Microeconomics

Market demand and revenue

Harald Wiese

Leipzig University

Structure

Introduction

- Household theory
  - Budget
  - Preferences, indifference curves, and utility functions
  - Household optimum
  - Comparative statics
  - Decisions on labor supply and saving
  - Uncertainty
  - **Market demand and revenue**
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

Overview

- Aggregation of individual demand curves
- Demand curves
    - Linear demand curves
    - Price elasticity of demand
    - Revenue and marginal revenue with respect to price
- Inverse demand function
    - From demand curve to inverse demand curve
    - Linear inverse demand curve
    - Again: price elasticity of demand
    - Marginal revenue
- Average and marginal values (excursus)
- Demand for murder, fast driving, theft (excursus)

Prohibitive price and satiation quantity

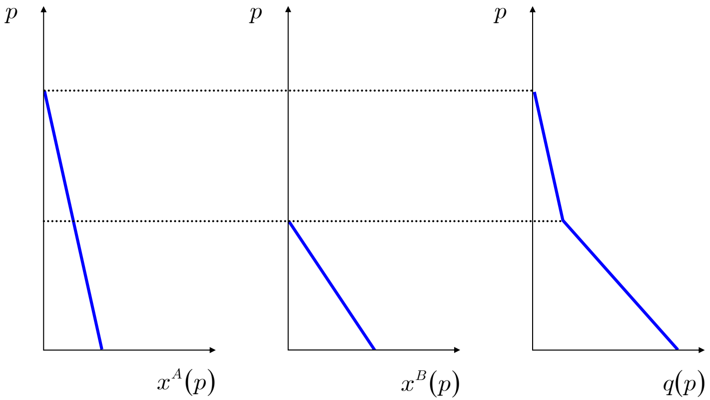
Definition (prohibitive price)

Price for which demand is just zero

Definition (satiation quantity)

Quantity demanded at price zero

Aggregation of individual demand curves



- Note prohibitive prices!
- Horizontal aggregation!

The linear model

Demand curve

demand function

$$X(p) = d - ep$$

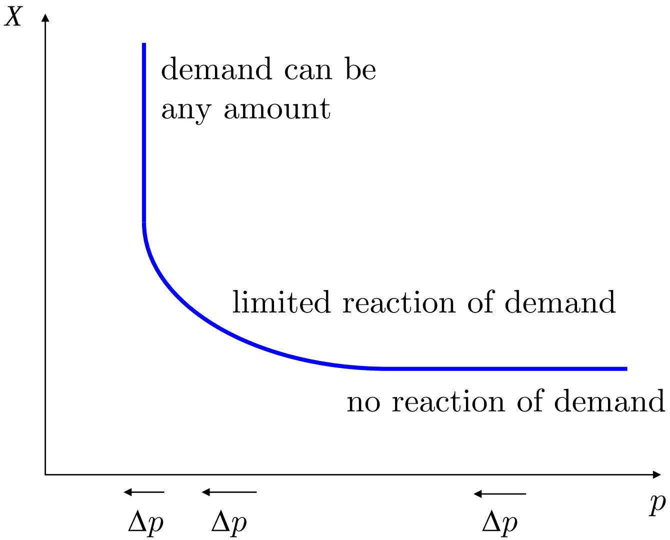
$d, e \geq 0, p \leq \frac{d}{e}$

Problem

Determine

- satiation quantity (demand at price zero) and
- prohibitive price (price that yields zero demand)

Demand function and price elasticity I



Demand function and price elasticity II

Definition (price elasticity)

$$\varepsilon_{X,p} = \frac{\frac{dX}{X}}{\frac{dp}{p}} = \frac{dX}{dp}\frac{p}{X}.$$

By how many percent does demand change if the price increases by 1 percent?

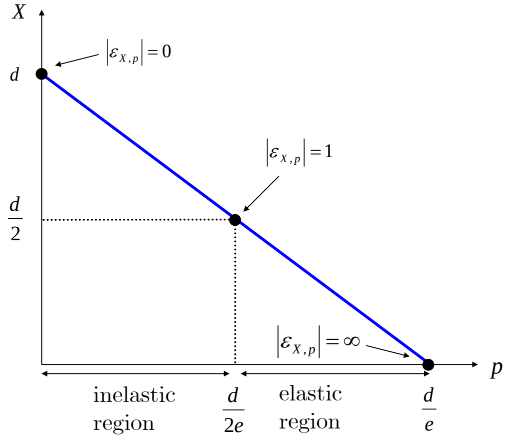
- Inelastic demand
$$\left|\varepsilon_{X,p}\right| < 1$$
- Elastic demand
$$\left|\varepsilon_{X,p}\right| > 1$$

