Microeconomics

Production theory

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Structure

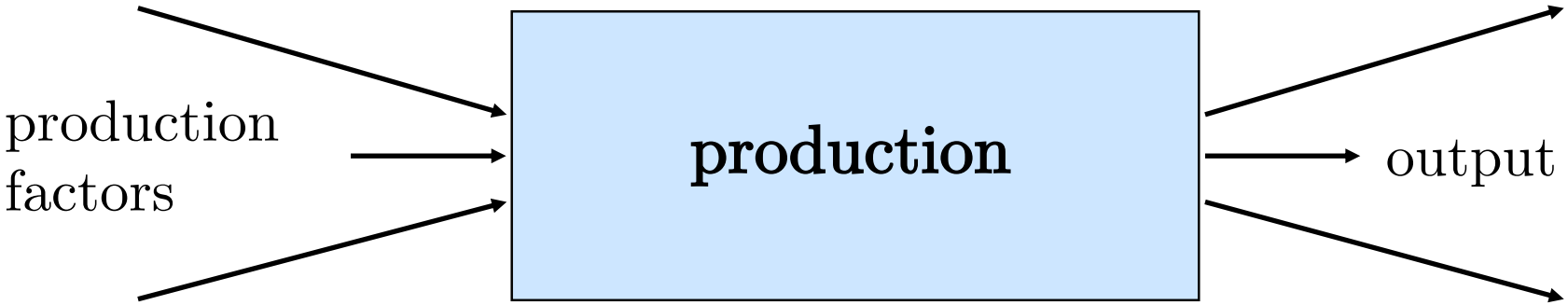
Introduction

- Household theory
- Theory of the firm (SH 43)
  - **Production theory**
  - Cost
  - Profit maximization
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

Introduction

Production process



(SH 29)

Overview

- Introduction
- Production function
- Partial factor variation
- Proportional factor variation
- Isoquants and marginal rate of technical substitution
- Overview: factor variations

Production function

- states the maximum output of a good that can be produced using given quantities of production factors:

