Microeconomic Analyses of Old Indian Texts

Decision theory for the *Bhagavad Gītā*

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Course overview

- Chapter I:
  Introduction: Old Indian literature and microeconomics

Part A. Decision theory

- Chapter II:
  Preferences
- Chapter III:
  Decisions
- **Chapter IV:**
  Decision theory for the Bhagavad Gita
- Chapter V:
  Monopoly theory and Kautilya's market tax

Chapter IV: Decision theory for the Bhagavad Gita

Introduction I

- Krishna in the Bhagavad Gita:

Don't let the actions's fruit be your motivation

- Savage (The Foundations of Statistics):

An act may [...] be identified with its possible consequences

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Nevertheless: A decision-theoretic interpretation of the Gita is possible.
As befitting a *ṭīkā*, we present our arguments in several small steps.

Introduction II

- Bhagavad Gita (Gita for short)
  = part of book 6 (out of 18 books) of the Mahabhárata.
- The setting is this:
  - Árjuna and his brothers are about to fight against their cousins.
  - Árjuna's charioteer is his friend Krishna who reveals himself as God Krishna later on.
  - **Árjuna realizes that many of his relatives and teachers can be found on the other side.**
  - **He tells Krishna that he does not want to fight.**
  - **Krishna argues that the warrior Árjuna should fight.**
  - Finally, Árjuna is convinced and the battle can begin.

Set theory I

Definition (set and elements)

Set – any collection of "elements" that can be distinguished from each other. Set can be empty: $\varnothing$.

Definition (set and subset)

Let M be a nonempty set. A set N is called a subset of M (denoted by $N \subseteq M$) if and only if every element from N is contained in M. We use curly brackets $\{\}$ to indicate sets.

Problem

True?

- $\{1, 2\} = \{2, 1\}$
- $\{1, 2, 3\} \subseteq \{1, 2\}$

Set theory II

Definition (cardinality)

Let M be a nonempty set. The cardinality of M is the number of elements in M and is denoted by $|M|$.

For example, the cardinality of $\{1, 2, 3\}$ is $|\{1, 2, 3\}| = 3$.

Definition

Let M be a nonempty set. A tuple on M is an **ordered** list of elements from M. $()$ are used to denote tuples.
$(a_1, ..., a_n) = (b_1, ..., b_m)$ if $n = m$ and $a_i = b_i$ for all $i = 1, ..., n$.

Problem

$(1, 2, 3) = (2, 1, 3)$? $(1, 2, 2) = (1, 2)$?

Set theory III

Definition (power set)

Let M be any set. The set of all subsets of M is called the power set of M and is denoted by 2^M.

$$2^M = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

- $M := \{1,2,3\}$ has eight elements which is equal to $2^3 = 2^{|\{1,2,3\}|}$.
- Why 2 in 2^M?

$$\text{subset } \{1\} \text{ corresponds to } (1,0,0)$$
$$\text{subset } \{1,3\} \text{ corresponds to } (1,0,1)$$

- For any set M, we have $\left|2^M\right| = 2^{|M|}$.

Decision theory
preference relation without completeness

Definition (preference relation)

$X =$ non-empty set (of "objects").
$\succsim$ is a (weak) preference relation on X
$x \succsim y$ means "x is at least as good (as preferable, as virtuous) as y".
$\succsim$ complete if $x \succsim y$ or $y \succsim x$ holds for all $x, y \in X$, $x \neq y$.

Completeness $=$ the agent "knows what he wants"

Also:

- strict preference, expressed by $x \succ y$
- indifference, expressed by $x \sim y$

Decision theory
basic microeconomic model

Definition

The basic microeconomic decision model consists of

- a set of actions A,
- a set of consequences C with relation $\succsim$ on C
- a consequence function $f : A \rightarrow C$

An agent chooses

- an action $a \in A$,
- earns the consequence $f(a)$.

The theoretical prediction is an action a^* that obeys

$$f(a^*) \succsim f(a) \ \text{ for all } a \in A.$$

Decision theory

definition: the set of best actions

$\succ$: an asymmetric relation on A (e.g., a strict preference relation).

