Microeconomics

Monopoly and monopsony

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Structure

Introduction

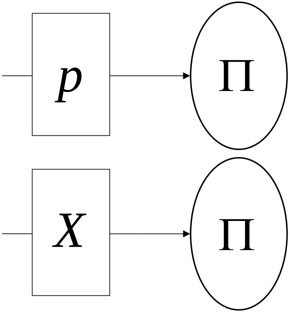
- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
  - **Monopoly and monopsony**
  - Game theory
  - Oligopoly
- External effects and public goods

Pareto-optimal review

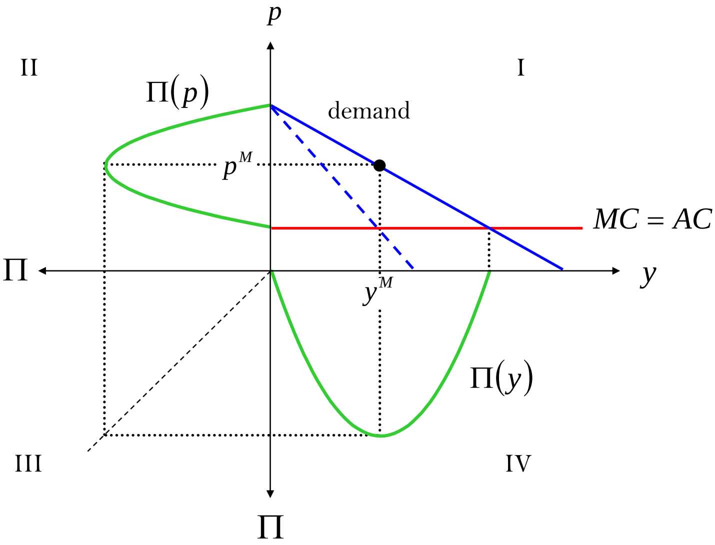
Definition monopoly and monopsony

- Monopoly: **one** firm sells
- Monopsony: **one** firm buys

- Monopoly:
  - Price setting
  - Quantity setting



Price versus quantity setting



Overview

- Definitions
- Price setting
  - Revenue and marginal revenue with respect to price
  - Profit
  - Profit maximization (without price differentiation)
- Quantity setting
  - Revenue and marginal revenue with respect to *price (?)*
  - Profit
  - Profit maximization without price differentiation
  - Profit maximization with price differentiation
- Quantity and profit taxes
- Welfare analysis
- Monopsony

Revenue and marginal revenue with respect to price

- Revenue for demand function $X(p)$:

$$R(p) = pX(p)$$

- Marginal revenue ($= MR$, here MR_p):

$$MR_p = \frac{dR}{dp} = X + p\frac{dX}{dp} \text{ (product rule)}$$

- If the price increases by one unit,
  - on the one hand, revenue increases by X (for every sold unit the firm obtains one Euro)
  - on the other hand, revenue decreases by $p\frac{dX}{dp}$ (the price increase decreases demand that is valued at price p)

Profit in the linear model

Definition

Let X be the demand function. Then

$$\underbrace{\Pi\left(p\right)}_{\text{profit}}:=\underbrace{R\left(p\right)}_{\text{revenue}}-\underbrace{C\left(p\right)}_{\text{cost}}$$

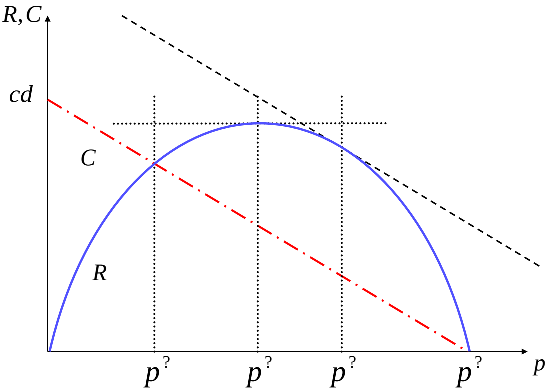
is profit depending on price p and

$$\begin{aligned}\Pi(p) &= p\left(d-ep\right)-c\left(\left(d-ep\right)\right),\\ c,d,e &\geq 0, p\leq\frac{d}{e}\end{aligned}$$

profit in the linear model.

