Microeconomics

Comparative statics

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Structure

Introduction

- Household theory
  - Budget
  - Preferences, indifference curves, and utility functions
  - Household optimum
  - **Comparative statics**
  - Decisions on labor supply and saving
  - Uncertainty
  - Market demand and revenue
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

- Parameters:
  describe the economic situation (input of economic models), e.g., preferences of a household
- Variables:
  are the result of economic models (after application of the equilibrium concept), e.g., profit-maximizing prices

- Comparative:
  Comparison of equilibria resulting from alternative parameter values
- Static:
  Adaption processes are not analyzed

Introduction

Comparative Statics in household theory

- The demand for good 1
  - for money budget:

$$x_1^G = x_1^G(p_1, p_2, m),$$

  - for endowment budget:

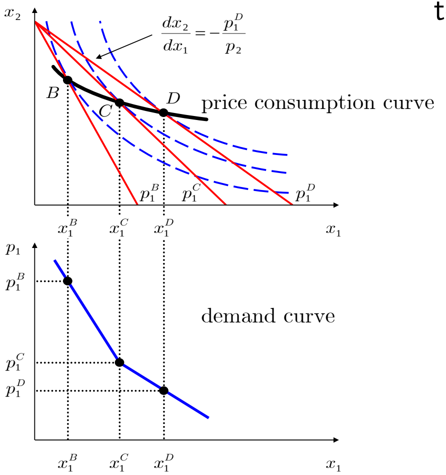
$$x_1^A = x_1^A(p_1, p_2, \omega_1, \omega_2).$$

- How does demand for good 1 change due to a change in
  - prices p_1, p_2,
  - income m,
  - endowment ω_1, ω_2.

Comparative statics

- Introduction
- Impact of the own price
  - price consumption curve and demand curve for money budget
  - price consumption curve and demand curve for endowment budget
  - price elasticity of demand
- Impact of the other good's price
- Impact of income
- Slutsky equations

Impact of the own price



Assign an optimum $\left(x_1^*(p_1), x_2^*(p_1)\right)$ to every p_1!

- Price consumption curve:
  Locus of these household optima expressed as a function $x_2 = h(x_1)$!
- Demand curve:
  Locus of $\left(x_1^*(p_1), p_1\right)$ expressed as a function $x_1^* = f(p_1)$!

Impact of the own price

Cobb-Douglas utility function

$U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ with the household optimum

$$x_1^* = \frac{1}{3}\frac{m}{p_1}, \;\; x_2^* = \frac{2}{3}\frac{m}{p_2}$$

Demand curve for good 1:

$$x_1^* = f(p_1) = \frac{1}{3}\frac{m}{p_1}$$

Price consumption curve:

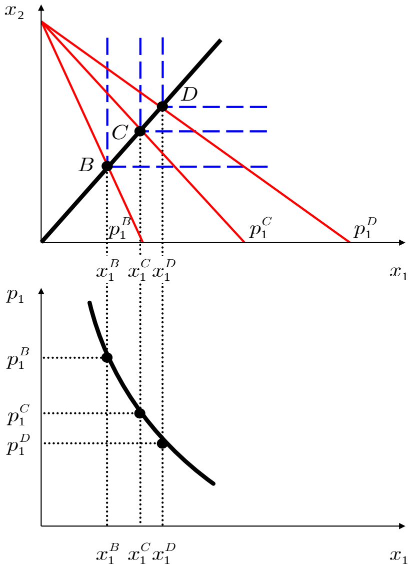
$$x_2 = h(x_1) = \frac{2}{3}\frac{m}{p_2}$$

Problem

What about perfect complements?

