Microeconomics

Public goods

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Structure

Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods
  - External effects and environmental economics
  - **Public goods**

Pareto-optimal review

Public goods and publicly available goods

- Public goods: no rivalry in consumption
- Pure public goods: no rivalry in consumption and no exclusion

Publicly available goods are not necessarily public goods!
University education:
Capacity limits of lecture halls (rivalry in consumption)
Exclusion is possible

Overview

- Public goods and publicly available goods
- Optimal provision of public goods
- Aggregation of individual willingness to pay
- Voluntary provision of public goods

Reminder: optimal coordination of production and consumption for private goods

Optimal product mix implies

$$\left|\frac{dx_2}{dx_1}\right|^{\text{production possibility frontier}} = MRT \overset{!}{=} MRS = \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}},$$

because if

$$\left|\frac{dx_2}{dx_1}\right|^{\text{production possibility frontier}} > \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}}$$

held, then one small unit of good 1 could be

- produced and consumed additionally (?) or
- produced and consumed less (?).

Optimal provision of public goods

Two individuals, A and B, consume

- the private good x in quantities of x_A and x_B and
- the public good G ($=$ good 1)

Optimal product mix implies

$$\left|\frac{dx_A}{dG}\right|^{\substack{\text{indifference}\\ \text{curve}}} + \left|\frac{dx_B}{dG}\right|^{\substack{\text{indifference}\\ \text{curve}}} \overset{!}{=} \left|\frac{d(x_A + x_B)}{dG}\right|^{\substack{\text{production possibility}\\ \text{frontier}}},$$

because if

$$\left|\frac{dx_A}{dG}\right|^{\substack{\text{indifference}\\ \text{curve}}} + \left|\frac{dx_B}{dG}\right|^{\substack{\text{indifference}\\ \text{curve}}} > \left|\frac{d(x_A + x_B)}{dG}\right|^{\substack{\text{production possibility}\\ \text{frontier}}}$$

held, then one small unit of good G could be

- produced and consumed additionally (?) or
- produced and consumed less (?).

Optimal provision of public goods: variants

- n individuals:

$$\sum_{i=1}^{n} MRS^i = \sum_{i=1}^{n} \left| \frac{dx_i}{dG} \right|^{\text{indifference curve}} \stackrel{!}{=} \left| \frac{d\left(\sum_{i=1}^{n} x_i\right)}{dG} \right|^{\text{production possibility frontier}}$$

- Prices p_G and p_x for goods:

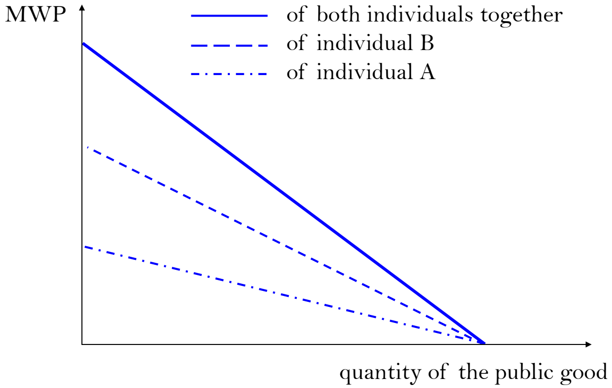
$$\sum_{i=1}^{n} MRS^i \stackrel{!}{=} \left| \frac{d\left(\sum_{i=1}^{n} x_i\right)}{dG} \right|^{\text{production possibility frontier}} = \frac{p_G}{p_x}$$

- x = money with $p_x = 1$:

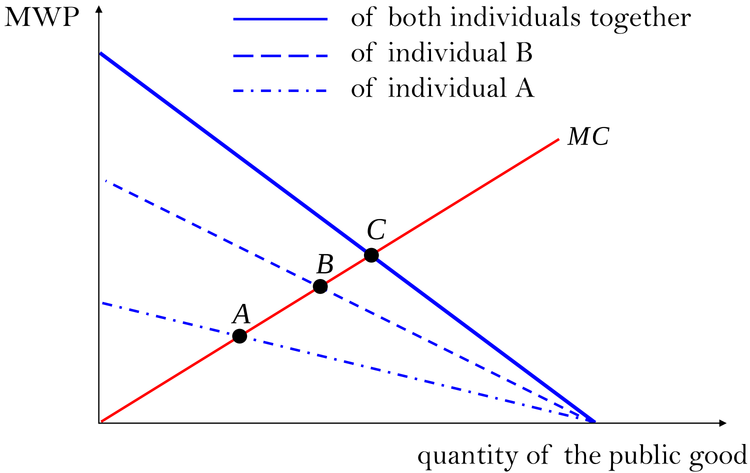
$$\sum_{i=1}^{n} \left| \frac{dx_i}{dG} \right|^{\text{indifference curve}} \stackrel{!}{=} \left| \frac{d\left(\sum_{i=1}^{n} x_i\right)}{dG} \right|^{\text{production possibility frontier}} = MC_G$$