Functions: price $\mapsto$ quantity $\mapsto$ cost

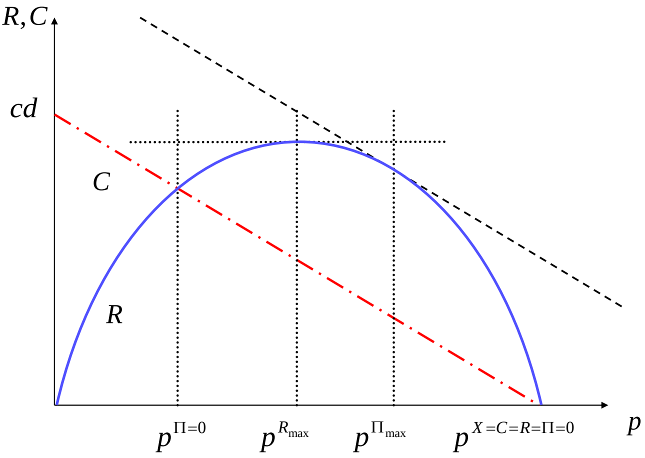
Revenue, cost and a question I



Problem

What is the economic meaning of the prices with question mark?

Revenue, cost and a question II



Marginal cost with respect to price and with respect to quantity

$\frac{dC}{dX}$: marginal cost (with respect to quantity)
$\frac{dC}{dp}$: marginal cost (with respect to price)

$$\frac{dC}{dp} = \underbrace{\frac{dC}{dX}}_{>0} \underbrace{\frac{dX}{dp}}_{<0} < 0.$$

Chain rule: differentiate $C(X(p))$ with respect to p means:

- first, differentiate C with respect to $X \Rightarrow$ marginal cost
- then, differentiate X with respect to $p \Rightarrow$ slope of demand function

Functions: price $\mapsto$ quantity $\mapsto$ cost

Profit maximization

Profit condition

$$\frac{d\Pi}{dp} \overset{!}{=} 0 \text{ or } \frac{dR}{dp} - \frac{dC}{dp} \overset{!}{=} 0 \text{ or}$$

$$\frac{dR}{dp} \overset{!}{=} \frac{dC}{dp}$$

Problem

Confirm: The profit-maximizing price in the linear model is $p^M = \frac{d+ce}{2e}$. Which price maximizes revenue?

We have

$$p^M = \frac{d + ce}{2e}.$$

How does p^M change if c increases?
Differentiation:

$$\frac{dp^M}{dc} = \frac{1}{2}$$

Exercises

Problem 1

Consider a monopolist with cost function $C(X) = cX$, $c > 0$, and demand function $X(p) = ap^{\varepsilon}$, $\varepsilon < -1$.

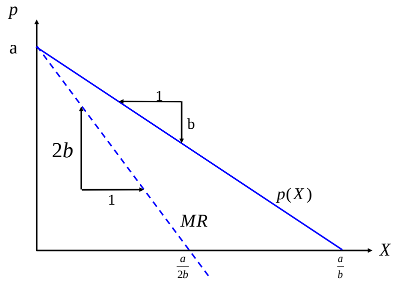
1. Determine
   - price elasticity of demand
   - marginal revenue with respect to price
2. Express the monopoly price as a function of ε!
3. Determine and interpret $\frac{dp^M}{d|\varepsilon|}$!

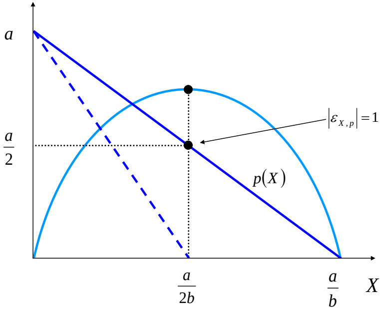
Problem 2

The demand function is given by $X(p) = 12 - 2p$ and the cost function of the monopolist by $C(X) = X^2 + 3$. Determine the profit-maximizing price!