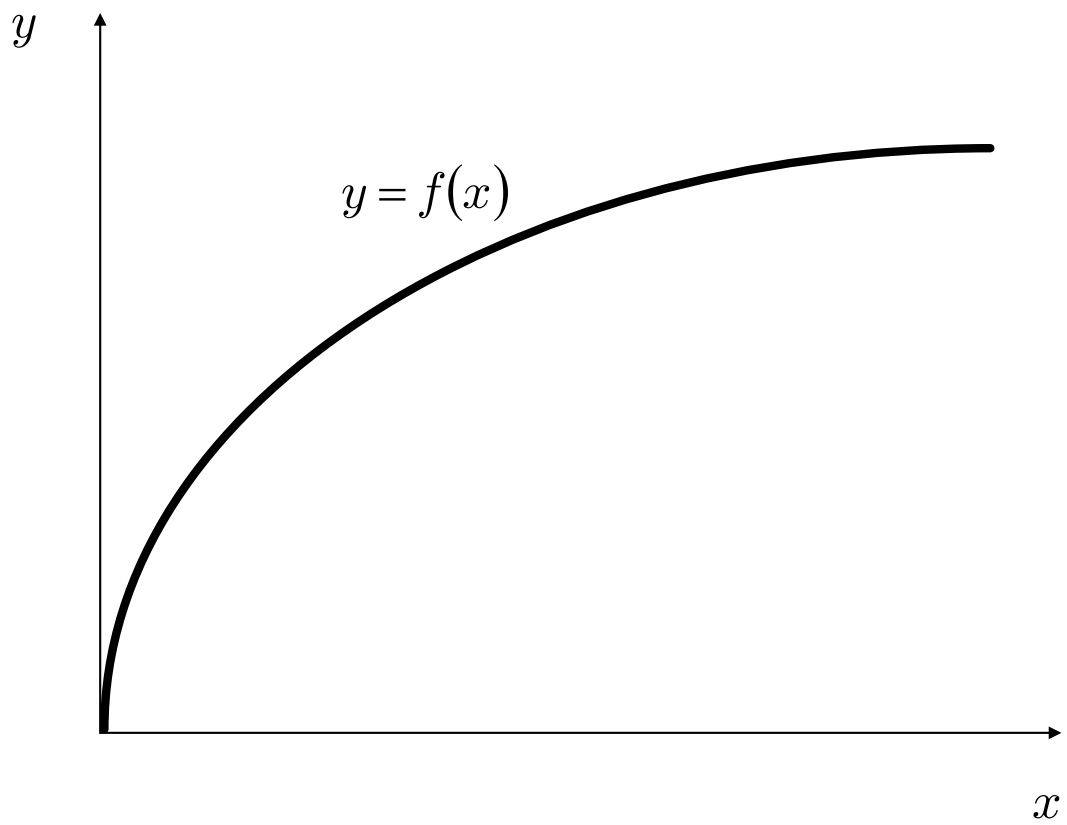
$$y = f\left(x_1, x_2\right).$$

- y : produced quantity
- x_1, x_2 : quantities of the (two) production factors

Problem

What is the difference between ordinal and cardinal utility theory? Is production theory ordinal or cardinal?

Production function



Production function

Axioms

Problem

Can the completeness axiom be transferred to production theory?

Problem

Is transitivity satisfied in production theory?

- Monotonicity is satisfied if throwing away production factors is for free.
- Convexity can also be transferred to production theory.

Partial factor variation

Terms

- Total factor variation: All factors are varied.
- Partial factor variation: Only one factor is varied.

- Marginal productivity (MP):

$$MP_1 = \frac{\partial y}{\partial x_1}$$

  Analogy in utility theory?
- Average productivity (AP):

$$AP_1 = \frac{y}{x_1}$$

Problem

1000 workers produce 5000 cars in one month.
Average productivity? Unit of measurement?

Partial factor variation

Problem

How should one define the production elasticity of a factor?

Problem

Express the production elasticity as a function of average productivity and marginal productivity!

Problem

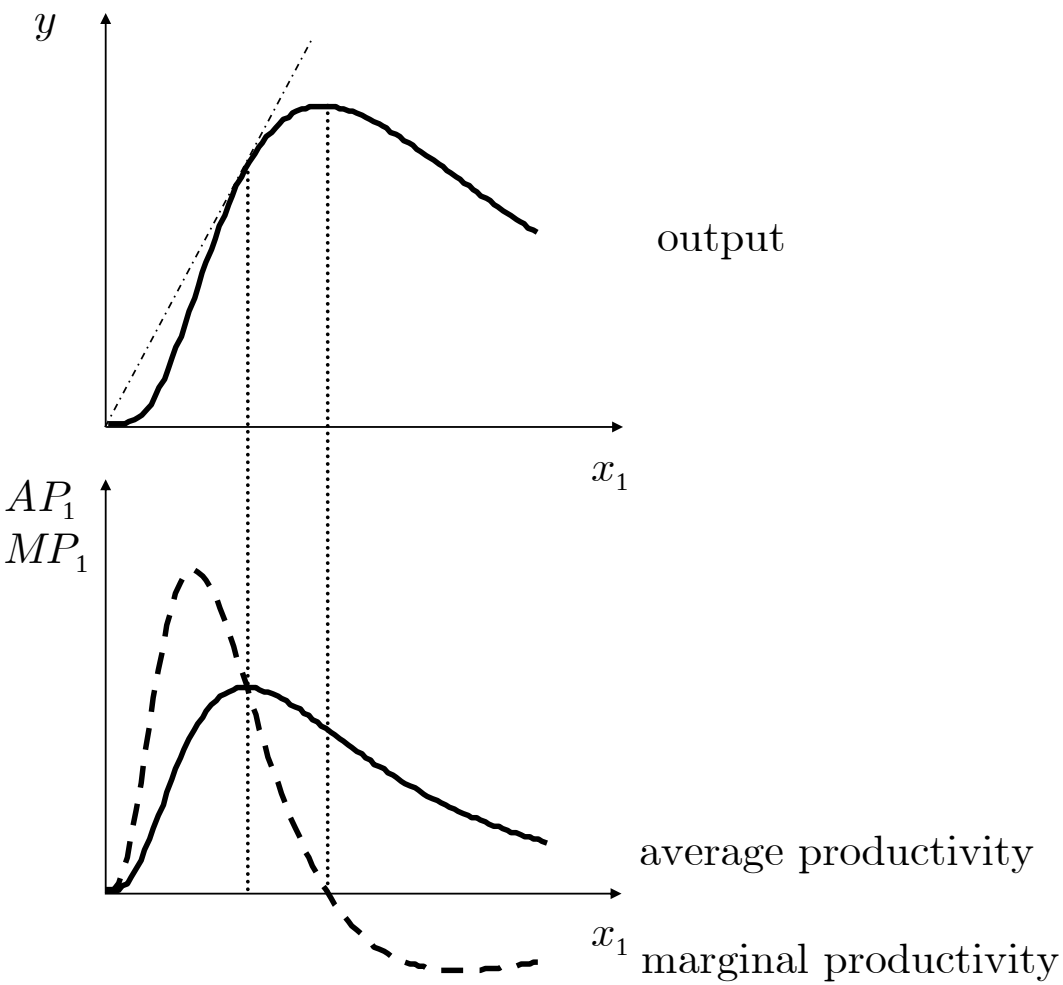
Production elasticity of the first factor for $y = cx_1^a x_2^b$ with $a, b, c > 0$?

