# Microeconomic Analyses of Old Indian Texts

## Monopoly theory and Kautilya's market tax

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# Course overview

- Chapter I:
  Introduction: Old Indian literature and microeconomics

**Part A. Decision theory**

- Chapter II:
  Preferences
- Chapter III:
  Decisions
- Chapter IV:
  Decision theory for the Bhagavad Gita
- **Chapter V:**
  **Monopoly theory and Kautilya's market tax**

# Chapter V: Monopoly theory and Kautilya's market tax

- Introduction
- Monopoly: pricing policy
- Monopoly: quantity policy
- The market tax
- Conclusions

# Introduction

## Arthaśāstra

- Kauṭilya's manual on "wise kingship" = Arthaśāstra
- roughly 2000 years ago
- taxation, diplomacy, warfare, and the management of spies
- here: peculiar market tax for foreign traders

# Introduction

## market tax I

"The Superintendent of Customs should set up the customs house along with the flag facing the east or the north near the main gate ... The traders should announce the quantity and the price of a commodity that has reached the foot of the flag: "Who will buy this commodity at this price for this quantity?" After it has been proclaimed aloud three times, he should give it to the bidders. If there is competition among buyers, the increase in price along with the customs duty goes to the treasury."

# Introduction

## market tax II

1. the trader sells a good with production cost $C$
2. the trader declares the value $V$ of the good
3. the good is auctioned off, final price $p$
4. tax $p - V$

# Introduction

## market tax III

- Case $p \geq V$
  - tax: $p - V$
  - revenue: $p - (p - V) = V$
  - profit: $V - C$
- otherwise: try again another time/at another market

# Introduction

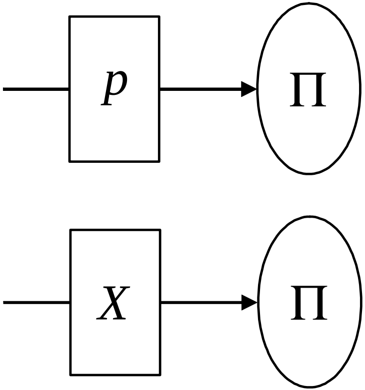
## market tax IV

A very honest trader might try $V = C$.

- profit is zero: $V - C = 0$
- producer's rent $p - C$ is taxed
- $V = C$ not a good idea.
- Trader faces an optimization problem.
  - high valuation $V$ in order to evade the market tax
  - low valuation so as to find a buyer (otherwise, duty and transportation cost incurring once again)

# Definitions

- Monopoly: **one** firm sells
- Monopsony: **one** firm buys




first: pricing policy for a monopolist

# Pricing policy
## demand properties

Demand function

$$X(p) = d - ep$$

$d, e \geq 0, p \leq \frac{d}{e}$

### Problem

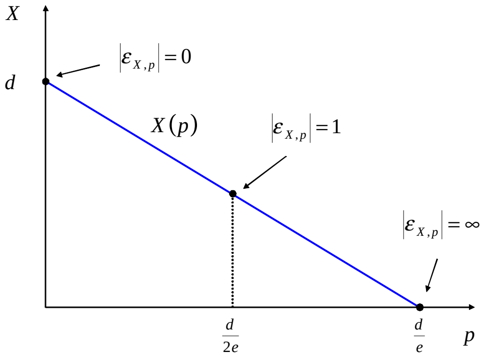
*Find*

- *the satiation quantity (quantity for price* $0$*),*
- *the prohibitive price (price which lets demand drop to* $0$*)*

# Pricing policy
## demand properties

**Solution**

- *satiation quantity* $X(0) = d$
- *prohibitive price* $\frac{d}{e}$ *(solve* $X(p) = 0$ *for the price)*
- *price elasticity of demand*



# Pricing policy

## profit

### Definition

$X$ is the demand function.

