Microeconomics

The first welfare theorem

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Structure

Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
  - Perfect competition
  - **The first welfare theorem**
  - Monetary assessment of environmental impacts
- Types of markets
- External effects and public goods

Pareto-optimal review

Overview

- Pareto improvements and Pareto optimality
- The first welfare theorem
  - Content of the first welfare theorem
  - Optimality in exchange
  - Optimal factor usage
  - Optimal coordination of production and consumption
  - Summary

Pareto improvement

How can we measure the aggregate effect of a law or of

- changes in taxation
- subsidies
- customs?

- add utilities?
  $\Rightarrow$ contradicts ordinal utility theory
- maybe everyone is better off $\Rightarrow$ Pareto improvement

more precisely:

Someone is better off,

and nobody is worse off.

Pareto improvement

Judgment of people is accepted.
No benevolent dictator

Problems:

1. No distinction whether the improvement concerns rich or poor people
2. Pareto-improving laws hardly ever exist if millions of people are affected.
   Therefore, applications in reality are limited. Applications in theory are not.

Pareto efficiency = Pareto optimality

= no Pareto improvement possible

Problem

Distribution of 10 bottles of lemonade

Distribution	Emily	Leonie	Moritz
A	2	4	4
B	1	5	3
C	5	5	0
D	1	4	3

- Distribution B is a Pareto improvement relative to D.
- Distributions B and C are Pareto efficient.
- A Pareto-efficient distribution cannot be a Pareto improvement relative to another Pareto-efficient distribution.

Pareto optimality

Problem

a) How do you assess redistributions of income from the perspective of the Pareto criterion if inequalities with respect to income distribution are thereby eliminated?
b) How about situations where one individual possesses everything?

- **Potential Pareto improvement**: beneficiaries can compensate the injured party and still stay beneficiaries.
- **Kaldor criterion:** Decision rule on the basis of potential Pareto improvements

Theorem

A system of perfectly competitive markets is Pareto efficient.

Proof in three parts:

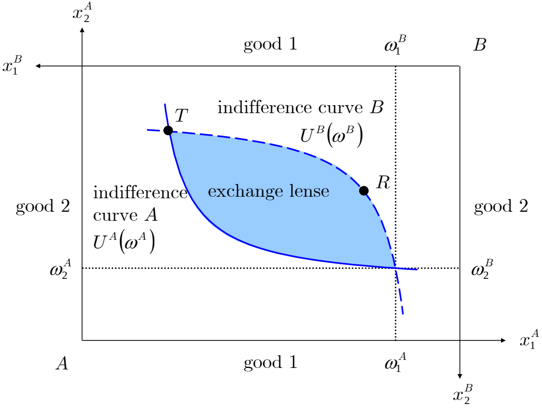
- Pareto optimality in exchange
- Pareto-optimal production
- Pareto-optimal product mix

Assumptions:

- "nicely shaped" indifference curves, isoquants, transformation frontiers, etc.
- monotonic preferences

Optimality in exchange
Assumptions

- 2 persons, 2 goods
- Endowments
  - Individual A owns $\omega^A = \left(\omega_1^A, \omega_2^A\right)$,
  - Individual B owns $\omega^B = \left(\omega_1^B, \omega_2^B\right)$.
- Utility functions U^A and U^B, respectively



- Point = allocation
- Width $= \omega_1^A + \omega_1^B$
- Height $= \omega_2^A + \omega_2^B$

- Preferences
  Which allocation
  is better (for whom?)
  than ω?

Optimality in exchange

Optimality in exchange implies

$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A \overset{!}{=} MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|$$

because if

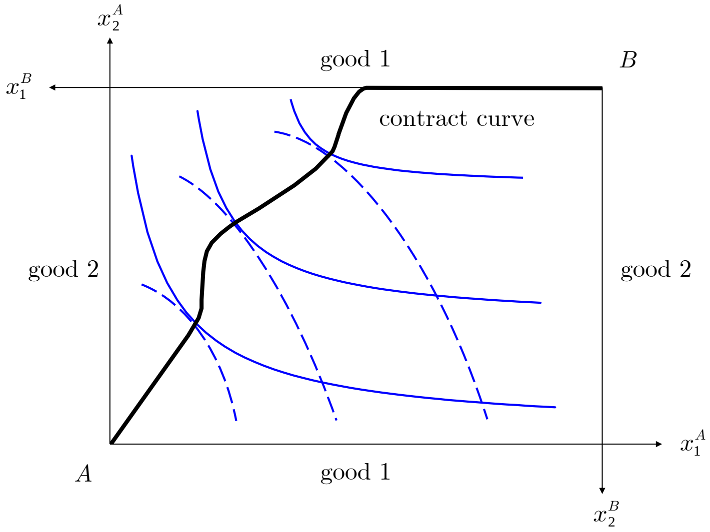
$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A < MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|.$$

held, A could

- give one small unit to B (?) or
- receive one small unit from B (?)