Problem

Under which conditions does average productivity increase?

Partial factor variation

Sato production function



Problem

Where do you see:

- marginal product max.
- average product max.
- marginal product $>$ average product
- marginal product $=$ average product
- marginal product 0

(SH 45f)

Partial factor variation

Diminishing returns

The marginal product of any production factor increases, remains constant and then decreases (can be negative).

Example

Sato production function

$$y = f(x_1, x_2) = \frac{x_1^a x_2^b}{(x_1 + x_2)^{a+b-1}},$$

where $a, b > 1$

Proportional factor variation

Returns to scale

Definition (constant returns to scale)

$$f(tx_1, tx_2) = tf(x_1, x_2) \qquad (t > 1)$$

Definition (increasing returns to scale)

$$f(tx_1, tx_2) > tf(x_1, x_2) \qquad (t > 1)$$

Definition (decreasing returns to scale)

$$f(tx_1, tx_2) < tf(x_1, x_2) \qquad (t > 1)$$

Problem

Returns to scale for $f(x_1, x_2) = 2x_1 + x_2$ or $f(x_1, x_2) = x_1 x_2$?

Proportional factor variation

Scale elasticity

Definition (scale elasticity)

$$\varepsilon_{y,t} = \left.\frac{\frac{df(tx_1,tx_2)}{f(tx_1,tx_2)}}{\frac{dt}{t}}\right|_{t=1} = \left.\frac{df\left(tx_1,\, tx_2\right)}{dt}\,\frac{t}{f\left(tx_1,\, tx_2\right)}\right|_{t=1}$$

Problem

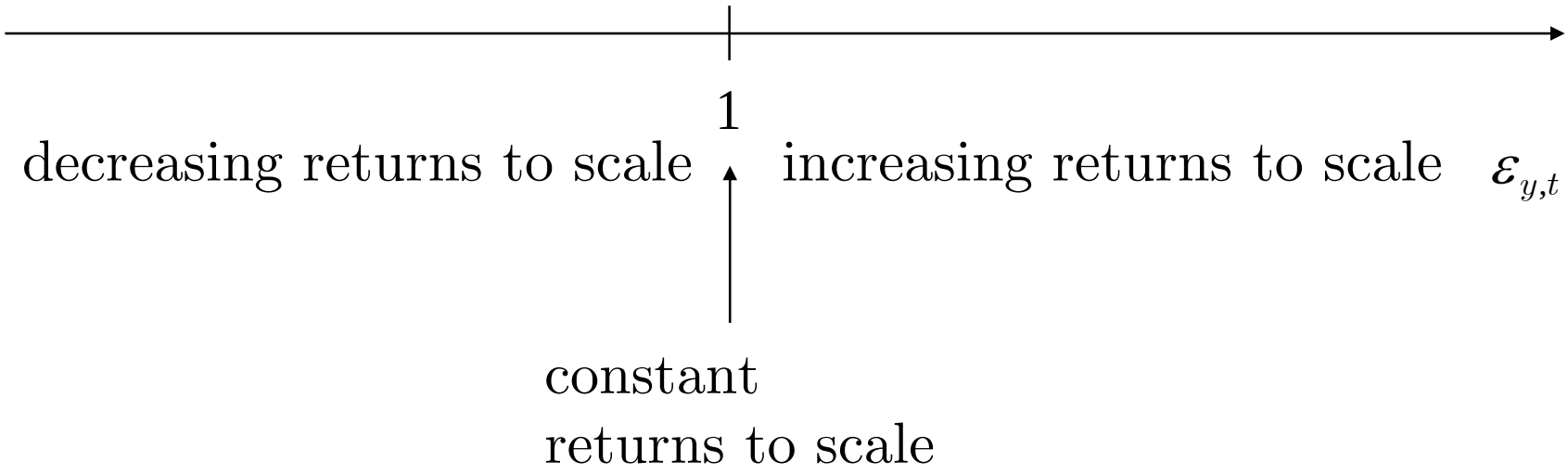
Scale elasticity for a Cobb-Douglas production function $y = x_1^a x_2^b$?