Demand function and price elasticity III

$$X(p) = d - ep$$

$$\begin{aligned} \varepsilon_{X,p} &= \frac{dX}{dp}\frac{p}{X} \\ &= (-e)\frac{p}{d-ep} \end{aligned}$$

- $|\varepsilon_{X,p}| = 0$ for $p = 0$
- $|\varepsilon_{X,p}| = \infty$ for $d - ep = 0$, hence, for $p = \frac{d}{e}$
- $|\varepsilon_{X,p}| = 1$ yields $e\frac{p}{d-ep} = 1$, $ep = d - ep$ and therefore $p = \frac{d}{2e}$



Expenditures and revenue

Price times quantity

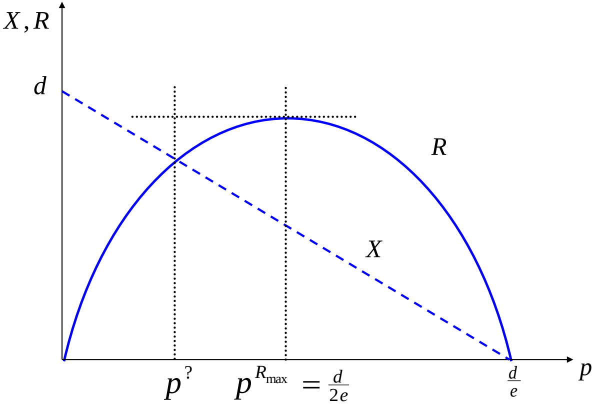
- from the household's perspective: expenditures
- from the firm's perspective: revenue

- Revenue for demand function $X(p)$:

$$R(p) = pX(p)$$

- Revenue equals 0
  - at the prohibitive price (why?) and
  - at the satiation quantity (why?).

Revenue curve and a question I

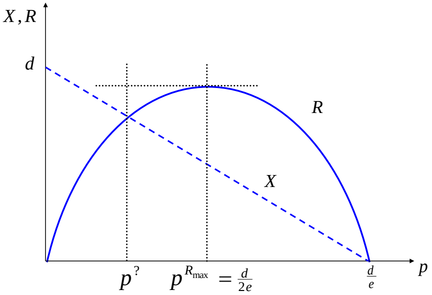


Problem

What is the economic interpretation of the price $p^?$?

Revenue curve and a question II

N.......!



Units:

- Prices:

$$\frac{\text{money units}}{\text{quantity units}}$$

- Revenue $=$ price $\times$ quantity:

$$\begin{aligned} & \frac{\text{money units}}{\text{quantity units}} \cdot \text{quantity units} \\ = \; & \text{quantity units} \end{aligned}$$

Marginal revenue with respect to price

- Revenue for demand function $X(p)$:

$$R(p) = pX(p)$$

- Marginal revenue ($= MR$, here MR_p):

$$MR_p = \frac{dR}{dp} = X + p\frac{dX}{dp} \text{ (product rule)}$$

- If the price increases by one unit,
  - revenue increases by X (for every unit sold the firms obtain one Euro)
  - revenue decreases by $p\frac{dX}{dp}$ (the increase in price decreases demand that is valued with the price)

Marginal revenue and price elasticity

Problem

Confirm the Amoroso-Robinson relation

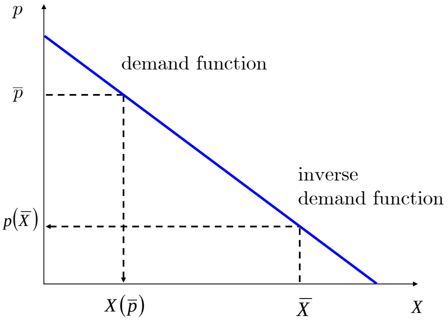
$$\frac{dR}{dp} = X\left(1+\varepsilon_{X,p}\right) = -X\left(\left|\varepsilon_{X,p}\right|-1\right)!$$

Problem

What is the price elasticity of demand if revenue reaches its maximum?

Inverse demand function

From demand function to inverse demand function



- demand function $X(p)$:
  Quantity depends on price.
- Inverse demand function $p(X)$:
  $p(X)$ is the price where quantity X can be sold.

Inverse demand function

Problem

Determine the inverse demand function for $X(p) = 100 - 2p$.

Problem

Confirm that average revenue is equal to the price (revenue equals $R(X) = p(X)X$).

Problem

Do you recognize $p(0)$ and $X(0)$?