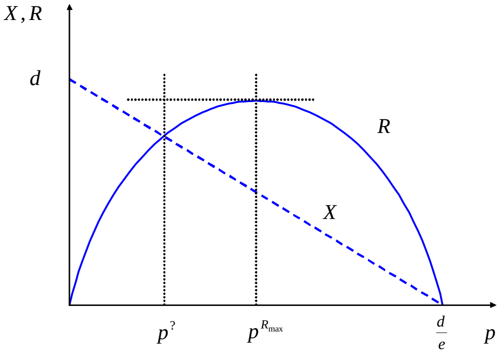
$$\begin{aligned} \underbrace{\Pi(p)}_{\text{profit}} : &= \underbrace{R(p)}_{\text{revenue}} - \underbrace{C(p)}_{\text{cost}} \\ &= pX(p) - C[X(p)] \end{aligned}$$

– monopoly's profit in terms of price $p$.

$$\begin{aligned} \Pi(p) &= p(d - ep) - c((d - ep)), \\ c, d, e &\geq 0, p \leq \frac{d}{e} \end{aligned}$$

– profit in linear model.

# Pricing policy
## decision situation: graph I



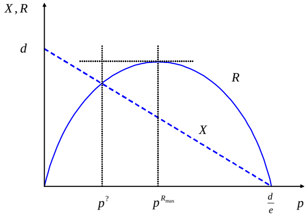

### Problem
*Find the economic meaning of the question mark!*

# Pricing policy

## decision situation: graph I

**Solution**

*No meaning!*




Units:

- Prices:

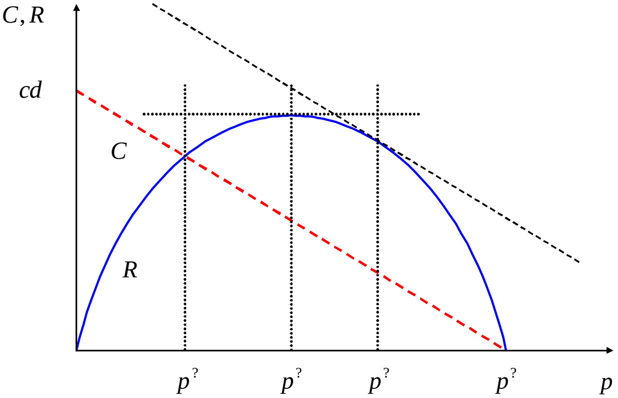
$$\frac{\text{monetary units}}{\text{quantity units}}$$

- Revenue $=$ price $\times$ quantity:

$$\frac{\text{monetary units}}{\text{quantity units}} \cdot \text{quantity units}$$
$$= \text{monetary units}$$

# Pricing policy

## decision situation: graph II



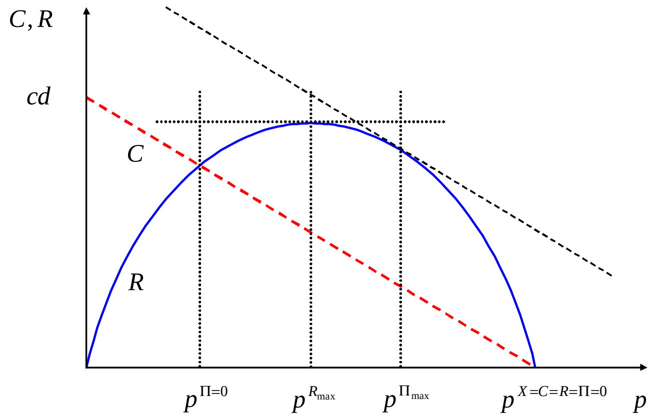

### Problem

*Find the economic meaning of the question marks!*

# Pricing policy

## decision situation: graph II

### Solution



# Pricing policy

## differentiating with respect to price

Marginal revenue with respect to price:

$$\frac{dR(p)}{dp} = \frac{d\left[pX(p)\right]}{dp} = X + p\frac{dX}{dp}$$

- A price increase by one Euro per unit of quantity increases revenue by $X$; for every unit sold the firm obtains an extra Euro.
- A price increase by one Euro per unit of quantity changes demand by $\frac{dX}{dp}$ (which is negative!) and hence revenue by $p\frac{dX}{dp}$.