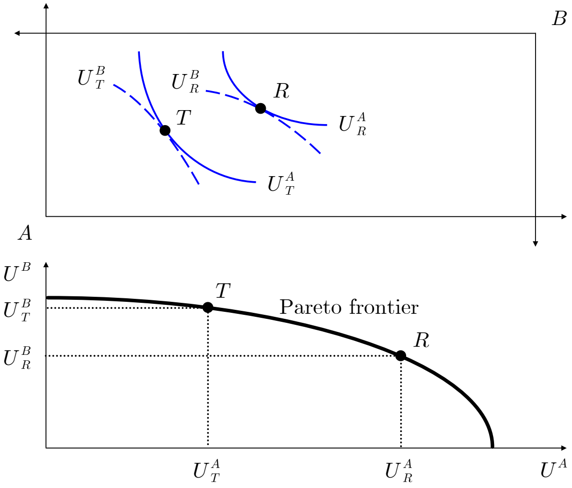
Contract curve or exchange line: Locus of all Pareto-optimal allocations in the exchange Edgeworth box

Optimality in exchange
Contract curve



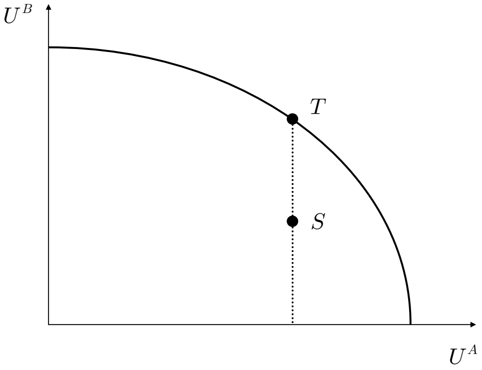
Optimality in exchange
Utility-possibility frontier

highest utility level for B if A realizes a particular fixed utility level



Optimality in exchange

Utility-possibility frontier



Problem

Are the points S and T Pareto optimal?

Optimality in exchange

- In the household optimum we have for both households:

$$\text{marginal rate of substitution} \overset{!}{=} \text{price ratio}$$

- This yields:

$$MRS^A \left(\overset{!}{=} \frac{p_1}{p_2} \right) \overset{!}{=} MRS^B.$$

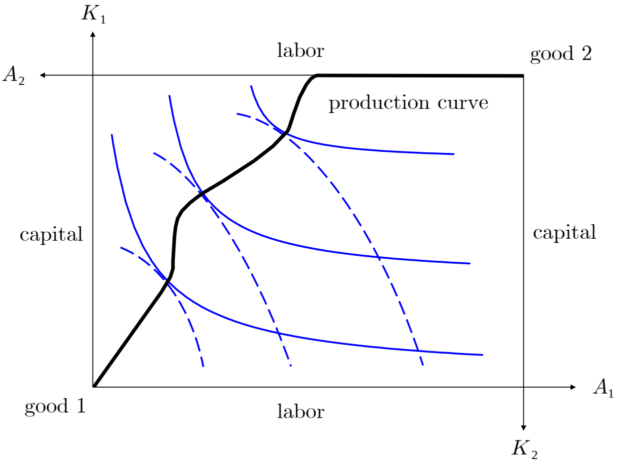
This equality is the condition for optimality in exchange!

- First part of the proof done!

Optimal factor usage

Production Edgeworth box

Efficient production: no further unit of good 1 can be produced without producing less of good 2.



Optimal factor usage

Optimal factor usage implies

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 \stackrel{!}{=} MRTS_2 = \left|\frac{dK_2}{dA_2}\right|,$$

because if

$$\left|\frac{dK_1}{dA_1}\right| = MRTS_1 > MRTS_2 = \left|\frac{dK_2}{dA_2}\right|,$$

held, then one small unit of labor could be used for

- production of good 1 instead of production of good 2 (?) or
- production of good 2 instead of production of good 1 (?).