Impact of the own price

Perfect complements



Impact of the own price

Demand curves for money budget

Definition (ordinary goods)

$$\frac{\partial x_1^G}{\partial p_1} < 0 \text{ or } \frac{\partial x_1^A}{\partial p_1} < 0$$

Definition (non-ordinary goods)

$$\frac{\partial x_1^G}{\partial p_1} > 0 \text{ or } \frac{\partial x_1^A}{\partial p_1} > 0$$

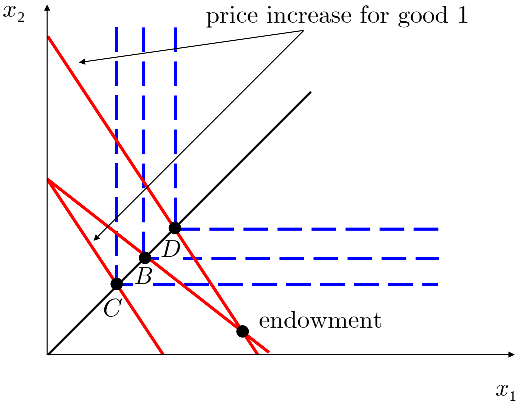
Giffen goods if the budget is a money budget

Problem

Sketch a price consumption curve for a Giffen good!

Impact of the own price

Price consumption curve and demand curve for endowment budget



Initial situation: budget line through B.

Is good 1 ordinary

- for money budget?
- for endowment budget?

Impact of the own price

Price consumption curve and demand curve for endowment budget

For a Cobb-Douglas utility function with parameter a, the demand for good 1 can be obtained as follows

$$
\begin{aligned}
x_1^* &= a\frac{m}{p_1} \\
&= \frac{p_1\omega_1 + p_2\omega_2}{p_1} \\
&= \omega_1 + \frac{p_2}{p_1}\omega_2
\end{aligned}
$$

Good 1 is ordinary

- for money budget and also
- for endowment budget.

Impact of the own price

Price consumption curve and demand curve for endowment budget

$$\frac{\partial x_1^G}{\partial p_1} \neq \frac{\partial x_1^A}{\partial p_1}?$$

Take the derivative of

$$x_1^A(p_1, p_2, \omega_1, \omega_2) = x_1^G(p_1, p_2, p_1\omega_1 + p_2\omega_2).$$

with respect to p_1:

$$\frac{\partial x_1^A}{\partial p_1} = \frac{\partial x_1^G}{\partial p_1} + \frac{\partial x_1^G}{\partial m} \cdot \underbrace{\frac{\partial\left(p_1\omega_1 + p_2\omega_2\right)}{\partial p_1}}_{\substack{p_1 \uparrow \text{ by one unit} \\ \text{increases value of endowment}}} = \frac{\partial x_1^G}{\partial p_1} + \frac{\partial x_1^G}{\partial m}\omega_1.$$

Endowment income effect: $\frac{\partial x_1^G}{\partial m}\omega_1$

Impact of the own price

Price elasticity of demand

Definition

$$\text{elasticity} = \frac{\text{relative change of effect [\%]}}{\text{relative change of cause [\%]}}$$

Elasticities for demand

- cause: change in
  - price of the same good
  - price of the other good
  - income
- effect: change in demand

Impact of the own price

Price elasticity of demand

$$\varepsilon_{x_1,p_1} = \frac{\frac{dx_1}{x_1}}{\frac{dp_1}{p_1}} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1},$$

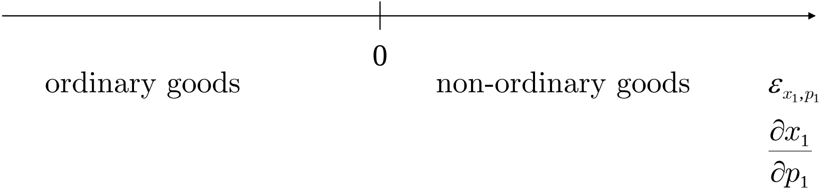
- for Cobb-Douglas utility function $x_1^* = f(p_1) = \frac{1}{3}mp_1^{-1}$

$$\begin{aligned}\varepsilon_{x_1,p_1} &= (-1)\frac{1}{3}mp_1^{-2}\frac{p_1}{x_1} \\ &= (-1)\frac{1}{3}mp_1^{-2}\frac{p_1}{\frac{1}{3}mp_1^{-1}} \\ &= -1.\end{aligned}$$

- and for $U(x_1, x_2) = \frac{1}{3}\ln x_1 + \frac{1}{2}\ln x_2$?