- the set of "best" actions from A':

$$\max\left(A'; \succ\right) \subseteq A'$$

- i.e., those actions a from A' for which no other action $b \in A'$ with $b \succ a$ exists

Decision theory
definition: choice function

Definition (choice function)

Let A be a set of actions with $|A| > 2$. A choice function γ on A is given by

$$\gamma \quad : \quad 2^A \to A, \text{ with}$$
$$\gamma\left(A'\right) \quad \in \quad A' \text{ for every } A' \in 2^A.$$

- strict preference relation $\succ$ on A is complete
  $\Rightarrow \gamma\left(A'\right) = \max\left(A'; \succ\right)$ defines a choice function
- subset $A' = \{a, b\}$ with neither $a \succ b$ nor $b \succ a$
  $\Rightarrow \max\left(A'; \succ\right) = A'$ and, hence, $\gamma\left(A'\right) := \max\left(A'; \succ\right)$ does not define a choice function.

Consider the set

$$A = \{(1,3), (2,2), (3,2), (3,5), (4,2)\}$$

- first entry = number of bananas
- second entry = number of grapes

1. The agent has monotonic preferences. He prefers bananas to grapes. Whenever there are more bananas than grapes in one bundle, he strictly prefers that bundle. If two bundles contain the same number of bananas, he lets the number of grapes decide.

2. The agent has monotonic preferences and is interested in the sum of fruit, only.

Which (if any) of these rules define a choice function γ by $\gamma(A') := \max(A'; \succ)$?

WARP definition:

- If action a is chosen in a situation where b is also feasible,
- then b cannot be chosen in a situation where both a and b are feasible.

Violation:

- you are in a pizzeria intent on ordering a cheese pizza.
- the waiter comes and apologizes: quatro formaggio is out today.
- you alter your choice: a pizza funghi, please.

Decision theory

RSM = Agents use two rationales in a prespecified order
Example: Buying a car:

1. reject all cars that cost more than € 10.000 and,
2. choose the one with the smallest milage

(Manzini and Mariotti, AER 2007).

Definition (rational shortlist method)

A choice function γ is an RSM if a pair of asymmetric relations $(\succ_1, \succ_2)$ exists such that

$$\gamma\left(A'\right) = \max\left(\max\left(A'; \succ_1\right); \succ_2\right)$$

holds for all $A' \in 2^A$.

Decision theory
RSM does not fulfill WARP

Three actions a, b, and c where

- $c \succ_P a$ (P stands for Pareto-dominatance, $\succ_1$ not complete)
- $a \succ_F b \succ_F c$ (F stands for fairness)

$$
\begin{aligned}
\gamma \left(\{a, b, c\} \right) &= \max \left(\max \left(\{a, b, c\} ; \succ_P \right) ; \succ_F \right) \\
&= \max \left(\{b, c\} ; \succ_F \right) \; (a \text{ is eliminated by } c) \\
&= b \; (b \text{ is fairer than } c)
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma \left(\{a, b\} \right) &= \max \left(\max \left(\{a, b\} ; \succ_P \right) ; \succ_F \right) \\
&= \max \left(\{a, b\} ; \succ_F \right) \; (\text{neither } a \succ_P b \text{ nor } b \succ_P a) \\
&= a \; (a \text{ is fairer than } b)
\end{aligned}
$$

Four kinds of preferences:

- a preference relation $\succsim_C$ on C
- a preference relation $\succsim_A$ on A and
- a preference relation $\succsim_{A \times C}$ on $A \times C$.

Since actions and consequences cannot be mixed arbitrarily, we can derive

- a fourth preference relation, $\succsim$ on A, by defining

$$a \succsim b \text{ as } [a, f(a)] \succsim_{A \times C} [b, f(b)]$$

Gita:

- $\succsim_C$ on C: Árjuna's argument against fighting and killing
- $\succsim_A$ on A: Krishna's insistance on svadharma
- $\succsim_{A \times C}$ on $A \times C$: Both actions and consequences relevant
- $\succsim$ on A: Which action is best?

Three levels of decision making

Reinhard Selten suggests three levels of decision making:

      (i) routine,

     (ii) imagination, and

   (iii) reasoning.