The linear model

A reminder





Marginal revenue

- Marginal revenue and elasticity (Amoroso-Robinson relation)

$$MR = \frac{dR}{dX} = p + X\frac{dp}{dX} \text{ (product rule)}$$
$$= p\left[1 + \frac{1}{\varepsilon_{X,p}}\right] = p\left[1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right] > 0 \text{ for } \left|\varepsilon_{X,p}\right| > 1$$

- Marginal revenue equals price $MR = p + X \cdot \frac{dp}{dX} = p$ in three cases:
  - horizontal (inverse) demand, $\frac{dp}{dX} = 0$: $MR = p + X \cdot \underbrace{\frac{dp}{dX}}_{=0} = p$
  - first „small" unit, $X = 0$: $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX} = p = \frac{R(X)}{X}$
  - first-degree price differentiation, $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX}$

    $\Rightarrow$ see below

Profit

Definition

For $X \geq 0$ and inverse demand function p monopoly profit depending on quantity is given by

$$\underbrace{\Pi(X)}_{\text{profit}} := \underbrace{R(X)}_{\text{revenue}} - \underbrace{C(X)}_{\text{cost}} = p(X)\,X - C(X)$$

Linear case:

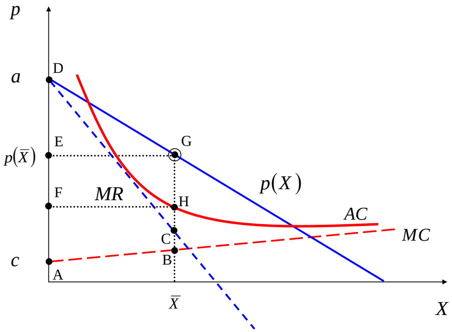
$$\Pi(X) = (a - bX)\,X - cX, \quad X \leq \frac{a}{b}$$

Profit

Average and marginal definition

profit for $\bar{X}$:

$$\begin{aligned} & \Pi(\bar{X}) \\ = \; & p(\bar{X})\bar{X} - C(\bar{X}) \\ = \; & [p(\bar{X}) - AC(\bar{X})]\,\bar{X} \\ & \text{(average definition)} \\ = \; & \int_0^{\bar{X}} [MR(X) - MC(X)]\, dX \\ & -F \text{ (if appropriate)} \\ & \text{(marginal definition)} \end{aligned}$$



Quantity setting with uniform price

- We have:
  - inverse demand function for the monopolist: $p(X)$
  - total cost: $C(X)$
- Monopolist's profit Π :

$$\begin{aligned}\Pi(X) &= R(X) - C(X) \\ &= p(X)X - C(X).\end{aligned}$$

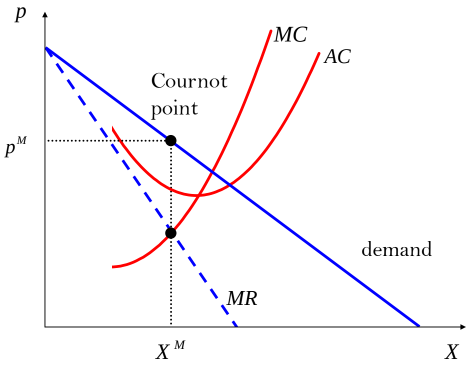
- Necessary condition for profit maximization:

$$\frac{d\Pi}{dX} = \frac{dR}{dX} - \frac{dC}{dX} \overset{!}{=} 0$$

or, equivalently,

$$MR \overset{!}{=} MC$$

Quantity setting with uniform price



Problem

Inverse demand function $p(X) = 27 - X^2$.
Revenue-maximizing and profit-maximizing price for $MC = 15$?

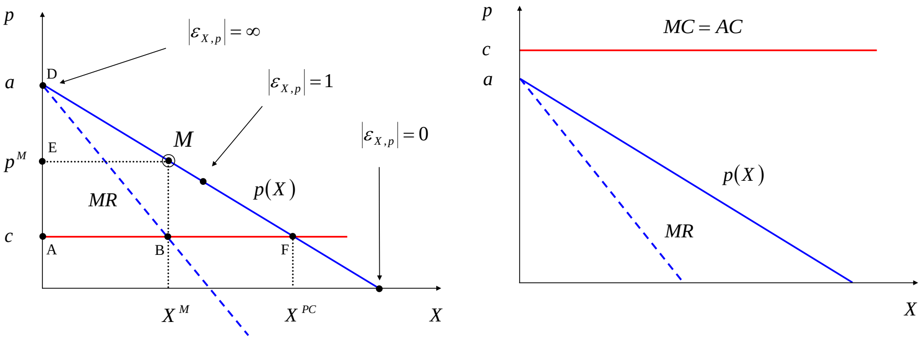
Clever man:

