Microeconomics Pareto-optimal review

Harald Wiese

Leipzig University

Structure

Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods
 - External effects and environmental economics
 - Public goods

Pareto-optimal review

Overview

- Identical marginal willingness to pay
- Identical marginal opportunity cost
- Equality between marginal willingness to pay and marginal opportunity cost

 $\begin{array}{l} \mathsf{MRS} = \mathsf{MRS} \\ \mathsf{Exchange Edgeworth \ box} \end{array}$

Optimality in exchange implies

$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A \stackrel{!}{=} MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|,$$

because if

$$\left|\frac{dx_{2}^{A}}{dx_{1}^{A}}\right| = MRS^{A} > MRS^{B} = \left|\frac{dx_{2}^{B}}{dx_{1}^{B}}\right|$$

held, then A could

- give one small unit of good 1 to B (?) or
- receive one small unit of good 1 from B(?)

Contract curve or exchange line: Locus of all Pareto-optimal allocations in the exchange Edgeworth box

$$\label{eq:MR} \begin{split} \mathsf{MR}(\mathsf{T})\mathsf{S} &= \mathsf{MR}(\mathsf{T})\mathsf{S} \\ \mathsf{Production \ Edgeworth \ box} \end{split}$$

Optimal factor usage implies

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 \stackrel{!}{=} MRTS_2 = \left|\frac{dK_2}{dA_2}\right|,$$

because if

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 > MRTS_2 = \left|\frac{dK_2}{dA_2}\right|$$

held, then one small unit of labor could be used for

- production of good 1 instead of production of good 2 or
- production of good 2 instead of production of good 1.

Production curve: locus of all combinations of capital and labor that satisfy equality of the marginal rates of technical substitution

- Marginal revenue $MR = \frac{dR}{dx_i}$ can be seen as the monetary marginal willingness to pay for selling one extra unit of good *i*.
 - $\bullet\,$ Denominator good \to good 1 or 2, respectively
 - Nominator good \rightarrow "money" (revenue).
- Profit maximization by a firm selling on two markets 1 and 2 implies

$$\left|\frac{dR}{dx_1}\right| = MR_1 \stackrel{!}{=} MR_2 = \left|\frac{dR}{dx_2}\right|$$

• Otherwise, if revenue on market 1 was larger than revenue on market 2, ...

- Marginal profit is the marginal willingness to pay for producing and selling one extra unit of good y.
- Two firms in a cartel maximize

$$\Pi_{1,2}(x_1, x_2) = \Pi_1(x_1, x_2) + \Pi_2(x_1, x_2)$$

with FOCs

$$\frac{\partial \Pi_{1,2}}{\partial x_1} \stackrel{!}{=} 0 \stackrel{!}{=} \frac{\partial \Pi_{1,2}}{\partial x_2}$$
• If $\frac{\partial \Pi_{1,2}}{\partial x_2}$ were higher than $\frac{\partial \Pi_{1,2}}{\partial x_1} \dots$
How about the Cournot duopoly with FOCs

$$\frac{\partial \Pi_1}{\partial x_1} \stackrel{!}{=} 0 \stackrel{!}{=} \frac{\partial \Pi_2}{\partial x_2}$$

MRT = MRTTwo factories – one market

• Marginal cost $MC = \frac{dC}{dy}$ is a monetary marginal opportunity cost of production

$$MRT = \left| rac{dx_2}{dx_1}
ight|^{ ext{transformation frontier}}$$

- $\bullet\,$ Denominator good \rightarrow good 1 or 2, respectively
- Nominator good \rightarrow "money" (cost).
- One firm with two factories or a cartel in case of homogeneous goods:

$$MC_1 \stackrel{!}{=} MC_2.$$

- Otherwise, if $MC_1 > MC_2$ held, then...
- Pareto improvements (optimality) have to be defined relative to a specific group of agents!

MRT = MRTInternational trade

- David Ricardo (1772–1823)
- "comparative cost advantage"

Marginal rates of transformation are identical:

$$MRT^{P} = \left|\frac{dW}{dCI}\right|^{P} \stackrel{!}{=} \left|\frac{dW}{dCI}\right|^{E} = MRT^{E}$$

because if

$$4 = MRT^{P} = \left|\frac{dW}{dCI}\right|^{P} > \left|\frac{dW}{dCI}\right|^{E} = MRT^{E} = 2$$

held... Reminder:

$$MRT = \left|\frac{df(x_1)}{dx_1}\right| = \frac{MC_1}{MC_2}.$$

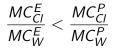
• Before Ricardo:

England exports cloth and imports wine if

$$MC_{CI}^{E} < MC_{CI}^{P}$$
 and $MC_{W}^{E} > MC_{W}^{P}$

hold.

Ricardo:



suffices for profitable international trade.

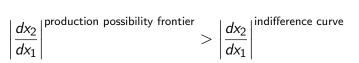
MRS = MRT

Base case

Optimal product mix implies

$$\left|\frac{dx_2}{dx_1}\right|^{\text{production possibility frontier}} = MRT \stackrel{!}{=} MRS = \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}}$$

because if



held, then one small unit of good 1 could be

- produced and consumed additionally (?) or
- produced and consumed less (?).

MRS = MRTPerfect competition – output space

• FOC output space

$$p \stackrel{!}{=} MC$$

- Let good 2 be money with price 1
- MRS is
 - consumer's monetary marginal willingness to pay for one additional unit of good 1
 - equal to p for marginal consumer
- *MRT* is the amount of money one has to forgo for producing one additional unit of good 1, i.e., the marginal cost
- Thus,

price = marginal willingness to pay $\stackrel{!}{=}$ marginal cost

which is also fulfilled by first-degree price discrimination.

$\label{eq:MRS} \mathsf{MRS} = \mathsf{MRT}$ Perfect competition – input space

FOC input space

$$MVP = p \frac{dy}{dx} \stackrel{!}{=} w$$

where

- the marginal value product *MVP* is the monetary marginal willingness to pay for the factor use and
- *w*, the factor price, is the monetary marginal opportunity cost of employing the factor.

For the Cournot monopolist, $MRS \stackrel{!}{=} MRT$ can be rephrased as the equality between

- the monetary marginal willingness to pay for selling this is the marginal revenue $MR = \frac{dR}{dv}$ and
- the monetary marginal opportunity cost of production, the marginal cost $MC = \frac{dC}{dy}$

MRS = MRTHousehold optimum

Consuming household "produces" goods by using his income to buy them, $m = p_1x_1 + p_2x_2$, which can be expressed with the transformation function

$$x_2 = f(x_1) = \frac{m}{p_2} - \frac{p_1}{p_2}x_1.$$

Hence,

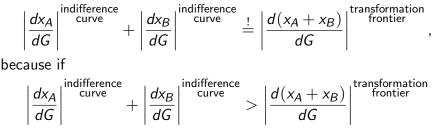
$$MRS \stackrel{!}{=} MRT = MOC = \frac{p_1}{p_2}$$

MRS = MRTPublic goods

Two individuals, A and B, consume

- the private good x in quantities of x_A and x_B and
- the public good G (attention: good 1)

Optimal product mix implies



held, then one small unit of good G could be

- produced and consumed additionally (?) or
- produced and consumed less (?).