(i) *Árjuna (i) is* willing to fight

Decision-theoretic interpretation:
not $A = \{\text{fight, not fight}\}$, but $A_{sv} = \{\text{fight}\}$
$sv = svadharmic$ where $dharma = \text{duty/religion}$ and $sva = \text{own}$

(ii) After inspecting the opposing side, the *early Árjuna* is horrified:

- "Krishna, at the sight of my own kin standing here ready to fight, my limbs feel tired and my mouth has gone dry, my body is trembling and my hair is standing on end."
- "It would be better for me if Dhrita·rashtra's sons, armed with weapons, were to kill me in battle unresisting and unarmed!"

Despondent Árjuna

imagination level (interpretation)

- Full action set $A = \{\text{fight, not fight}\}$.
- Set of states of the world $W = \{\text{good luck, bad luck}\}$
- Uncertain-consequence function $g : A \times W \rightarrow C$, defined by

		state of the world	
		good luck	bad luck
Arjuna	fighting	victory and family destr.	defeat and family destr.
	not fighting	defeat without family destr.	defeat without family destr.

For Árjuna (ii), not fighting is a dominant action.

Despondent Árjuna

incomplete preferences

- The routine level (*kṣatradharma*) and
- the imagination level (*kuladharma*)

militate for contradictory recommendations.
Árjuna turns to Krishna for help:
"... my mind confused over my duty [*dharma*, not *svadharma*],
I ask you to tell me for sure what would be best."

Decision-theoretic interpretation:
"what would be best" is clearly to be understood in terms of $\succsim$ on A.

Could Árjuna not just consult his preferences $\succsim$ on A?
No, they are incomplete–he does not know what to prefer.

Krishna's counter-arguments

Kishna:
"Whoever thinks this soul can kill or be killed, doesn't understand. It neither kills, nor is it killed. It isn't born; it never dies ... .

Just as a man casts off his worn-out clothes and puts on other new ones, so the embodied soul [*deh-in*] casts off its worn-out bodies and takes other new ones."

Decision-theoretic interpretation:
Krishna argues against Árjuna's preferences $\succsim_C$ on C.

1. Chance to attain heaven:
   "You should attend to your own duty [*svadharma*] and stand firm, for there is nothing better for a warrior than a legitimate battle. Happy the warriors who find such a battle, Partha–an open door to heaven …"

2. Danger of suffering reputational damage:
   "The great warriors will think you withdrew from the battle out of fear, and though highly regarded by them before, you will be slighted. Your enemies too will say many unseemly things, disparaging your ability; and what could be more painful than that? Get up, son of Kunti [Kunti is Árjuna's mother], and resolve to fight! For you will either be killed and attain heaven, or you will prevail and enjoy the earth."

Krishna's counter-arguments

A dominance argument II

Decision-theoretic interpretation:
Krishna corrects Árjuna's view of the consequence function g:

		state of the world	
		good luck	bad luck
	fighting	prevail and enjoy the earth	be killed and attain heaven
Arjuna			
	not fighting	shameful loss of reputation	shameful loss of reputation

For Krishna, fighting is a dominant action.

Árjuna is afraid of his bad conscience:
"Better in this world to live an alms without killing the mighty elders;
for were I to kill the elders, eager though they are for worldly gain, in
this very world I would taste pleasures smeared with blood."

Krishna exculpates him:
"I am Time, the world destroyer, ripened, and here I am busy
crushing the worlds. Even without you, all the warriors drawn up in
the opposing ranks will cease to exist. ... I have myself long since
doomed them to perish; you just be the instrument ... ".

Decision-theoretic interpretation:
Krishna tells Árjuna that he is wrong about the consequences.
Árjuna cannot prevent family destruction.

Krishna:

- "Don't let the actions's fruit be your motivation"
- "He whose mind is unperturbed in times of sorrow, who has lost the craving for pleasures, and who is rid of passion, fear and anger, is called a sage of steadied thought. His wisdom is secure who is free of any affections and neither rejoices nor recoils on obtaining anything good or bad."