Antoine Augustin Cournot

- Antoine Augustin Cournot (1801-1877) was a French philosopher, mathematician, and economist.
- In his main work "Recherches sur les principes mathématiques de la théorie des richesses", 1838, Cournot presents essential elements of monopoly theory (this chapter) and oligopoly theory (next chapter)
- Inventor *(?)* of the Nash equilibrium

Quantity setting with uniform price

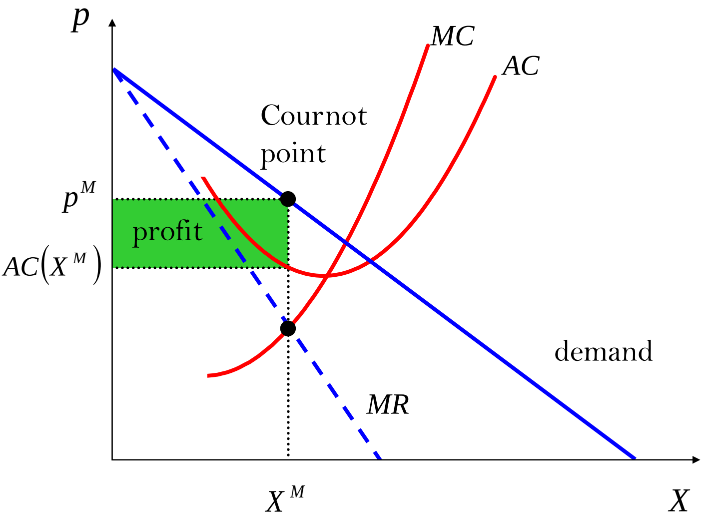
The linear model



$$X^M = X^M(c, a, b) = \begin{cases} \frac{1}{2}\frac{(a-c)}{b}, & c \leq a \\ 0, & c > a \end{cases}$$

Quantity setting with uniform price

Maximum profit



Quantity setting with uniform price

Comparative statics I

$$X^M(a,b,c) = \frac{1}{2}\frac{(a-c)}{b}, \text{ where } \frac{\partial X^M}{\partial c} < 0;\ \frac{\partial X^M}{\partial a} > 0;\ \frac{\partial X^M}{\partial b} < 0,$$
$$p^M(a,b,c) = \frac{1}{2}(a+c), \text{ where } \frac{\partial p^M}{\partial c} > 0;\ \frac{\partial p^M}{\partial a} > 0;\ \frac{\partial p^M}{\partial b} = 0,$$
$$\Pi^M(a,b,c) = \frac{1}{4}\frac{(a-c)^2}{b}, \text{ where } \frac{\partial \Pi^M}{\partial c} < 0;\ \frac{\partial \Pi^M}{\partial a} > 0;\ \frac{\partial \Pi^M}{\partial b} < 0.$$

Problem

Show $\Pi^M(c) = \frac{1}{4}\frac{(a-c)^2}{b}$ and determine $\frac{d\Pi^M}{dc}$! Hint: Use the chain rule.

Quantity setting with uniform price

Comparative statics I

Solution

$$\begin{aligned}\frac{d\Pi^M}{dc} &= \frac{d\left(\frac{1}{4}\frac{(a-c)^2}{b}\right)}{dc} \\ &= \frac{1}{4b}2(a-c)(-1) \\ &= -\frac{a-c}{2b}\end{aligned}$$

Alternative expressions for profit maximization

$$MC \overset{!}{=} MR = p\left[1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right]$$

$$p \overset{!}{=} \frac{1}{1 - \frac{1}{\left|\varepsilon_{X,p}\right|}} MC = \frac{\left|\varepsilon_{X,p}\right|}{\left|\varepsilon_{X,p}\right| - 1} MC$$

$$\frac{p - MC}{p} \overset{!}{=} \frac{p - p\left[1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right]}{p} = \frac{1}{\left|\varepsilon_{X,p}\right|}$$

Monopoly power

- perfect competition:
  Profit maximization implies "price = marginal cost"
  Explanation: With perfect competition every firm is "small" and has no influence on price. Inverse demand is then horizontal, hence $MR = p$.
- Monopoly:
  The optimal price is in general above marginal cost.