Impact of the own price

Price elasticity of demand



For ordinary goods:

$$\varepsilon_{x_1,p_1} > -1 \Leftrightarrow \left|\varepsilon_{x_1,p_1}\right| < 1$$

Definition

Demand is called inelastic if $\left|\varepsilon_{x_1,p_1}\right| < 1$ holds.

Acquisitive crime

$\left|\varepsilon_{x,p}\right| < 1 \Rightarrow$ expenditures increase with respect to price:

$$\begin{aligned}
\frac{d\left(px\left(p\right)\right)}{dp} &= x + p\frac{dx}{dp} \\
&= x\left(1 + \frac{p}{x}\frac{dx}{dp}\right) \\
&= x\left(1 + \varepsilon_{x,p}\right) \\
&= x\left(1 - \left|\varepsilon_{x,p}\right|\right) > 0.
\end{aligned}$$

- Drug addicts have inelastic demand.
- Higher taxes or criminalization
  - increase prices and hence
  - increase expenditures for drugs leading to

a potential increase in acquisitive crime.

Impact of the other good's price

If the price of butter increases, many consumers buy margarine instead. Hence,

$$\frac{\partial x_{\text{margarine}}}{\partial p_{\text{butter}}} > 0$$

Examples

- Substitutes:
  - butter and margarine
  - car and bike
- Complements:
  - cinema and popcorn
  - left shoe and right shoe

Impact of the other good's price

Problem

Sketch a household diagram with

- x-axis: heroin
- y-axis: methadone

The price of heroin increases and hence ... (K 69)

Problem

Sketch demand curves with

- x-axis: number of visits to cinema
- y-axis: price of visits to cinema

Price of popcorn increases and hence ... (K 70)
Price of theater increases and hence ... (K 71)

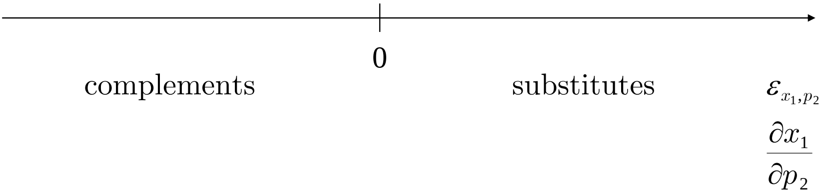
Impact of the other good's price

Cross price elasticity of demand is given by

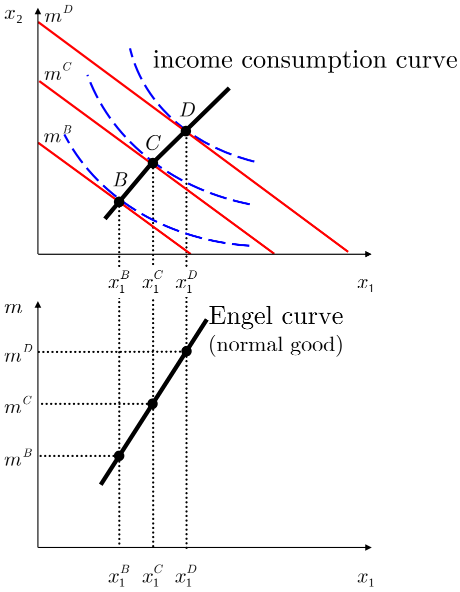
$$\varepsilon_{x_1,p_2} = \frac{\frac{dx_1}{x_1}}{\frac{dp_2}{p_2}} = \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1}$$

and for a Cobb-Douglas utility function we have $x_1^* = \frac{1}{3}\frac{m}{p_1}p_2^0$ and hence

$$\varepsilon_{x_1,p_2} = \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1} = 0 \cdot \frac{p_2}{x_1} = 0$$



Impact of income



Impact of income

Cobb-Douglas utility function

$U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ with household optimum

$$x_1^* = \frac{1}{3}\frac{m}{p_1},\ x_2^* = \frac{2}{3}\frac{m}{p_2}$$

Engel curve for good 1:

$$x_1^* = q\,(m) = \frac{1}{3}\frac{m}{p_1}$$

Income consumption curve:

$$x_2^* = \frac{2}{3}\frac{m}{p_2} = \frac{2}{3}\frac{3p_1x_1^*}{p_2} = 2\frac{p_1}{p_2}x_1^* = g\,(x_1^*)$$

Problem

And for $U\,(x_1, x_2) = \min\,(x_1, 2x_2)$?