$$\begin{aligned} R(p) &= p(d - ep) = pd - ep^2 \\ p^{R_{\max}} &= \frac{d}{2e} \end{aligned}$$

# Marginal cost

## w.r.t. price and w.r.t. quantity

$\frac{dC}{dX}$ : marginal cost (with respect to quantity)
$\frac{dC}{dp}$ : marginal cost with respect to price

$$\frac{dC}{dp} = \underbrace{\frac{dC}{dX}}_{>0} \underbrace{\frac{dX}{dp}}_{<0} < 0.$$

# Profit maximization

First-order condition (condition for maximizing):

$$\frac{dR}{dp} \stackrel{!}{=} \frac{dC}{dp}$$

## Problem

*Confirm: For linear demand* $p^M = \frac{d+ce}{2e}$*. What price maximizes revenue? How does* $p^M$ *change if* $c$ *changes?*

# Quantity policy

preliminaries

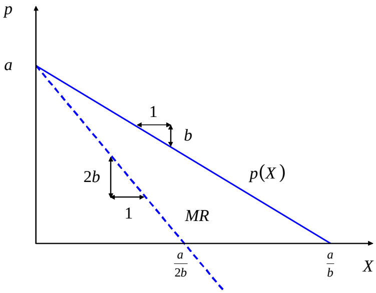
## Problem

*Assume inverse linear demand* $p(X) = a - bX$, $a, b > 0$. *Determine*

1. *the slope of the inverse linear demand function,*
2. *the slope of its marginal-revenue curve,*
3. *satiation quantity and*
4. *prohibitive price.*

## Solution

1. *The slope of the inverse demand curve is* $dp/dX = -b$
2. *Revenue:* $R(X) = p(X)X = aX - bX^2$
   *MR:* $dR(X)/dX = a - 2bX$.
   *Slope:* $-2b$
3. *satiation quantity:* $a/b$
4. *prohibitive price:* $a$



# Quantity policy

## definition profit function

### Definition

$X \geq 0$; $p$ inverse demand function.

$$\underbrace{\Pi(X)}_{\text{profit}} := \underbrace{R(X)}_{\text{revenue}} - \underbrace{C(X)}_{\text{cost}} = p(X)X - C(X)$$

– monopoly's profit in terms of quantity

Linear model:

$$\Pi(X) = (a - bX)X - cX, \quad X \leq \frac{a}{b},$$

# Marginal revenue

Marginal revenue (with respect to quantity)

$$MR = p + X\frac{dp}{dX}$$

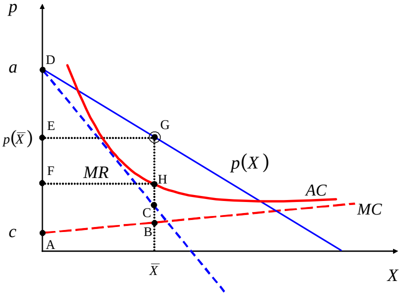
- If the monopolist increases his quantity by one unit, he obtains the current price for that last unit sold.
- The bad news is that a quantity increase decreases the price by $\frac{dp}{dX}$. Without price discrimination, this price decrease applies to all units sold. Thus, in case of a negatively sloped inverse demand curve, revenue is changed by $X\frac{dp}{dX} \leq 0$.

# Quantity policy

## average versus marginal definition of profit

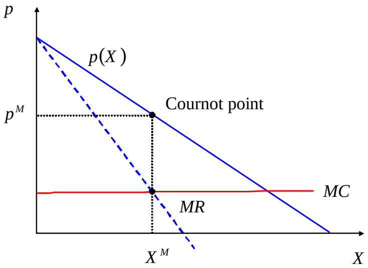
Profit at $\bar{X}$ :

$$
\begin{aligned}
& \Pi\left(\bar{X}\right) \\
= \; & p(\bar{X})\bar{X} - C\left(\bar{X}\right) \\
= \; & \left[p(\bar{X}) - AC\left(\bar{X}\right)\right]\bar{X} \\
= \; & \text{profit (average definition)} \\
= \; & \int_0^{\bar{X}} \left[MR\left(X\right) - MC\left(X\right)\right] dX \\
& -F \text{ (perhaps)} \\
= \; & \text{profit (marginal definition)}
\end{aligned}
$$



# Quantity policy
## first order condition

FOC (w.r.t. $X$): $MC \stackrel{!}{=} MR$.