Definition (Lerner index)

$$\frac{p - MC}{p}$$

Monopoly power

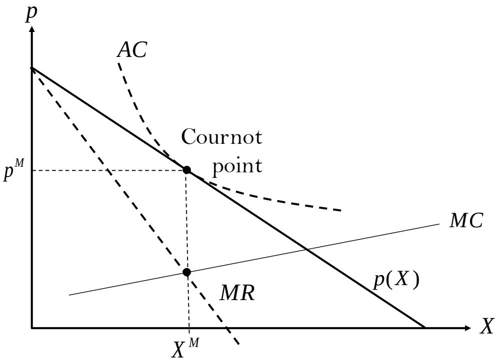
Lerner index

- Perfect competition: $p \overset{!}{=} MC$ and hence $\frac{p-MC}{p} \overset{!}{=} 0$
- Monopoly: $MC \overset{!}{=} MR = p\left[1 - \frac{1}{|\varepsilon_{X,p}|}\right]$ and hence

$$\frac{p - MC}{p} \overset{!}{=} \frac{p - MR}{p} = \frac{p - p\left[1 - \frac{1}{|\varepsilon_{X,p}|}\right]}{p} = \frac{1}{|\varepsilon_{X,p}|}$$

Interpretation: If demand reacts strongly to price increases, the monopolist wants to choose a price close to marginal cost.

Monopoly power, but zero monopoly profit



$$p > MC, \text{ but } AC\left(X^M\right) = \frac{C\left(X^M\right)}{X^M} = p^M$$

Forms of price differentiation

- **First-degree** price differentiation:
  Every consumer pays his willingness to pay
  $\Longrightarrow$ complete absorption of consumer surplus
- **Second-degree** price differentiation:
  The firm requires different prices for different quantities (e.g., quantity discount)
  $\Longrightarrow$ different prices for high-intensity users and low-intensity users
- **Third-degree** price differentiation:
  Consumers are grouped in different categories.
  $\Longrightarrow$ uniform price only within a category

First-degree price differentiation

Every consumer pays his willingness to pay:

$$MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX} = p$$

A price decrease resulting from an extension of output concerns

- only the marginal consumer,
- but not inframarginal consumers (those with a higher willingness to pay)

First-degree price differentiation

Formally: Take the derivative of revenue with respect to quantity

$$MR = \frac{d\left(\int_0^y p\left(q\right)dq\right)}{dy} = p\left(y\right)$$

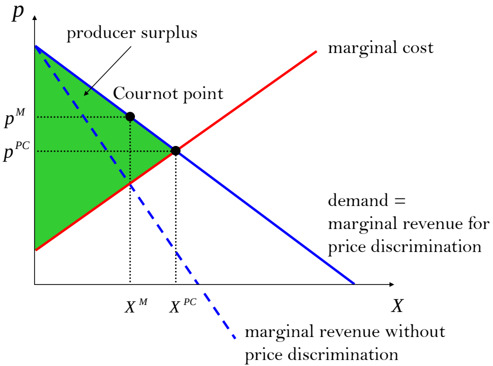
Hint: Differentiating an integral with respect to the upper bound of integration yields the value of the integrand (here $p\left(q\right)$) at the upper bound.

Optimality condition:

$$p = MR \stackrel{!}{=} MC$$

First-degree price differentiation

Marginal revenue

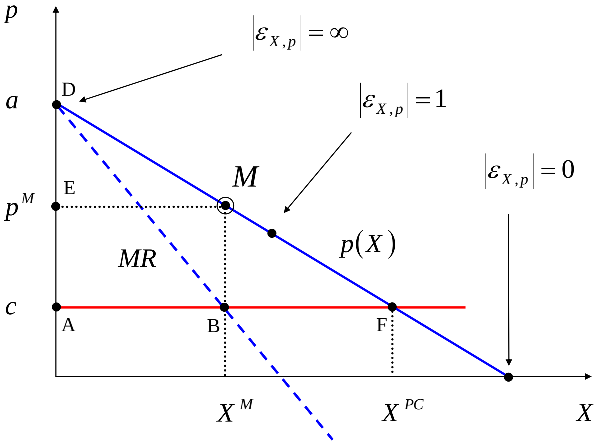


Problem

$X(p) = 12 - \frac{1}{2}p,\ C(X) = X^2 + 2$

First-degree price differentiation

Comparison of profits



First-degree price differentiation

Exercise

A book shop can produce a book at constant marginal cost of 8 (no fixed cost). 11 potential buyers have a maximum willingness to pay of 55, 50, 45, ..., 10, and 5.

a) No price differentiation:
   Price, number of books, profit?
b) First-degree price differentiation:
   Price, number of books, profit?