Decision-theoretic interpretation:

Krishna seems to advocate a preference relation $\succsim_C$ with

$$\text{pleasure} \sim_C \text{sorrow}.$$

Here, pleasure or sorrow do not only refer to Árjuna's egotistic motives but also to Árjuna's preferences for his *kula*.

Problem: Can people choose preferences?

Krishna's counter-arguments

Svadharma and paradharma I

Krishna suggests equanimity with respect to $\succsim_C$, but certainly not with respect to $\succsim$ on A.

"You have a right to the action alone, never to its fruits. Don't let the action's fruit be your motivation, and don't be attached to inactivity. ... the wise ones of disciplined understanding renounce the fruit produced by action and ... attain the perfect state."

"One's own duty [*svadharma*], even if done imperfectly, is better than another's [*paradharma*], even if done well. The duty of others is fraught with danger; better to die while fulfilling one's own."

Krishna's counter-arguments
Svadharma and paradharma II

Decision-theoretic interpretation:

Definition

Preferences $\succsim_{A \times C}$ on $A \times C$ are purely consequentialist, if there is a preference relation $\succsim_C$ on C such that

$$[a, c] \succsim_{A \times C} [a', c']$$
$$\Leftrightarrow \ c \succsim_C c'.$$

$\succsim_{A \times C}$ are purely action-oriented if a preference relation $\succsim_A$ on A exists with

$$[a, c] \succsim_{A \times C} [a', c']$$
$$\Leftrightarrow \ a \succsim_A a'.$$

Krishna's counter-arguments
Svadharma and paradharma III

Krishna's insistence on *svadharma* can be expressed by

$$
\begin{aligned}
A &= A_{sv} \cup A_{pa}, \\
A_{sv} \cap A_{pa} &= \varnothing
\end{aligned}
$$

where *pa* refers to *paradharma* or laws for others and

$$
\left[a_{sv}, c \right] \succ_{A \times C} \left[a_{pa}, c' \right]
$$

whenever $a_{sv} \in A_{sv}$ and $a_{pa} \in A_{pa}$, for any c and $c' \in C$.

Krishna's counter-arguments

RSM = rational shortlist method

- RSM in general: Agents use two rationales in a prespecified order. Example: Buy a car by
    1. reject all cars that cost more than € 10.000 and,
    2. choose the one with the smallest milage
  (Manzini and Mariotti, AER 2007).
- Svadharmic RSM:
    1. discard all actions that are not svadharmic.
    2. choose the one with the best consequences.

Theorem

The svadharmic RSM always fulfills the weak axiom of revealed preference.

Details in manual.

Conclusions I

- A decision-theoretic reconstruction of some parts of the Bhagavad Gita possible.
- Krishna does not shy away from consequentialists arguments.
- Krishna: actions are not only relevant because of their consequences.
- Krishna's *svadharmic* point of view an example of the *Rational Shortlist Method*.
- Milder forms of *svadharma* (duty in line with one's social standing, behavior "befitting one's station") are in use in all societies.
- *Svadharmic* decision theory closely related to research on identity undertaken by psychologists, sociologists, and even economists.

Conclusions II

- Standard decision theory could also take *svadharma* aspects into account.
  If an action is especially fitting or unfitting to a particular person, this fact (known to the agent and/or known to others) may be counted among the consequences of that action.
- *Svadharmic* decision theory and rule-based consequentialism may be close cousins:
  - When we argue for rules or *svadharma*, consequences are important.
  - Whenever an individual decision maker has to act, he should be guided by these rules, not by consequences.

Conclusions III

- Alternative citations from the Gita might have been chosen.
  See Krishna's teachings on *sattva, rajas*, and *tamas*.
  Krishna warns Árjuna against the *rajas* mode.

- Similar arguments in the fifth book: Yudhishthira's doubts and
  arguments focus on *kuladharma*
  while Krishna himself, Kuntī and even Duryodhana
  advocate the *kṣatradharma* and *svadharma* point of view.

- The Indian Nobel price winner of 1998, Amartya Sen takes
  Árjuna's side:
  "one must take responsibility for the consequences
  of one's actions and choices, and [...] this responsibility
  cannot be obliterated by any pointer
  to a consequence-independent duty or obligation."