Linear inverse demand function

A problem

Problem

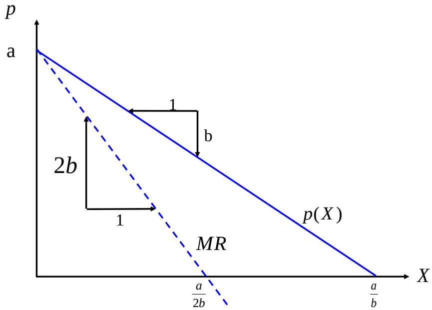
Assume the linear inverse demand function $p(X) = a - bX$, $a, b > 0$, and determine

1. the slope of the inverse demand function
2. the slope of marginal revenue $dR(X)/dX$
3. the satiation quantity and
4. the prohibitive price

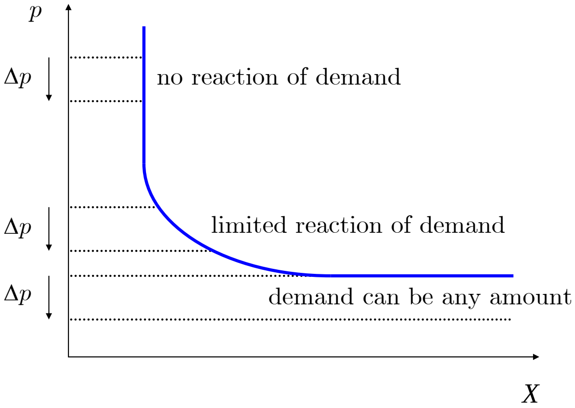
The linear model

The solution

1. $dp/dX = -b$
2. Revenue: $R(X)$
   $= p(X)X = aX - bX^2$
   marginal revenue:
   $dR(X)/dX$
   $= a - 2bX.$
   slope: $-2b$
3. Satiation quantity: a/b
4. a is the prohibitive price



Again: price elasticity of demand



$$\varepsilon_{X,p} = \frac{\frac{dX}{X}}{\frac{dp}{p}} = \frac{dX}{dp}\frac{p}{X}$$

Again: price elasticity of demand

Problem

Calculate price elasticity of demand for the linear demand function $p(X) = a - bX$! Which price and which quantity yields an elasticity of -1? Which price yields an elasticity of zero?

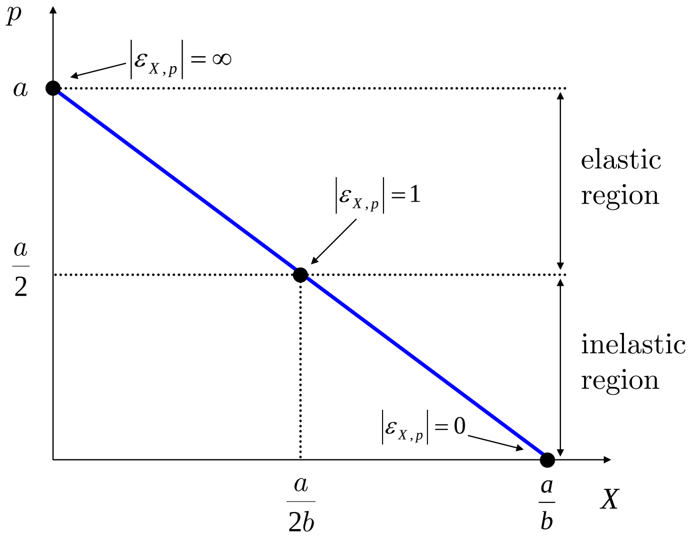
- Inelastic demand

$$\left|\varepsilon_{X,p}\right| < 1$$

- Elastic demand

$$\left|\varepsilon_{X,p}\right| > 1$$

Again: price elasticity of demand



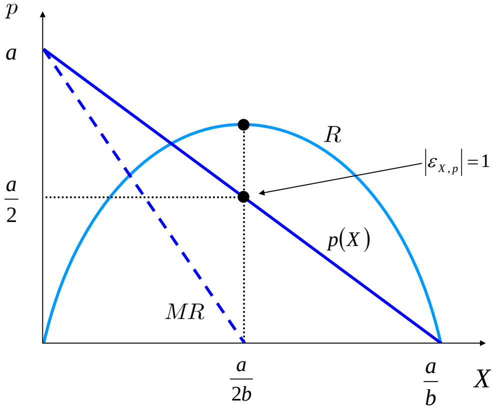
for an inverse demand function:

$$MR = p\left(1 + \frac{1}{\varepsilon_{X,p}}\right) = p\left(1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right).$$

Problem

Derive the Amoroso-Robinson relation above, this time by factoring out p!

Demand function and Revenue



If the absolute value of the elasticity equals 1, then an increase of the quantity by one percent yields a reduction of the achievable price by one percent. Revenue does not change in this case.