Third-degree price differentiation

Two markets, one production site I

Students, pensioners, children, day versus night demand

- Profit

$$\Pi(x_1, x_2) = p_1(x_1)\, x_1 + p_2(x_2)\, x_2 - C(x_1 + x_2),$$

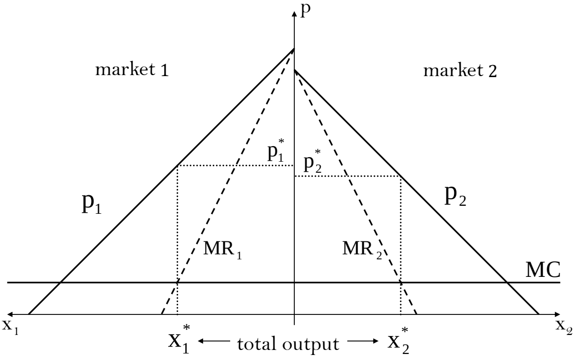
- Maximization condition

$$\begin{aligned} \frac{\partial \Pi(x_1, x_2)}{\partial x_1} &= MR_1(x_1) - MC(x_1 + x_2) \overset{!}{=} 0, \\ \frac{\partial \Pi(x_1, x_2)}{\partial x_2} &= MR_2(x_2) - MC(x_1 + x_2) \overset{!}{=} 0. \end{aligned}$$

- $MR_1(x_1) \overset{!}{=} MR_2(x_2)$
- Assume $MR_1 < MR_2$. Then ...

Third-degree price differentiation

Two markets, one production site II



If $MC\left(x_1^* + x_2^*\right) < MR_1\left(x_1^*\right) = MR_2\left(x_2^*\right)$ then produce more *(not in german slides!)*

Third-degree price differentiation

Two markets, one production site III

- $MR_1\left(x_1^*\right) = MR_2\left(x_2^*\right)$:

$$p_1^M \left[1 - \frac{1}{|\varepsilon_1|}\right] \overset{!}{=} p_2^M \left[1 - \frac{1}{|\varepsilon_2|}\right]$$

-

$$|\varepsilon_1| > |\varepsilon_2| \Rightarrow p_1^M < p_2^M.$$

Hence: inverse elasticity rule

One market, two production sites

- Profit:

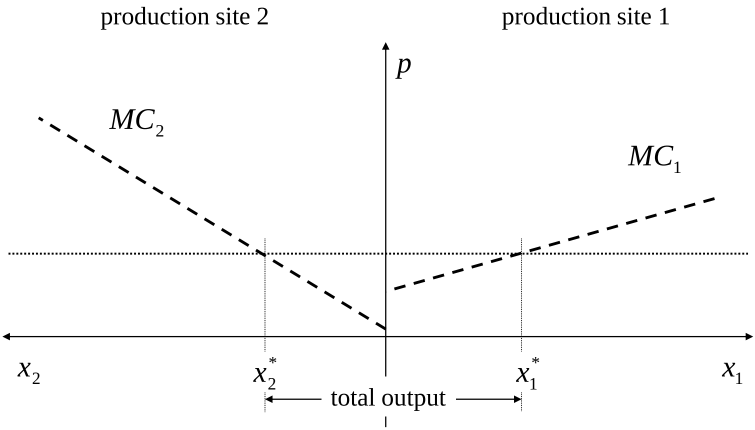
$$\Pi(x_1, x_2) = p(x_1 + x_2)(x_1 + x_2) - C_1(x_1) - C_2(x_2)$$

- Maximization conditions:

$$\begin{aligned}\frac{\partial \Pi(x_1, x_2)}{\partial x_1} &= MR(x_1 + x_2) - MC_1(x_1) \stackrel{!}{=} 0 \\ \frac{\partial \Pi(x_1, x_2)}{\partial x_2} &= MR(x_1 + x_2) - MC_2(x_2) \stackrel{!}{=} 0\end{aligned}$$

- $MC_1 \stackrel{!}{=} MC_2$
- Assume $MC_1 < MC_2$. Then ...

One market, two production sites



Exercises

Problem 1

Assume that price differentiation is not possible. Determine X^M for $p(X) = 24 - X$ and constant marginal cost $c = 2$! Moreover, determine X^M for $p(X) = \frac{1}{X}$ and constant marginal cost c!

