Microeconomics

Cost

Harald Wiese

Leipzig University

Structure

Introduction

- Household theory
- Theory of the firm
  - Production theory
  - **Cost**
  - Profit maximization
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

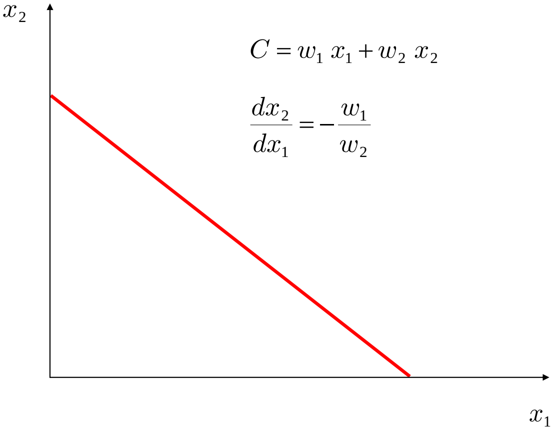
Pareto-optimal review

Overview

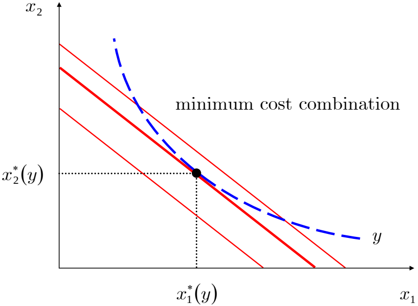
- Household theory and theory of the firm
- Cost function
- Marginal and average cost
- Short-run and long-run cost
- Fixed, quasifixed and variable cost

Household theory and theory of the firm

household theory	**Theory of the firm**
good	factor
utility function	production function
indifference curve	isoquant
budget line	isocost line
maximization of utility for a given income	maximization of output for a given cost budget
minimization of expenditures for given utility	minimization of expenditures for given output
expenditure function	cost function



x_2
$C = w_1 x_1 + w_2 x_2$
$\frac{dx_2}{dx_1} = -\frac{w_1}{w_2}$
x_1



Utility maximization:
Highest reachable indifference curve, i.e.,

$$MRS \overset{!}{=} \frac{p_1}{p_2} \text{ and } p_1x_1 + p_2x_2 \overset{!}{=} m$$

Cost minimization:
Lowest possible isocost line, i.e.,

$$MRTS \overset{!}{=} \frac{w_1}{w_2} \text{ and } f(x_1, x_2) \overset{!}{=} y$$

Cost function

Definition (Cost function)

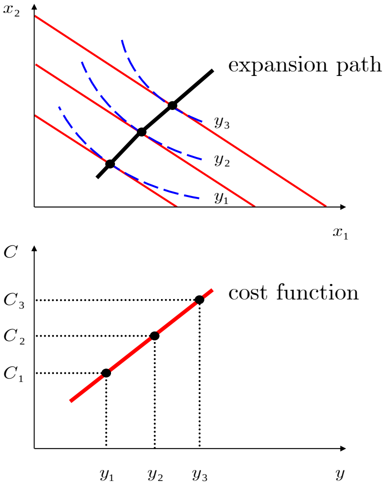
$$C(y) := \min_{\substack{x_1, x_2 \\ \text{with } y = f(x_1, x_2)}} (w_1 x_1 + w_2 x_2)$$

Hence: Cost for y are the expenditures for the factor inputs necessary to produce y.

Problem

What is the income consumption curve?

Cost function



Assign an optimum $\left(x_1^*\left(y\right), x_2^*\left(y\right)\right)$ to every y!

- Expansion path:
  Express the locus of
  these cost-minimizing variables
  in a function $x_2 = h\left(x_1\right)$!
- Cost function:
  Describe the locus of $\left(y, C\left(y\right)\right)$

Cost function

Derivation

- Cobb-Douglas production function $y = f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$, $w_1 = 1$, $w_2 = 2$
- Condition for cost minimization:

$$MRTS = \frac{MP_1}{MP_2} = \frac{\frac{1}{3} x_1^{\frac{1}{3}-1} x_2^{\frac{2}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{\frac{2}{3}-1}} = \frac{1}{2}\frac{x_2}{x_1} \stackrel{!}{=} \frac{1}{2}$$

$\Longrightarrow x_2 \stackrel{!}{=} x_1$

- Thereby, we obtain output

$$y = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} = x_1^{\frac{1}{3}} x_1^{\frac{2}{3}} = x_1.$$

- Cost function

$$C(y) = w_1 x_1(y) + w_2 x_2(y) = 3y$$

Marginal and average cost

- Marginal cost $(= MC)$

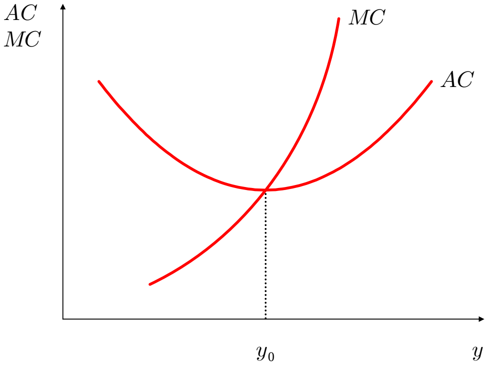
$$MC = \frac{dC}{dy}$$

- Average cost $(= AC)$

$$AC = \frac{C(y)}{y}$$

Hence, again: marginal ... versus average ...

Marginal and average cost



Problem

Marginal and average cost for $C(y) = y^3 - 10y^2 + 29y$?

Problem

Marginal and average cost for $C(y) = 2y + 3$ at $y = 0$!