Marginal revenue I

$MR := \frac{dR}{dX}$ consists of two parts

- At the one hand, if the quantity increases by one unit, revenue increases by the price of this unit ($p > 0$).
- At the other hand, revenue decreases because consumers are not willing to buy the increased demand at the old price (for a negatively sloped market demand).
  Loss in revenue = product of
  - price discount due to the increase of quantity $\frac{dp}{dX}$ and
  - number of units X sold so far

  Hence: Marginal revenue equals

$$MR = p + X\frac{dp}{dX}.$$

Marginal revenue II

- Marginal revenue and elasticity (Amoroso-Robinson relation)

$$\begin{aligned} MR &= \frac{dR}{dX} = p + X\frac{dp}{dX} \text{ (product rule)} \\ &= p\left[1 + \frac{1}{\varepsilon_{X,p}}\right] = p\left[1 - \frac{1}{\left|\varepsilon_{X,p}\right|}\right] > 0 \text{ for } \left|\varepsilon_{X,p}\right| > 1 \end{aligned}$$

- Marginal revenue equals price $MR = p + X \cdot \frac{dp}{dX} = p$ for
  - $\frac{dp}{dX} = 0$ horizontal (inverse) demand: $MR = p + X \cdot \underbrace{\frac{dp}{dX}}_{=0} = p$
  - first "small" unit, $X = 0$: $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX} = p = \frac{R(X)}{X}$
  - first-degree price discrimination: $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX}$

    $\rightarrow$ see chapter "Monopoly and monopsony"

Average and marginal values (excursus)

Reminder: For the first "small" unit:

$$X = 0 : MR = p + \underset{=0}{X} \cdot \frac{dp}{dX} = p = \frac{R(X)}{X} = AR$$

- We look for condition such that the following holds:

$$\frac{df}{dx} = \frac{f(x)}{x}.$$

- The conditions are

$$\begin{array}{ll} \text{1. condition:} & x > 0 \text{ and } \frac{d\frac{f(x)}{x}}{dx} = 0, \\ \text{2. condition:} & x = 0 \text{ and } f(0) = 0. \end{array}$$

Average and marginal values (excursus)

The proof of the first condition follows from

$$\begin{aligned}\frac{d\frac{f(x)}{x}}{dx} &= \frac{\frac{df}{dx}x - 1\cdot f(x)}{x^2} \\ &= \frac{1}{x}\left(\frac{\frac{df}{dx}x}{x} - \frac{f(x)}{x}\right) \\ &= \frac{1}{x}\left(\frac{df}{dx} - \frac{f(x)}{x}\right).\end{aligned}$$

By $\frac{d\frac{f(x)}{x}}{dx} = 0$ and $x \neq 0$ we get the equality of the first derivative (e.g., marginal revenue) and the average (e.e., average revenue).

Average and marginal values (excursus)

For the proof of the second condition we can use de l'Hospital's rule

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{\frac{df}{dx}}{\frac{dg}{dx}}.$$

Problem

Calculate $\lim_{x \to 0} \frac{f(x)}{g(x)}$ for $f(x) = e^x - 1$ and $g(x) = \sqrt{x}$!

In our case we have $g(x) = x$ and hence obtain

$$\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} \frac{\frac{df}{dx}}{1} = \left.\frac{df}{dx}\right|_{x=0}.$$

$\Longrightarrow$ The average at zero is equal to the derivative at zero.

Demand for murder, fast driving, theft

Fine as price for

- parking in no-parking zones
- overrunning of velocity

Prison (in years) and death penalty (probability of sentence) for murder
Empirical analysis: Demand for murder is negatively sloped according to Isaac Ehrlich:

- USA 1935-1969: an additional execution would have prevented 8 deaths

Probability of severe injury due to fast driving
is reduced by an airbag; increase in the demand for fast driving
Empirical analysis: total effect (number of traffic fatalities) is close to zero.

Central tutorial I

Problem H.7.1.
Demand function $q(p) = a - bp$
Show
$$\varepsilon_{q,p} = -\frac{p}{\text{prohibitive price} - p}.$$

Problem H.7.2.
Inverse demand function $p(q) = 30 - 3q$

- Marginal revenue?
- Draw demand function and marginal revenue!

Central tutorial II

Problem H.7.3.

Inverse demand function $p(q) = 200 - 8q$
Number of consumers doubles;
For every consumer a "twin" appears

- New demand function?
- Price elasticity at $p = 3$?
- Marginal revenue according to the Amoroso-Robinson relation?

Central tutorial III

Problem H.7.4.
Inverse demand functions

$$p\left(x^A\right) = 5 - \frac{1}{2}x^A \text{ und } p\left(x^B\right) = 3 - \frac{1}{3}x^B$$

Draw and aggregate (graphically)!
Then aggregate the demand functions analytically (not the inverse demand functions)!

Problem H.7.5.
Price elasticity of demand for

a) $q(p) = 40p^{-2}$

b) $q(p) = (p+3)^{-2}$