Impact of income

Definition (normal goods)

$$\frac{\partial x_1}{\partial m} > 0$$

Definition (inferior goods)

$$\frac{\partial x_1}{\partial m} < 0$$

Problem

Sketch the Engel curve for an inferior good!

Problem

(K 73)

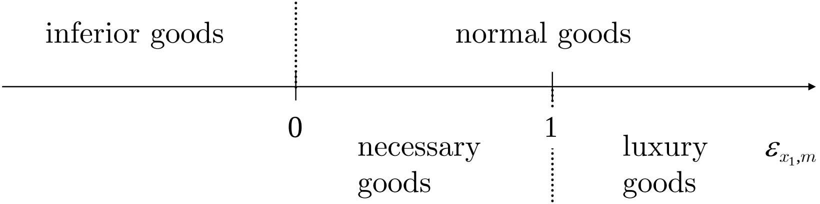
Impact of income

Income elasticity is given by

$$\varepsilon_{x_1,m} = \frac{\frac{dx_1}{x_1}}{\frac{dm}{m}} = \frac{\partial x_1}{\partial m}\frac{m}{x_1}$$

and for a Cobb-Douglas utility function $x_1^* = \frac{1}{3p_1}m$ and hence

$$\varepsilon_{x_1,m} = \frac{\partial x_1}{\partial m}\frac{m}{x_1} = \frac{1}{3p_1}\frac{m}{\frac{1}{3p_1}m} = 1$$



Impact of income

Monotonic preferences $\Rightarrow$ all income is spend on consumption of goods.

- single good $\Rightarrow x(p, m) = \frac{m}{p}$,

$$\varepsilon_{x,m} = \frac{\partial x}{\partial m}\frac{m}{x} = \frac{1}{p}\frac{m}{\frac{m}{p}} = 1$$

- several goods $\Rightarrow s_1 := \frac{p_1 x_1}{m}$ and $s_2 := \frac{p_2 x_2}{m}$:

$$s_1 \varepsilon_{x_1,m} + s_2 \varepsilon_{x_2,m} = 1.$$

(proof in the book)

Impact of income

Good 1 is called **normal** if:

$$\frac{\partial x_1^A}{\partial \omega_1} > 0,\ \ \frac{\partial x_1^A}{\partial \omega_2} > 0 \text{ oder } \frac{\partial x_1^G}{\partial m} > 0.$$

Normality is equivalent for money budget and endowment budget.

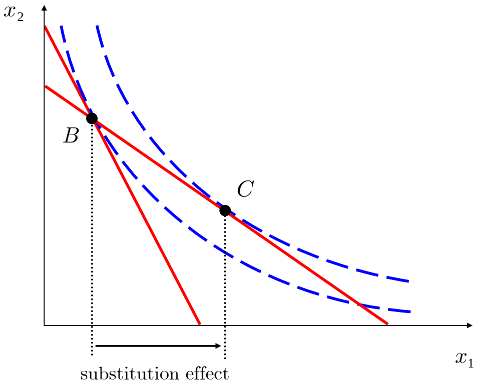
Slutsky equations

An intuitive explanation for the three effects

1. **Substitution effect** or opportunity-cost effect: $p_1 \uparrow$
   - $\Rightarrow p_1 / p_2 \uparrow$
   - $\Rightarrow x_1 \downarrow$ and $x_2 \uparrow$
2. **Consumption income effect** (monetary e.): $p_1 \uparrow$
   - $\Rightarrow$ overall consumption possibilities decrease
   - $\Rightarrow x_1 \downarrow$ if 1 is a normal good
3. **Endowment income effect**: $p_1 \uparrow$
   - $\Rightarrow$ value of endowment increases
   - $\Rightarrow x_1 \uparrow$ if 1 is a normal good