Problem

For a Cobb-Douglas production function the scale elasticity equals the sum of ...?

Proportional factor variation

Returns to scale and scale elasticity



Proportional factor variation

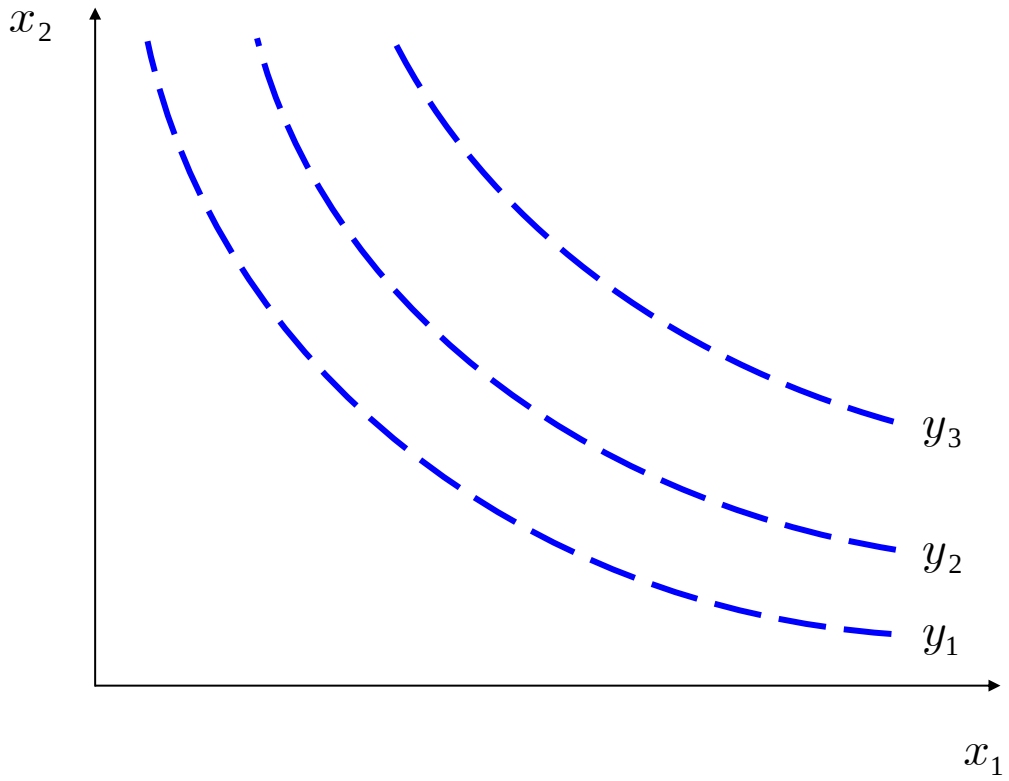
Homogeneity

- A production function is called homogeneous of degree ν if

$$f\left(tx_1, tx_2\right) = t^{\nu} f\left(x_1, x_2\right).$$

- Homogeneous production functions with $\nu = 1$ are called linearly homogeneous. (= constant returns to scale)
- Scale elasticity for homogeneous production functions $= \nu$.

Isoquants



Problem

Illustrate increasing returns to scale!
Illustrate technological progress! (K 295)

Marginal rate of technical substitution

- is the absolute value of the slope of an isoquant.
- states how many units of factor 2 can be waived if one additional unit of factor 1 is used and if output is held constant.

Problem

Household theory:

