac{3}{1+3}\cdot 100 = 75, \end{aligned}$$

By

$$conf = \frac{C_F^*+C_S^*}{C} = \frac{\frac{200}{3}+75}{100} = \frac{17}{12}$$

we have a mild egoistic conflict.

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