Slutsky equation

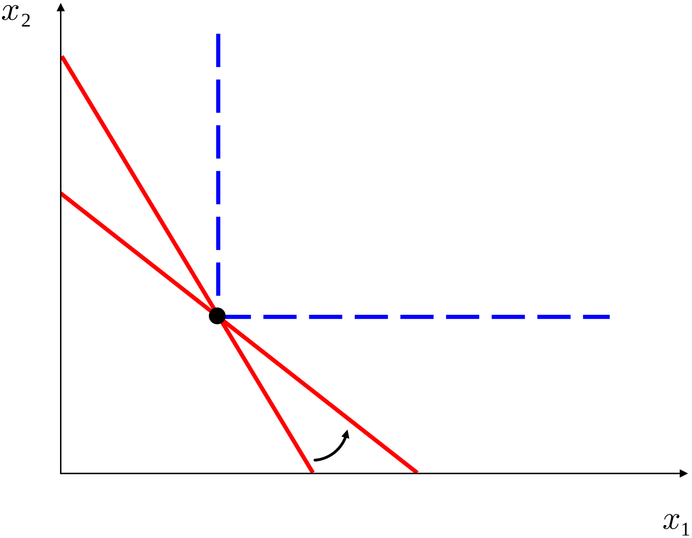
Substitution effect



- absolute substitution effect in the figure
- relative substitution effect $\frac{\Delta x_1^S}{\Delta p_1}$ or $\frac{dx_1^S}{dp_1}$

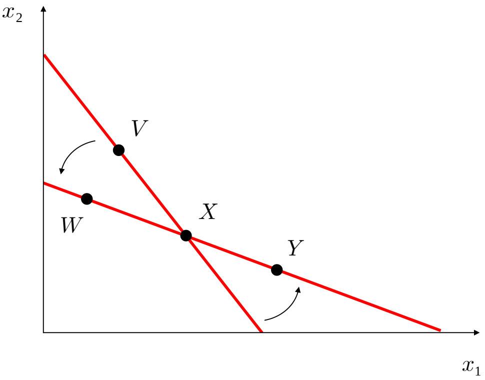
Slutsky equation

Absolute substitution effect for perfect complements



Slutsky equation

Relative substitution effect is never positive



- Starting point
  old household optimum X
  and $p_1 \downarrow$
- Impossible:
  - W optimal or
  - $\frac{\Delta x_1^S}{\Delta p_1} > 0$
- because of
  - W affordable before
- for strict monotonicity
  $X \succsim V \succ W$

Slutsky equation

Income effects

- Endowment income effect:

$$\frac{\partial x_1^G}{\partial m} \cdot \underbrace{\frac{\partial p\left(p_1\omega_1 + p_2\omega_2\right)}{\partial p_1}}_{\substack{p_1 \uparrow \text{ by one unit} \\ \text{increases value of endowment}}} = \frac{\partial x_1^G}{\partial m}\omega_1$$

- Consumption income effect:

$$\frac{\partial x_1^G}{\partial m} \underbrace{\left(-x_1\right)}_{\substack{p_1 \uparrow \text{ by one unit} \\ \text{increases expenditures} \\ \text{and thereby decreases} \\ \text{disposable income}}}$$

Slutsky equation for money budget

$$\underbrace{\frac{\partial x_1^G}{\partial p_1}}_{\text{overall effect}} = \underbrace{\frac{\partial x_1^S}{\partial p_1}}_{\text{substitution effect}} \underbrace{- \frac{\partial x_1^G}{\partial m} x_1^B}_{\substack{\text{consumption} \\ \text{income effect}}}.$$

Problem

- If the Engel curve of a good is increasing, then the demand curve needs to be decreasing.
- For money budget, is every ordinary good normal?