Marginal cost and marginal product

One production factor only:

$$C\left(y\right)=wx\left(y\right)$$

and hence

$$MC=\frac{dC}{dy}=w\frac{dx}{dy}=\frac{w}{\frac{dy}{dx}}=\frac{w}{MP}$$

Robert Sanden (Schwester Helga 46f.):

Soll sich, was du produzierst, auch lohnen,
achte auf deine Kostenfunktionen,
sei vor allem auf dem Posten,
in Bezug auf die Grenzkosten,
denn, wie sollt' es anders sein,
sind sie des Grenzertrages Widerschein.

Short-run and long-run cost

- Not all production factors are variable in the short run, e.g.,
  - Machines and buildings
  - Number of employees
- long-run cost function: optimal choice of all production factors
- short-run cost function: optimal choice of those production factors that are variable in the short run

Short-run and long-run cost

- Short-run cost lie above long-run cost.
- Short-run cost equal long-run cost for the output that is optimal for the firm size

Fixed factors: factors that are not variable in the short run

Definition (short-run cost function for fixed factor 2)

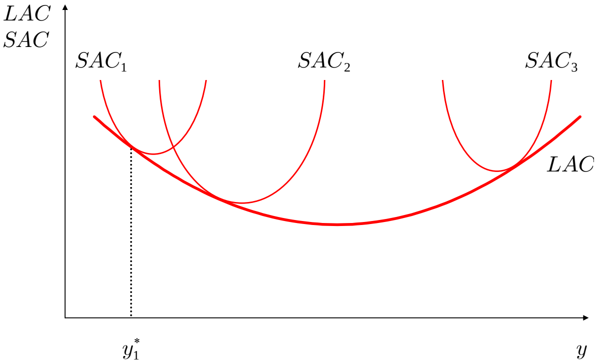
$$C_s(y) = C_{\overline{x_2}}(y) = \min_{\substack{x_1 \\ \text{with } y=f(x_1,\overline{x_2})}} \left(w_1 x_1 + w_2 \overline{x_2}\right).$$

Problem

$y = f(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ and $\overline{x_2} = 8$
$w_1 = 2$, $w_2 = 3$
Short-run cost function?

Short-run and long-run cost

Short-run and long-run average cost



Non-optimal choice:
short-run average cost > long-run average cost

Problem

$y = f(K, A)$

K is the fixed factor with K_1, K_2 or $K_3 = 0$

- $K = K_1 \Rightarrow SAC_1 = y^2 - 4y + 6$
- $K = K_2 \Rightarrow SAC_2 = y^2 - 8y + 18$
- $K = 0 \Rightarrow$ no output > 0 is optimal

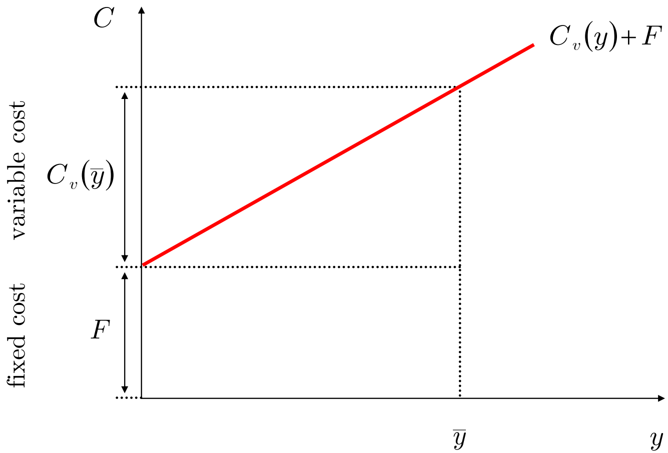
Sketch the long-run average-cost function!

Fixed and variable cost

- Fixed cost (F): independent of output.
- Variable cost $C_v(y)$: varying in output.
- Short-run cost can be split in fixed and variable cost:

$$C_s(y) = C_v(y) + F.$$

Fixed and variable cost



Fixed and variable cost

Problem

Factor labor, A, and capital, K, per year, respectively
labor variable
For $K_0 = 1000$ the production function is given by $y = \sqrt{A}$
$r = 5\%$ (price of capital, interest rate)
$w = 20\,000$ € per year

- short-run cost function?
- fixed and variable cost?

Fixed, quasifixed and variable cost

In the long run all factors are variable by definition.

Definition (quasifixed cost)

Cost that are not varying in output as long as output is positive.

Problem

Is the cost of 5 fixed or quasifixed cost for the following cost functions?

$$C(y) = \begin{cases} 0 & \text{for } y = 0 \\ 2y + 5 & \text{for } y > 0 \end{cases}$$

$$C_s(y) = \begin{cases} 5 & \text{for } y = 0 \\ 2y + 5 & \text{for } y > 0 \end{cases}$$

Central tutorial I

Problem J.7.1.
$MC = 2y$
variable cost for producing 10 units?

- continuous case (integration!) and
- discrete case (the cost of the first unit are 2)

Problem J.7.2.
$y = f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$
long-run marginal-cost function?

Problem J.7.3.
$y = f(x_1, x_2) = \min(x_1, 2x_2)$
long-run cost function?

Central tutorial II

Problem J.7.4.

A firm with two production sites

Marginal cost:

		1. unit	2. unit	3. unit	4. unit
marginal	production site 1	2	3	4	5
cost	production site 2	4	5	6	7

How is production of 2 units distributed? How is production of 4 units distributed?

Central tutorial III

Problem J.7.5.
$y = f(x_1, x_2) = x_1^{\frac{1}{2}} x_2$
short run $x_2 = 50$
$w_1 = 250$, $w_2 = 3$
The firm's short-run marginal-cost function (SMC)?

Problem J.7.6.
$C_s(y) = 1 + y^2$
Short-run variable average cost?
Short-run average cost?
Short-run marginal-cost function?
Draw a figure!