Production curve: locus of all combinations of capital and labor that satisfy equality of marginal rates of technical substitution

Optimal factor usage

- Cost minimization implies

$$\text{marginal rate of technical substitution} \overset{!}{=} \text{factor-price ratio}$$

- This yields:

$$MRTS_1 = \left|\frac{dK_1}{dA_1}\right| \left(\overset{!}{=} \frac{w_A}{w_K}\right) \overset{!}{=} \left|\frac{dK_2}{dA_2}\right| = MRTS_2$$

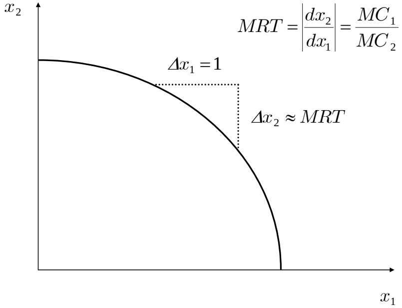
This equality is the condition for optimal factor usage!

- Second part of the proof done!

Optimal coordination of production and consumption

Production-possibility frontier=transformation frontier

- Contract curve ⇒ utility-possibility frontier
- Production curve ⇒ production-possibility frontier
  example: $x_2 = 1.000 - \frac{1}{5}x_1$



Optimal coordination of production and consumption

Production-possibility frontier=transformation frontier

Marginal rate of transformation $(= MRT)$:
Interpretation: How many units of good 2 have to be given up if one additional unit of good 1 is to be produced?
MRT = ratio of marginal cost:

- No change in cost along the production curve:
  $C(x_1, x_2) = C(x_1, f(x_1)) =$ constant
- Differentiating yields

$$\frac{\partial C}{\partial x_1} + \frac{\partial C}{\partial x_2}\frac{df(x_1)}{dx_1} = 0$$

  and hence

$$MRT = \left|\frac{df(x_1)}{dx_1}\right| = \frac{MC_1}{MC_2}.$$

Optimal coordination of production and consumption

Optimal mix of production implies

$$\left|\frac{dx_2}{dx_1}\right|^{\text{transformation frontier}} = MRT \overset{!}{=} MRS = \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}}$$

because if

$$\left|\frac{dx_2}{dx_1}\right|^{\text{transformation frontier}} > \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}}$$

held, then production and consumption of good 1 could

- increase by one small unit (?) or
- decrease by one small unit (?).

Optimal coordination of production and consumption

- In case of perfect competition

$$\textbf{price} \overset{!}{=} \textbf{marginal cost}$$

- This yields:

$$MRT = \frac{MC_1}{MC_2} \left(\overset{!}{=} \frac{p_1}{p_2} \right) \overset{!}{=} MRS.$$

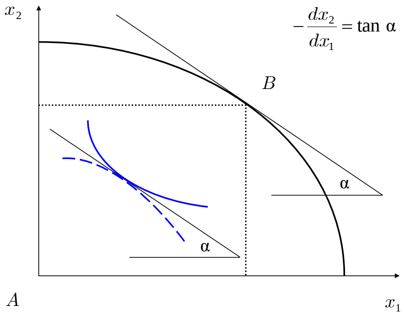
This equality is the condition for optimal coordination of production and consumption!

- Third part of the proof done!

Summary of the proof

Pareto optimality requires	**Perfect competition yields**
$MRS^A \overset{!}{=} MRS^B$ optimality in exchange	$MRS^A \overset{!}{=} \frac{p_1}{p_2} \overset{!}{=} MRS^B$
$MRTS_1 \overset{!}{=} MRTS_2$ optimal factor usage	$MRTS_1 \overset{!}{=} \frac{w_1}{w_2} \overset{!}{=} MRTS_2$
$MRS \overset{!}{=} MRT$ optimal product mix	$MRS \overset{!}{=} \frac{p_1}{p_2} \overset{!}{=} \frac{MC_1}{MC_2} \overset{!}{=} MRT$

Exchange Edgeworth box and transformation frontier in case of Pareto efficiency



- Optimality in exchange
  Indifference curves are tangent:
  $MRS^A \overset{!}{=} MRS^B$
- Optimal factor usage
  B lies on the transformation frontier, derived from the production curve:
  $MRTS_1 \overset{!}{=} MRTS_2$
- Optimal product mix
  Angle α :
  $MRS \overset{!}{=} MRT$

So what?

Central tutorial

Problem M.4.1.
Edgeworth box with identical utility functions $U(x_1, x_2) = x_1 x_2$
$\omega^A = (10, 90)$, $\omega^B = (90, 10)$.

a) Draw the Edgeworth box!
b) Determine and draw the contract curve!
c) Starting at the endowment, what is the best bundle that B can reach by voluntary trade?
d) Graphically, starting from the endowment:
   - set of Pareto improvements (exchange lens),
   - set of Pareto-efficient Pareto improvements
e) Pareto frontier analytically?