Antoine Augustin Cournot (1801-1877) was a French philosopher, mathematician and economist.

### Problem

*Find $X^M$ for $p(X) = 24 - X$ and constant unit cost $c = 2$! (a haon déag)*

# Quantity policy

## comparative statics

$$
\begin{aligned}
X^M(a,b,c) &= \frac{1}{2}\frac{(a-c)}{b}, \quad \text{where } \frac{\partial X^M}{\partial c} < 0;\ \frac{\partial X^M}{\partial a} > 0;\ \frac{\partial X^M}{\partial b} < 0, \\
p^M(a,b,c) &= \frac{1}{2}(a+c), \quad \text{where } \frac{\partial p^M}{\partial c} > 0;\ \frac{\partial p^M}{\partial a} > 0;\ \frac{\partial p^M}{\partial b} = 0, \\
\Pi^M(a,b,c) &= \frac{1}{4}\frac{(a-c)^2}{b}, \quad \text{where } \frac{\partial \Pi^M}{\partial c} < 0;\ \frac{\partial \Pi^M}{\partial a} > 0;\ \frac{\partial \Pi^M}{\partial b} < 0.
\end{aligned}
$$

# Quantity policy

## exercise

Consider a monopolist with

- the inverse demand function $p(X) = 26 - 2X$ and
- the cost function $C(X) = X^3 - 14X^2 + 47X + 13$

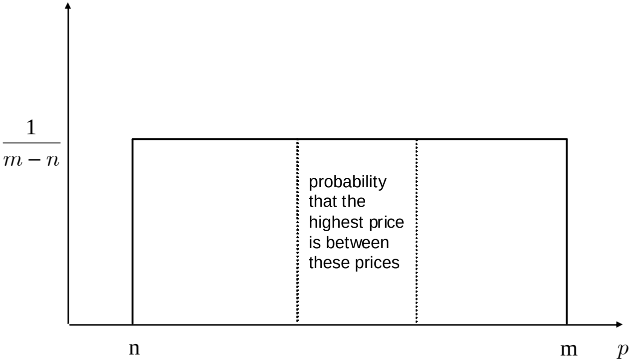
Find the profit-maximizing price!

# The market tax

## Density for the highest bids

The highest bid $p$ has the density distribution $f$ given by

$$f(p) = \begin{cases} \frac{1}{m-n}, & p \in [n, m] \\ 0, & p \notin [n, m] \end{cases}$$



# The market tax

## Assumptions

$$\underbrace{n}_{\text{lowest possible bid}} \leq \underbrace{C}_{\text{cost of production}} \leq \underbrace{m}_{\text{highest possible bid}}$$

The optimal $V$ obeys

- $V \geq C$ (otherwise profit negative)
- $V < m$ (otherwise no buyer will be found)

# The market tax

## profit function I

$$\Pi = -\underbrace{F}_{\substack{\text{cost of}\\ \text{market entry}}} + \underbrace{\delta\Pi \underbrace{\int_n^V f(p)\,dp}_{\substack{\text{probability}\\ \text{for bid below } V}}}_{\text{exp. disc. profit in next period}}$$

$$+ \underbrace{\int_V^m \underbrace{(p - C - T)}_{\substack{\text{unit profit after tax}\\ \text{for bids above } V}} f(p)\,dp}_{\text{exp. profit in current period}}$$

# The market tax

## profit function II

Solving for $\Pi$ yields

$$\Pi(V) = \frac{-F + \int\limits_V^m (V - C) f(p)\, dp}{1 - \delta \int\limits_n^V f(p)\, dp} = \frac{(V - C)\frac{m-V}{m-n} - F}{1 - \delta\frac{V-n}{m-n}}$$

where

- $\frac{m-V}{m-n}$ = probability for selling the object in the current period
- $\frac{V-n}{m-n}$ = probability for not selling in the current period.