Slutsky equation for money budget

Cobb-Douglas utility function I

$$U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \text{ with } x_1^G = \frac{1}{3}\frac{m}{p_1}$$

- Overall effect
$$\frac{\partial x_1^G}{\partial p_1} = -\frac{1}{3} m p_1^{-2},$$
- Substitution effect

$$\begin{aligned}
\frac{\partial x_1^S}{\partial p_1} &= \frac{\partial x_1^G\left(p_1, p_1 x_1^B + p_2 x_2^B\right)}{\partial p_1} = \frac{\partial\left(\frac{1}{3}\frac{p_1 x_1^B + p_2 x_2^B}{p_1}\right)}{\partial p_1} \text{ (replace } m\text{)} \\
&= \frac{1}{3}\frac{x_1^B p_1 - 1 \cdot \left(p_1 x_1^B + p_2 x_2^B\right)}{p_1^2} \text{ (quotient rule)} \\
&= \frac{1}{3}\frac{x_1^B}{p_1} - \frac{1}{3}\frac{p_1 x_1^B + p_2 x_2^B}{p_1^2}
\end{aligned}$$

Slutsky equation for money budget

Cobb-Douglas utility function II

- Income effect

$$\frac{\partial x_1^G}{\partial m} x_1^B = \frac{1}{3} \frac{1}{p_1} x_1^B.$$

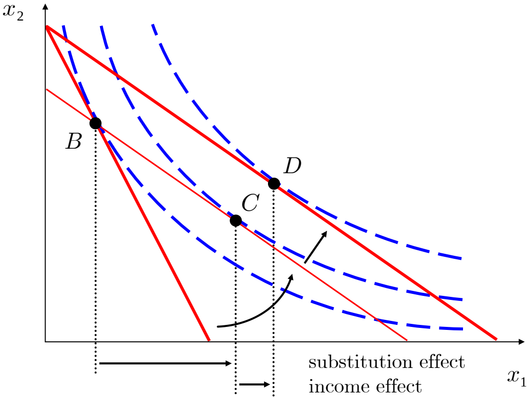
Luckily, we can confirm the Slutsky equation:

$$\begin{aligned}
\frac{\partial x_1^S}{\partial p_1} - \frac{\partial x_1^G}{\partial m} x_1^B &= \left(\frac{1}{3} \frac{x_1^B}{p_1} - \frac{1}{3} \frac{p_1 x_1^B + p_2 x_2^B}{p_1^2} \right) - \frac{1}{3} \frac{1}{p_1} x_1^B \\
&= -\frac{1}{3} \frac{p_1 x_1^B + p_2 x_2^B}{p_1^2} \\
&= \frac{\partial x_1^G}{\partial p_1} \left(\text{for } m = p_1 x_1^B + p_2 x_2^B \right).
\end{aligned}$$

Slutsky equation for money budget

<table>
<tr><td colspan="3">variation in income</td></tr>
<tr><td colspan="2">inferior good</td><td>normal good</td></tr>
<tr><td colspan="2">$\frac{\partial x_1^G}{\partial m} < 0$</td><td>$\frac{\partial x_1^G}{\partial m} > 0$</td></tr>
<tr><td>$x_1 \left| \frac{\partial x_1^G}{\partial m} \right| > \left| \frac{\partial x_1^S}{\partial p_1} \right|$</td><td>$x_1 \left| \frac{\partial x_1^G}{\partial m} \right| < \left| \frac{\partial x_1^S}{\partial p_1} \right|$</td><td></td></tr>
<tr><td>$\frac{\partial x_1^G}{\partial p_1} > 0$</td><td colspan="2">$\frac{\partial x_1^G}{\partial p_1} < 0$</td></tr>
<tr><td>non-ordinary good</td><td colspan="2">ordinary good</td></tr>
<tr><td colspan="3">variation in price</td></tr>
</table>

Slutsky equation for money budget
Substitution and income effect for a price decrease



And for

- a Giffen good?
- perfect complements?
- perfect substitutes?