Problem 2

On the first submarket, inverse demand is given by $p_1 = 12 - 4x_1$, on the second submarket by $p_2 = 8 - \frac{1}{2}x_2$. Marginal cost equal 4. Determine prices on the two submarkets. Can you confirm the inverse elasticity rule?

- increases the cost of producing one unit by tax rate t for every unit
- increases marginal cost from MC to $MC + t$

$$\begin{aligned} MR &= a - 2bX \overset{!}{=} MC + t \\ &\Rightarrow X^M(t) = \frac{a - MC - t}{2b} \\ &\Rightarrow p^M(t) = a - bX^M(t) \\ &= \frac{a + MC + t}{2} \end{aligned}$$

Half of the tax is passed on to consumers

Problem

Draw a figure!

Quantity and profit taxes

Profit tax I

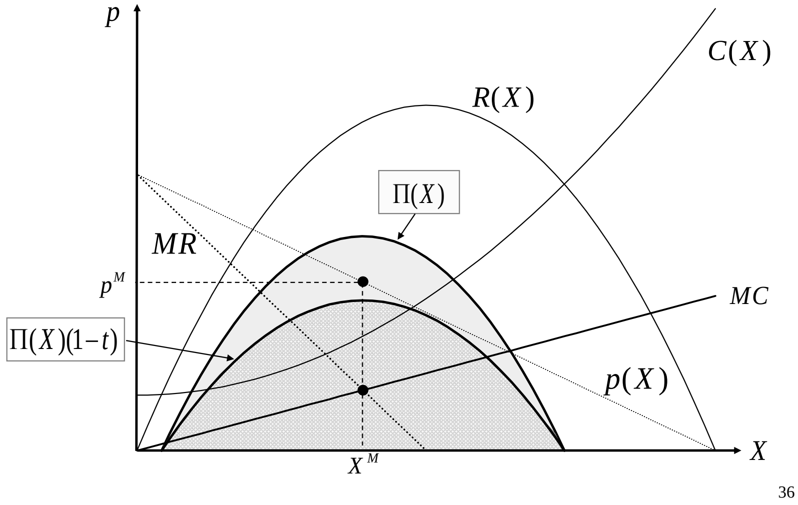
- A share of profit is payed to the state.
- If this share, τ , is constant, then instead of profit before tax $R(X) - C(X)$ the firm obtains only profit after tax

$$(1-\tau)\left[R(X) - C(X)\right].$$

$\Longrightarrow$ introduction of a profit tax does not change the profit-maximizing quantity

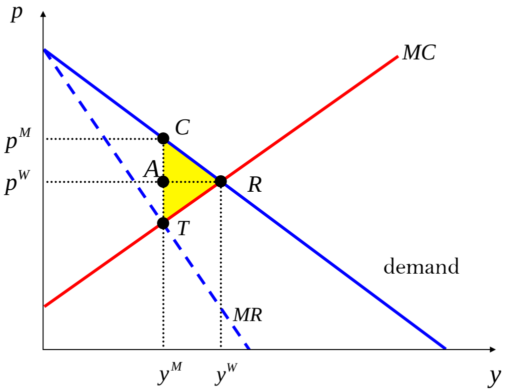
Quantity and profit taxes

Profit tax II



Monopoly with uniform price

Welfare loss



Problem

In which case do we obtain the largest sum of consumer surplus and producer surplus?

Problem

Transition $C \to R$ Pareto improvement?

Problem

$D(q) = -2q + 12,\ MC(q) = 2q$

Exercises

$y(p) = 8 - \frac{1}{2}p$
$MC = 4$, no fixed cost
Quantity tax $t = 4$

a) Price, consumer surplus, and producer's profit before introduction of the tax?
b) Price, consumer surplus, and producer's profit after introduction of the tax?
c) Tax revenue?
d) Sketch welfare loss!