$$MRS = \frac{MU_1}{MU_2}$$

Hence, production theory

$$MRTS = \text{——}$$

Production functions without factor substitution

correspond to perfect complements in household theory

Problem

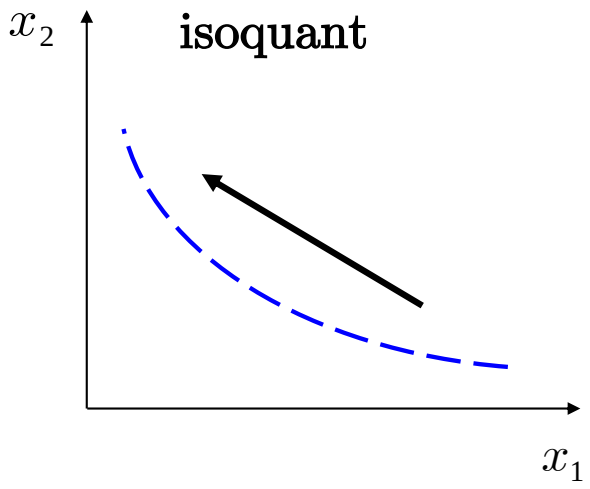
Bartender Harley needs

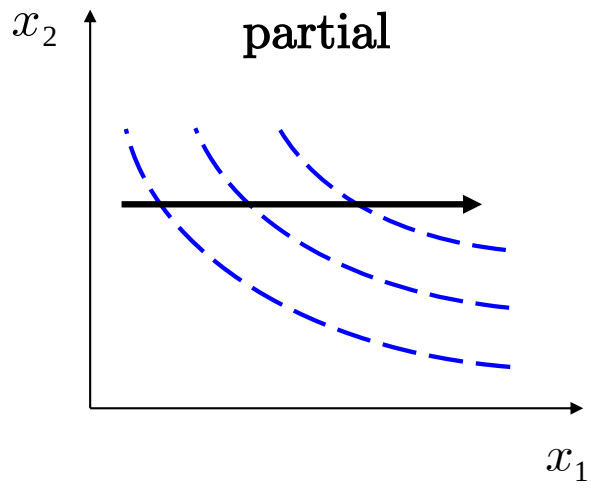
- 2 deciliter rum (x_1) and
- 6 deciliter cola (x_2)

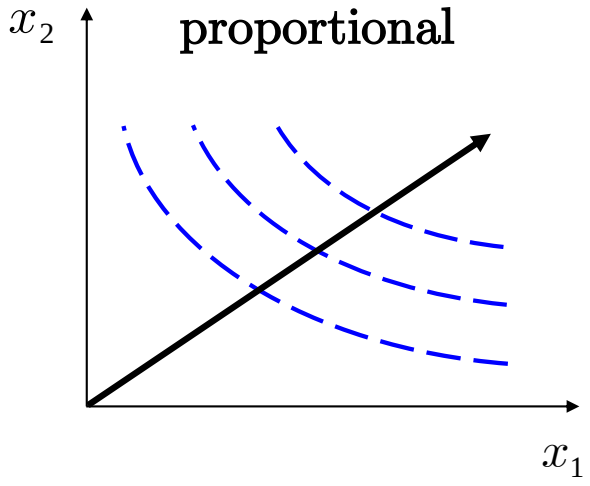
for a big cola with rum (y)

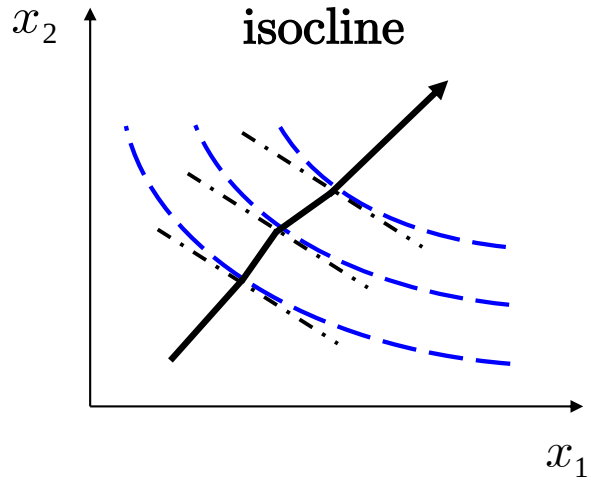
1. Isoquants for 2 big colas with rum?
2. Production function?

Overview factor variations









Central tutorial I

Problem I.7.1.

Constant returns to scale?

a) $y = f(K, L) = K^{\frac{1}{2}} L^{\frac{2}{3}}$

b) $y = f(K, L) = 3K^{\frac{1}{2}} L^{\frac{1}{2}}$

c) $y = f(K, L) = K^{\frac{1}{2}} + L^{\frac{1}{4}}$

d) $y = f(K, L) = 2K + 3L$

Problem I.7.2.

Production function $f(x_1, x_2) = (2x_1 + 4x_2)^{\frac{1}{2}}$

Marginal rate of technical substitution?

Central tutorial II

Problem I.7.3.
Cobb-Douglas production function $y = f(x_1, x_2) = Ax_1^a x_2^b$ with $A, a, b > 0$

a) Marginal product of factor 1?
b) Production elasticity for factor 1?
c) Scale elasticity?
d) MRTS?
e) Parameter values for
   - constant,
   - decreasing, or
   - increasing returns to scale?