# The market tax

## proposition

Assuming $F \leq \frac{(m-C)^2}{4(m-n)}$, the foreign trader chooses

$$V^* = n + \frac{m - n - \sqrt{R}}{\delta} \text{ with}$$

$$R := (m-n)\left[(1-\delta)(m-n-\delta(C-n)) + \delta^2 F\right] > 0.$$

If the unsuccessful trader does not try again in the kingdom at hand, the expected tax payment is

$$\begin{aligned}\int_V^m (p - V) f(p)\, dp &= \frac{1}{2}\frac{(m - V^*)^2}{m - n} \\ &= \frac{1}{2}\frac{\left(\sqrt{R} - (1-\delta)(m-n)\right)^2}{(m-n)\delta^2}\end{aligned}$$

# The market tax

## comparative-statics results

- $\frac{\partial V^*}{\partial C} > 0$: The seller's valuation has to be a positive function of the cost of producing (or buying) the good.
- Assume $\delta = 1$ and $F = 0$. In that extreme case, waiting for the next period has no cost. Indeed, we find

$$R = 0 \text{ and } V^* = m$$

  and the trader can risk to ask for the highest possible bid.
- Related to the second bullet item, we find
  - $\frac{\partial V^*}{\partial F} < 0$: If the trader faces high entry costs, he is less prepared to risk the no-sales case.
  - $\frac{\partial V^*}{\partial \delta} \geq 0$ : If the discount factor is low, future profits are discounted heavily and the trader prefers not to risk waiting for the next period.

# The market tax

## main result

The trader has to find a call price or value $V$ that is

- sufficiently high (so that the market tax can be avoided as much as possible)
- sufficiently low (so that at least one bidder can be found and the cost of market entry $F$ is not to be paid again)

# The market tax

## further provisions: duty

"When a man, fearing customs duty, declares a lower quantity or price, the king shall confiscate the amount in excess of that; or he should pay eight times the customs duty. ..."

Comment:
Apparently, duties also depended on the valuation. This gave the trader (if he were to sell dutiable goods) an additional incentive to state a low $V$. According to Kauṭilya, a fine of "eight times the customs duty" should give him a proper disincentive.

# The market tax

## further provisions: market tax I

"Or, when a man, fearing competing buyers, increases the price beyond the normal price of a commodity, the king shall confiscate the increase in price or assess twice the customs duty ... . "

Olivelle (translator):

- Why fear competing buyers?
- Perhaps, competing trader who "may sell their goods at a higher price than he".

  Comment:
  - This interpretation is not impossible–a trader may be jealous of other traders.
  - However, competition by other traders tends to reduce the price (for reasons of expected-profit maximization).
  - In contrast, it is the absence of competitors that allows a trader to increase his price.

# The market tax

## further provisions: market tax II

Comment:

If the trader expects many eager bidders, it would be in his interest to drive up $V$.

Inversely, if he has chosen a low $V$, he may indeed fear many bidders that would make him regret his decision.

Analogy: You take an umbrella with you, but “fear” it might not rain after all (in that case you would have taken the umbrella without good cause).

# The market tax

## conclusions I

- Practical implementation:
  - the seller and the final buyer have a very clear motivation to report a lower bid to the tax authorities, for some side-payment from the buyer to the seller.
  - sale near the "foot of the flag" facilitates supervision
- Unusual tax:
  Was Kauṭilya's Arthaśāstra
  - a historical document (telling us a lot about actual diplomacy, spying and taxing etc.) or
  - a teaching manual on statecraft?

# The market tax

## conclusions II

- Mystery of the lost Artha-śāstra:
  While the existence of the text was well-known through the centuries, a manuscript turned up "sometime before 1905", after a gap of about 1000 years. Maybe, the Artha-śāstra was destroyed along with the administrations that favored Kauṭilya's approach to economics?
- Modern-day implementation:
  Auctioneering houses like Sotheby's or electronic trading platforms like ebay do not encounter the supervision problems since $p$ is readily available for these market makers. Therefore, Kauṭilya's market tax (without punishment for too low or too high valuations) may still await realization in modern times.