Deductibility of donations

- x : donations
- y : other expenses

- without taxes: $x + y = m$
- with income tax: $x + y = m - tm = (1 - t)\, m$
- with income tax and deductibility:

$$\begin{aligned} x + y &= (1 - t)\, m + tx = m - t\,(m - x) \text{ oder} \\ (1 - t)\, x + y &= (1 - t)\, m \end{aligned}$$

- absolute slope of the budget line:

$$1 = \frac{1}{1} \to \frac{1 - t}{1} = 1 - t < 1$$

Deductibility of donations

- Slutsky equation:

$$\underbrace{\frac{\partial x}{\partial p_x}}_{\text{overall effect}} = \underbrace{\frac{\partial x^S}{\partial p_x}}_{\text{substitution effect}} \underbrace{-\frac{\partial x}{\partial m}x}_{\substack{\text{consumption} \\ \text{income effect}}}$$

- Since
  - winners of the lottery donate more (plausible assumption),
  - donation is a normal good,

  the deductibility (reduction of the price for x) leads to a higher donation amount.
- Empirically confirmed (K 81)

(Problem K 132)

Slutsky equation for endowment budget

$$\underbrace{\frac{\partial x_1^A}{\partial p_1}}_{\text{overall effect}} = \underbrace{\frac{\partial x_1^S}{\partial p_1}}_{\text{substitution effect}} \underbrace{-\frac{\partial x_1^G}{\partial m} x_1}_{\substack{\text{consumption} \\ \text{income effect}}} \underbrace{+\frac{\partial x_1^G}{\partial m} \omega_1}_{\substack{\text{endowment} \\ \text{income effect}}}$$

$$= \underbrace{\frac{\partial x_1^S}{\partial p_1}}_{\text{substitution effect}} + \underbrace{\frac{\partial x_1^G}{\partial m} (\omega_1 - x_1)}_{\substack{\text{overall} \\ \text{income effect}}}$$

Slutsky equation for endowment budget

	net demand $\omega_1 - x_1 < 0$	net supply $\omega_1 - x_1 > 0$
good 1 is normal	$\frac{\partial x_1^G}{\partial m}(\omega_1 - x_1) < 0$ *unambiguous overall effect*	$\frac{\partial x_1^G}{\partial m}(\omega_1 - x_1) > 0$ *ambiguous overall effect*
good 1 is inferior	$\frac{\partial x_1^G}{\partial m}(\omega_1 - x_1) > 0$ *ambiguous overall effect*	$\frac{\partial x_1^G}{\partial m}(\omega_1 - x_1) < 0$ *unambiguous overall effect*

Central tutorial I

Problem E.7.1.
$U(x_1, x_2) = \ln x_1 + x_2$
m, p_1 and p_2.
Engel curve for the first good!
Hint: case distinction at $\frac{m}{p_2} = 1$*!*

Problem E.7.2.
A household consumes two goods.

a) Assume that good 1 is a luxury good. Show that the share of income that is spend for this good is increasing with respect to income!
b) Is it possible that good 1 and good 2 are both luxury goods? *Hint: Use* $s_1 \varepsilon_{x_1,m} + s_2 \varepsilon_{x_2,m} = 1$, *where* s_1 *and* s_2 *are the shares of expenditure for good 1 and 2, respectively (hence,* $s_1 + s_2 = 1$ *holds).*

Central tutorial II

Problem E.7.3.
$U(x_1, x_2) = x_1$
m, p_1 and p_2.
income consumption curve!

Problem E.7.4.
A consumer grows tomatoes. He harvests fewer tomatoes than he consumes. Examine whether he consumes more or fewer tomatoes after an increase in the price for tomatoes! Use the Slutsky equation for endowment budget! Assume that tomatoes are inferior.

Central tutorial III

Problem E.7.5.

$U(x_1, x_2) = 2x_1 + x_2$

$m = 12$, $p_2 = 2$ and

a) $p_1 = 3$; $p_1 = 4$ and $p_1 = 5$
household optima

b) household optima depending on p_1

c) Draw the price consumption curve in a $x_1 - x_2 -$diagram