Microeconomics

Pareto-optimal review

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Structure

Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods
  - External effects and environmental economics
  - Public goods

Pareto-optimal review

Overview

1. Identical marginal willingness to pay
2. Identical marginal opportunity cost
3. Equality between marginal willingness to pay and marginal opportunity cost

MRS = MRS

Exchange Edgeworth box

Optimality in exchange implies

$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A \overset{!}{=} MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|,$$

because if

$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A > MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|$$

held, then A could

- give one small unit of good 1 to B (?) or
- receive one small unit of good 1 from B (?)

Contract curve or exchange line: Locus of all Pareto-optimal allocations in the exchange Edgeworth box

MR(T)S = MR(T)S

Production Edgeworth box

Optimal factor usage implies

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 \stackrel{!}{=} MRTS_2 = \left|\frac{dK_2}{dA_2}\right|,$$

because if

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 > MRTS_2 = \left|\frac{dK_2}{dA_2}\right|$$

held, then one small unit of labor could be used for

- production of good 1 instead of production of good 2 or
- production of good 2 instead of production of good 1.

Production curve: locus of all combinations of capital and labor that satisfy equality of the marginal rates of technical substitution

MRS = MRS

Two markets – one production site

- **Marginal revenue** $MR = \frac{dR}{dx_i}$ can be seen as the monetary marginal willingness to pay for **selling** one extra unit of good i.
  - Denominator good $\rightarrow$ good 1 or 2, respectively
  - Nominator good $\rightarrow$ "money" (revenue).
- Profit maximization by a firm selling on two markets 1 and 2 implies

$$\left|\frac{dR}{dx_1}\right| = MR_1 \overset{!}{=} MR_2 = \left|\frac{dR}{dx_2}\right|$$

- Otherwise, if revenue on market 1 was larger than revenue on market 2, ...

MRS = MRS

Two firms in a cartel

- **Marginal profit** is the marginal willingness to pay for **producing and selling** one extra unit of good y.
- Two firms in a cartel maximize

$$\Pi_{1,2}(x_1, x_2) = \Pi_1(x_1, x_2) + \Pi_2(x_1, x_2)$$

with FOCs

$$\frac{\partial \Pi_{1,2}}{\partial x_1} \stackrel{!}{=} 0 \stackrel{!}{=} \frac{\partial \Pi_{1,2}}{\partial x_2}$$

- If $\frac{\partial \Pi_{1,2}}{\partial x_2}$ were higher than $\frac{\partial \Pi_{1,2}}{\partial x_1}$ ...

How about the Cournot duopoly with FOCs

$$\frac{\partial \Pi_1}{\partial x_1} \stackrel{!}{=} 0 \stackrel{!}{=} \frac{\partial \Pi_2}{\partial x_2}$$

MRT = MRT

Two factories – one market

- Marginal cost $MC = \frac{dC}{dy}$ is a monetary marginal opportunity cost of production

$$MRT = \left| \frac{dx_2}{dx_1} \right|^{\text{transformation frontier}}$$

  - Denominator good $\rightarrow$ good 1 or 2, respectively
  - Nominator good $\rightarrow$ "money" (cost).
- One firm with two factories or a cartel in case of homogeneous goods:

$$MC_1 \overset{!}{=} MC_2.$$

- Otherwise, if $MC_1 > MC_2$ held, then...
- Pareto improvements (optimality) have to be defined relative to a specific group of agents!

- David Ricardo (1772–1823)
- "comparative cost advantage"

Marginal rates of transformation are identical:

$$MRT^P = \left|\frac{dW}{dCl}\right|^P \overset{!}{=} \left|\frac{dW}{dCl}\right|^E = MRT^E$$

because if

$$4 = MRT^P = \left|\frac{dW}{dCl}\right|^P > \left|\frac{dW}{dCl}\right|^E = MRT^E = 2$$

held...

Reminder:

$$MRT = \left|\frac{df\left(x_1\right)}{dx_1}\right| = \frac{MC_1}{MC_2}.$$

MRT = MRT

International trade

- Before Ricardo:
  England exports cloth and imports wine if

  $$\begin{array}{rcl} MC^E_{Cl} & < & MC^P_{Cl} \text{ and} \\ MC^E_W & > & MC^P_W \end{array}$$

  hold.
- Ricardo:

  $$\frac{MC^E_{Cl}}{MC^E_W} < \frac{MC^P_{Cl}}{MC^P_W}$$

  suffices for profitable international trade.

MRS = MRT

Base case

Optimal product mix implies

$$\left|\frac{dx_2}{dx_1}\right|^{\text{production possibility frontier}} = MRT \stackrel{!}{=} MRS = \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}},$$

because if

$$\left|\frac{dx_2}{dx_1}\right|^{\text{production possibility frontier}} > \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}}$$

held, then one small unit of good 1 could be

- produced and consumed additionally (?) or
- produced and consumed less (?).

MRS = MRT

Perfect competition – output space

- FOC output space

$$p \stackrel{!}{=} MC$$

- Let good 2 be money with price 1
- *MRS* is
  - consumer's monetary marginal willingness to pay for one additional unit of good 1
  - equal to p for marginal consumer
- *MRT* is the amount of money one has to forgo for producing one additional unit of good 1, i.e., the marginal cost
- Thus,

$$\text{price} = \text{marginal willingness to pay} \stackrel{!}{=} \text{marginal cost}$$

which is also fulfilled by first-degree price discrimination.

MRS = MRT

Perfect competition – input space

FOC input space

$$MVP = p\frac{dy}{dx} \overset{!}{=} w$$

where

- the marginal value product MVP is the monetary marginal willingness to pay for the factor use and
- w, the factor price, is the monetary marginal opportunity cost of employing the factor.

$MRS = MRT$

Cournot monopoly

For the Cournot monopolist, $MRS \overset{!}{=} MRT$ can be rephrased as the equality between

- the monetary marginal willingness to pay for selling – this is the marginal revenue $MR = \frac{dR}{dy}$ and
- the monetary marginal opportunity cost of production, the marginal cost $MC = \frac{dC}{dy}$

MRS = MRT

Household optimum

Consuming household "produces" goods by using his income to buy them, $m = p_1 x_1 + p_2 x_2$, which can be expressed with the transformation function

$$x_2 = f(x_1) = \frac{m}{p_2} - \frac{p_1}{p_2} x_1.$$

Hence,

$$MRS \stackrel{!}{=} MRT = MOC = \frac{p_1}{p_2}$$

MRS = MRT

Public goods

Two individuals, A and B, consume

- the private good x in quantities of x_A and x_B and
- the public good G (attention: good 1)

Optimal product mix implies

$$\left|\frac{dx_A}{dG}\right|^{\text{indifference curve}} + \left|\frac{dx_B}{dG}\right|^{\text{indifference curve}} \stackrel{!}{=} \left|\frac{d(x_A + x_B)}{dG}\right|^{\text{transformation frontier}},$$

because if

$$\left|\frac{dx_A}{dG}\right|^{\text{indifference curve}} + \left|\frac{dx_B}{dG}\right|^{\text{indifference curve}} > \left|\frac{d(x_A + x_B)}{dG}\right|^{\text{transformation frontier}}$$

held, then one small unit of good G could be

- produced and consumed additionally (?) or
- produced and consumed less (?).