Monopsony

- $y = f(x_1, x_2)$: Output resulting from factor-input combination (x_1, x_2)
- Profit:

$$\Pi(x_1, x_2) = \underbrace{p(f(x_1, x_2)) \cdot f(x_1, x_2)}_{\text{revenue}} - \underbrace{(w_1(x_1)\, x_1 + w_2(x_2)\, x_2)}_{\text{cost}}$$

- A necessary condition for a profit maximum is:

$$\begin{aligned}
\frac{\partial \Pi(x_1, x_2)}{\partial x_1} &= \frac{dp}{dy}\frac{\partial y}{\partial x_1} y + p(y)\frac{\partial y}{\partial x_1} - \left(w_1(x_1) + \frac{dw_1(x_1)}{dx_1} x_1 \right) \\
&= \left(\frac{dp}{dy} y + p(y) \right) \frac{\partial y}{\partial x_1} - MC_1 \\
&= MR \cdot MP_1 - MC_1 \\
&= \text{marginal revenue product} - \text{marginal cost} \stackrel{!}{=} 0
\end{aligned}$$

Monopsony

- Necessary conditions for profit maximization:

$$MR_1 \overset{!}{=} MC_1$$
$$MR_2 \overset{!}{=} MC_2$$

- The marginal revenue product is given by

$$MR_1 = \frac{dR}{dy}\frac{\partial y}{\partial x_1} = MR \cdot MP_1.$$

Problem

How do you determine the factor-demand curve in case of a monopsony?

Problem

Why is marginal revenue not equal to price?

Monopsony

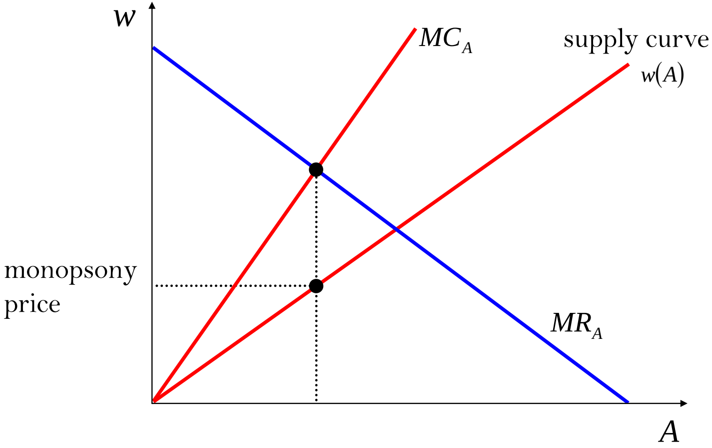
- Marginal cost of a factor is different from that factor's price.
- Differentiating the cost for factor 1 with respect to the number of factor units yields marginal cost of factor 1:

$$MC_1 = \frac{\partial C}{\partial x_1} = w_1 + \frac{dw_1}{dx_1}x_1.$$

Problem

Determine the marginal cost function of labor (A) for the inverse factor-demand function $w(A) = a + bA$.

Monopsony



Cost of labor graphically?
Exploitation?

Monopsony

Problem

How would you define supply elasticity of labor? How does the marginal cost of labor relate to its supply elasticity?
Hence, again Amoroso-Robinson ...

Monopsony

market for goods	factor market
optimality condition for factor usage	
$$MR_1 = \frac{\partial R}{\partial x_1} = \frac{dR}{dy}\frac{\partial y}{\partial x_1} = MR \cdot MP_1$$	$$MC_1 = \frac{\partial C}{\partial x_1} = w_1 + x_1 \frac{dw_1}{dx_1}$$
special case: price taker on market for goods $(MR = p)$ $$MR_1 = p \cdot MP_1 = MVP_1$$	special case: Price taker on factor market $\left(\frac{dw_1}{dx_1} = 0\right)$ $$MC_1 = w_1$$

Central tutorial I

Problem O.6.1.

$C(y) = \frac{1}{2}y^2$, $p(y) = 18 - y$

Cournot monopoly quantity?

Problem O.6.2.

$y(p) = 100 - p$

Two production sites, $y = y_1 + y_2$, with

- $MC_1 = y_1 - 5$
- $MC_2 = \frac{1}{2}y_2 - 5$

Optimal outputs?

Problem O.6.3.

Swimming pool with x visitors

$C(x) = 1.500.000$

Demand adults: $x_E = 400.000 - 40.000 p_E$

Demand children: $x_K = 400.000 - 200.000 p_K$

Third-degree price differentiation

Central tutorial II

Problem O.6.4.
$C(y) = y^2 + 2$
$D(p) = 10 - 2p$
First-degree differentiation

Problem O.6.5.
Banana Co. is the only employer on the island Banana
Inverse supply function for labor: $w(L) = 10 + L$
Production function: $f(L) = 10L$
World-market price for Bananas $= 2$

- How many workers does Banana Co. hire?
- Wage?