Vertical aggregation of individual willingness to pay

private goods: horizontal aggregation
public goods: vertical aggregation



Aggregation of individual willingness to pay



Comparison private and public goods

	private goods	public goods
definition	rivalry in consumption	no rivalry in consumption
examples	apples	broadcasted TV series
aggregation	horizontal (quantities)	vertical (MWP)
optimality	$MRS \stackrel{!}{=} MRT$	$\sum MRS \stackrel{!}{=} MRT$
quantities	different	equal (?)
MRS	equal	different

Voluntary provision of public goods

- Two residents ($i = 1, 2$) think of purchasing a street lamp.
- Initial budget: w_1 and w_2
- Willingness to pay: $r_1 = 20$ and $r_2 = 30$
- Cost of a street lamp: C
  - If **only one** resident contributes, she has to pay $b = C$;
  - If **both** residents contribute, each one pays a share of $b = \frac{1}{2}C$.
- Utility function: $u_1\left(w_1 - b, S\right)$
  - $S = 0$: the street lamp is not provided
  - $S = 1$: the street lamp is provided

Voluntary provision of public goods

situation	utility for resident 1	utility for resident2
no one contributes	$u_1(w_1, 0)$	$u_2(w_2, 0)$
resident 1 contributes, resident 2 does not	$u_1(w_1 - C, 1)$	$u_2(w_2, 1)$
resident 2 contributes, resident 1 does not	$u_1(w_1, 1)$	$u_2(w_2 - C, 1)$
both residents contribute	$u_1\left(w_1 - \frac{1}{2}C, 1\right)$	$u_2\left(w_2 - \frac{1}{2}C, 1\right)$

Voluntary provision of public goods

Strategic game

		resident 2: contribute	resident 2: not contribute
res. 1	contribute	$\left(u_1\left(w_1 - \frac{1}{2}C, 1\right), u_2\left(w_2 - \frac{1}{2}C, 1\right)\right)$	$\left(u_1\left(w_1 - C, 1\right), u_2\left(w_2, 1\right)\right)$
	not contribute	$\left(u_1\left(w_1, 1\right), u_2\left(w_2 - C, 1\right)\right)$	$\left(u_1\left(w_1, 0\right), u_2\left(w_2, 0\right)\right)$

Voluntary provision of public goods

Strategic game

$r_1 = 20$ and $r_2 = 30$, $C = 10$

resident 2

		contribute	not contribute
res. 1	contribute	$(u_1(w_1 - 5, 1), u_2(w_2 - 5, 1))$	$(u_1(w_1 - 10, 1), u_2(w_2, 1))$
	not contribute	$(u_1(w_1, 1), u_2(w_2 - 10, 1))$	$(u_1(w_1, 0), u_2(w_2, 0))$

Voluntary provision of public goods

Strategic game

$r_1 = 20$ and $r_2 = 30$, $K = 24$

resident 2

		contribute	not contribute
res. 1	contribute	$(u_1(w_1 - 12, 1), u_2(w_2 - 12, 1))$	$(u_1(w_1 - 24, 1), u_2(w_2, 1))$
	not contribute	$(u_1(w_1, 1), u_2(w_2 - 24, 1))$	$(u_1(w_1, 0), u_2(w_2, 0))$

Voluntary provision of public goods

Strategic game

$r_1 = 20$ and $r_2 = 30$, $K = 36$

resident 2

		contribute	not contribute
res. 1	contribute	$(u_1(w_1 - 18, 1), u_2(w_2 - 18, 1))$	$(u_1(w_1 - 36, 1), u_2(w_2, 1))$
	not contribute	$(u_1(w_1, 1), u_2(w_2 - 36, 1))$	$(u_1(w_1, 0), u_2(w_2, 0))$

Voluntary provision of public goods

Strategic game

$r_1 = 20$ and $r_2 = 30$, $K = 70$

		resident 2: contribute	resident 2: not contribute
res. 1	contribute	$(u_1(w_1 - 35, 1), u_2(w_2 - 35, 1))$	$(u_1(w_1 - 70, 1), u_2(w_2, 1))$
	not contribute	$(u_1(w_1, 1), u_2(w_2 - 70, 1))$	$(u_1(w_1, 0), u_2(w_2, 0))$

Central tutorial

Problem S.6.1.
10 people with MWP € 2 for each additional street lamp
Cost for x street lamps $C(x) = x^2$
Pareto-optimal number of additional street lamps?

Problem S.6.2.
200 people with $U(x_i, y) = x_i + y^{\frac{1}{2}}$, where

- x_i : quantity of the private good
- y : quantity of the public good

$p_x = 1,\ p_y = 10$
Pareto-optimal quantity of the public good?

Problem S.6.3.
Blubber's willingness to pay for y units glitter dust: $WP_B(y) = 4y^2$
Callisto's willingness to pay for y units glitter dust: $WP_C(y) = 8y$
Pareto-optimal amount of glitter dust for $C(y) = 8y^2$?